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# Honeycomb subdivision*

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**Abstract:** A novel class of subdivision schemes based on hexagonal meshes is presented. It is called honeycomb subdivision which enlarges the field of subdivision surfaces. By introducing the concept of central control vertices into the schemes, the honeycomb subdivision has the advantages of flexible coefficients selection, easy control of shape, less complexity of large meshes, and applicability. Its properties are analyzed and appropriate subdivision rules are given to obtain limit surfaces with tangent continuity. It can interpolate the given control vertices under certain conditions. The schemes are applicable for shape modeling in computer animation and industrial prototype design.
**Key words:** surface modeling; subdivision surfaces; honeycomb subdivision; dual operator

Complicated surface modeling and rendering are huge challenges in computer graphics. Recently, the subdivision surfaces, due to its unique simple calculation and arbitrary topological modeling ability, become the one of the research hotspots in the computer graphics realm, and have been widely accepted in the computer animation applications. Generally, subdivision surfaces are obtained as the limit of a recursive split and refinement process applied to initial polygonal mesh. By selecting suitable refinement masks we can achieve special geometric continuity. There are several typical types algorithm in the literature: Catmull-Clark's[1] and Doo-Sabin's[2] uniform B-spline tensor product extended subdivision schemes; Loop's box spline extended subdivision schemes[3]; the famous butterfly algorithm[4] and Kobbelt's interpolatory schemes[5], based on four point algorithm; and recently the $\sqrt{3}$ -subdivision[6]. Doo and Sabin[2], Ball and Storry[7], Halstead and his cooperators[8], and Reif[9] et. al., worked on strict mathematics analysis of the local limit properties of subdivision surfaces, and developed several optimal smooth subdivision rules. In the current researches, various new methods are created and many modern techniques are employed. But from our viewpoint and analysis there are three key problems in subdivision algorithms, and which greatly arouse our interesting.

Firstly, all subdivision schemes are based on quadrangular or triangular mesh till now. Though they are simple and natural, there are still a kind of mesh, hexagonal mesh, in the nature, e.g., honeycomb and snowflake. The hexagon, always act as a reasonable approximation of ellipse and circle, it can be used in texture synthesize and texture mapping, wavelet and finite element analysis. Moreover, notice if the 2D plan is seamless covered without overlap by regular polygons; the only three choices are regular triangle, square and regular hexagon. So the

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importance of hexagonal mesh is self-evident. On the other hand, it is well-known that the dual operator is a key operator in subdivision schemes. The famous Doo-Sabin scheme can be viewed as a combination of bisection and dual operation over original meshes. In a triangular mesh, every regular vertex has the valence 6, and the polygons on the dual mesh are mostly hexagonal. Thus some questions may be naturally putted forward as follows: How to develop reasonable subdivision schemes over hexagonal based mesh? And how about the continuity of the limit surfaces relative to the special subdivision rules? But there are few papers discussed the hexagonal based subdivision surfaces. They are not the simple extension of existing subdivision schemes. In fact, the topological structure of meshes greatly influences the subdivision results. In [10], Zorin pointed out that some triangular meshes may result fairless shapes by employing quad meshed based Catmull-Clark scheme, with large numbers of irregular vertices. The main reason is the regular cases of different subdivision schemes are different. Thus, it is necessary for us to research the special reasonable schemes over hexagonal meshes to reach the requirements for computer animation and industrial modeling.

Secondly, several researchers have gone deep into the mathematics analysis of the continuity and smoothness of the subdivision surface. But the many results of them are generic or limited in the traditional subdivision rules. For instance, Reif[9] gave a theoretical framework of local analysis. However, the new subdivision schemes, distinguished from the classical schemes, do not fit the conditions given by [9]. Moreover, when more coefficients are added, the precise representation of eigein-structure of subdivision matrix is hard to be obtained, which cause the analysis difficulty. But in our hexagonal subdivision schemes, the eigenvalues still can be precisely computed out by applying some transforms and due to the good properties of the circulant matrix. And from this, we can determine the coefficients to reach the tangent continuity for the limit subdivision surface.

