Applied mathematics in computer science and technology

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Main purpose of the course

• To **improve** the mathematical theory of **self-cultivation**
  – Intuitively understand the mathematical thinking in the papers

• To **train skilled ability to express** mathematical concepts
  – How to formulate mathematical problems
  – How to abstract mathematical methods

• To **cultivate outstanding ability** to use mathematical models
  – The use of mathematical models to solve computer-related research issues
Lear to think in mathematical way

MAIN PURPOSE OF THE COURSE
The final examination

• Question type
  – Concept (30%)
  – Calculation and proofs (30%)
  – Math modeling (30%)
  – Survey (10%)
孔子曰：智者乐山，仁者乐水
The final examination

- **Innovative Score System**

<table>
<thead>
<tr>
<th>Type</th>
<th>Questions</th>
<th>Required</th>
<th>Discount of over-answered</th>
<th>Budget</th>
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<tbody>
<tr>
<td>Concept</td>
<td>6</td>
<td>3</td>
<td>50%</td>
<td>30</td>
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<tr>
<td>Computing</td>
<td>6</td>
<td>3</td>
<td>50%</td>
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<td>Modeling</td>
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<td>Survey</td>
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The final examination

- Example of ISS: normal

<table>
<thead>
<tr>
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<th>Correct</th>
<th>Gain</th>
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<tr>
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<tr>
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<tr>
<td>Survey</td>
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The final examination

• Example of ISS: love computing

<table>
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<th>Correct</th>
<th>Gain</th>
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<td>Survey</td>
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The final examination

- Example of ISS: love concept

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<td>5=(3+2)</td>
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<tr>
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<tr>
<td>Survey</td>
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<td>1</td>
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<tr>
<td>Total</td>
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Final Review

道可道，非常道
名可名，非常名

《道德经》开篇语
Similar course at top universities

• Princeton:
  – Mathematical methods in Computer Science
  – 讲授图论、拓扑初步、线性规划、矩阵论、统计初步等

• Cambridge:
  – Mathematical methods for Computer Science
  – http://www.cl.cam.ac.uk/teaching/0809/CST/node38.html
  – 讲授傅立叶方法、小波分析、不等式与极限理论、Markov链等统计理论

• 均以计算机图形学、计算机视觉、图像处理、人工智能、人机交互等计算机科学中的问题为背景进行讲解
Our course

• Fundamentals of 4 math topics:
  – Statistical learning
  – Variational methods
  – Partial differential equations
  – Optimization methods
Concepts in Statistical learning

• What is machine learning?

• The categories of learning methods
  – Supervised learning
  – Unsupervised learning

• Fundamental statistical concept
  – Prior, likelihood, Posterior
  – Markov chain
Computing methods in learning

- **Point estimations**
  - Bayesian formula
  - Binary distribution, Gaussian distribution

- **Clustering**
  - K-means, MOG, spectral clustering

- **Time variance data**
  - Hidden Markov Chain
Data modeling

• Geometric description:
  – Dimensional reduction
  – Kernel methods

• Algebra description:
  – Classification v.s. regression
  – How to overcome over-fitting?
Concepts in variational methods

• variational problems:
  – 两点间的最短连线问题
  – 最速降线(brachistochrone)问题
  – 测地线(geodesic line)问题
变分法中的符号

- **给定函数** $y(x)$
  - 宗量：$x$
  - 函数：$y(x)$
  - 宗量的增量：$\Delta x$
  - 函数的增量：
    - $\Delta y = y(x+\Delta x) - y(x)$
  - 当两点无限接近：
    - $\Delta x \to dx, \Delta y \to dy$
  - 略去高阶微量：
    - $dy = y'(x)dx$
  - 当在$x$处取得函数极值
    - $dy = 0$

- **给定泛函** $\Pi(y)$
  - 宗量：$y$
  - 泛函：$\Pi(y)$
  - 函数的变分：$\delta y$
  - 泛函的变分：
    - $\delta \Pi = \Pi(y+\delta y) - \Pi(y)$
  - 在计算$\delta \Pi$时可以展开$\Pi(y+\delta y)$中的被积函数只保留线性项
  - 当在$y$处取得泛函极值
    - $\delta \Pi = 0$

函数$y(x)$在定义域内与$y(x)+\delta y(x)$处处无限接近
Partial differential equation

• Different types of PDE:
  – Can you distinguish them?
  – Laplacian equation, Poisson equation

• Basic concepts:
  – Curve/surface representation
    • Parametric or implicit definition
  – Tangent, normal, curvature
  – Gradience, Divergence
Computing in PDE

• Laplacian operator
• Discrete operators used in PDE
• How to numerically solve special PDEs

• Applications of PDE
Optimization methods

- Linear methods
- Non-linear methods
How to prepare the final examination

读
读书
读数学书
很认真地读数学书

Read
Read book
Read math book
Read math book seriously
It is just a new beginning

大音希声
大象无形

——摘自老子《道德经》

建议更多了解几何、代数、拓扑方面的数学内容，他们是相通的。
Thank you!

Please keep on learning mathematics in your life.