Thirdly, the traditional subdivision schemes based on the bisection discrete algorithms of uniform spline have relative fixed coefficient. Thus the result shapes are relative fix. But in computer animation modeling, especially in the biology modeling, the main principle is that lifelikeness models do not have to much fair and just act in their natural way. The original Catmull-Clark algorithm, always create surfaces which seem much artificial and unnatural. This makes users have to do more post-processing for bump effects. So we introduce the central control vertices into our scheme. By the controls of them, combined with the neatly coefficient tuning and the level of details, the artificial problem are well solved.

In this paper we first review some terms and conceptions in the subdivision schemes. Then we give a set of subdivision topological rules over hexagonal meshes and the geometric smooth coefficients based on the continuity and limit properties of the subdivision surface. Finally, we list some discussions of implementation and several examples are shown in section 4. As the faces of the initial mesh used in our algorithms are mostly hexagonal, we call them honeycomb meshes, and relative schemes are called honeycomb subdivision.

## 1 Honeycomb mesh

In graph theory, a polygonal mesh can be viewed as a simple graph $G=G(V,E)$ defined on a none-empty vertex set $V=\{v_i \mid i\in I\}$ and an edge set $E=\{e_{ij},\ldots,e_{rs}\}$, where edge $e_{ij}=\{v_i,v_j\}$. Then the valence of a vertex is the number of vertices connected to it, i.e., $Valence(v_i)=\left|\{v_i \middle| e_{ij}\in E\}\right|$. A polygonal face $F(V,E)$ in $G$ is a sub-graph which only have one connected branch, and the valence of each vertex are 2. The valence $F(V,E)$ of is the number of its vertices, and it is obviously equal to the number of its edges. If the valence of each face in graph $G(V,E)$ is 6, then it is called a *hexagonal mesh*, or a *honeycomb mesh*. Similarly, we say it is a triangular mesh if all faces are valence 3, and a quad mesh if valence 4. For convenience, we imprecisely say it is an *n*-mesh if in which most faces are valence *n*. The set of vertices in the faces which contain the vertex *v* is called the 1-neighborhood of *v*, denoted by $N_1(v)$.

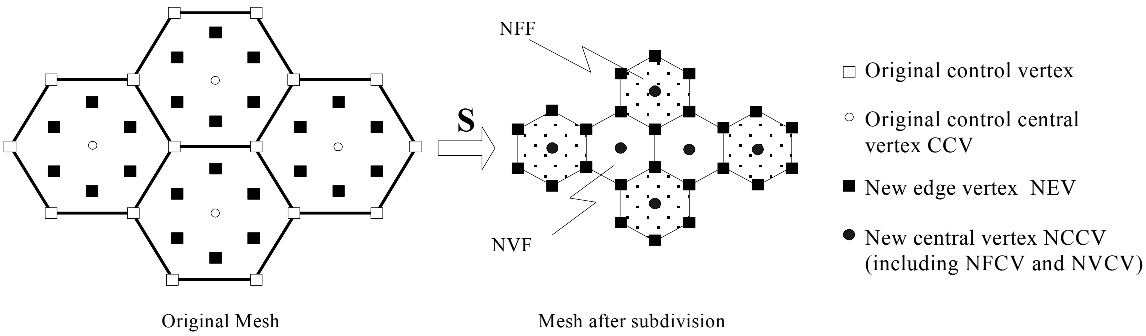



Fig. 1 Topological rules of the honeycomb subdivision

Given a mesh $G = G(V, E)$ , we can construct a new mesh(graph), in which the vertices is corresponding to the original faces, and the edges is defined as the connectivity of the faces. The new mesh is called the dual graph of original mesh. Obviously, each vertex in the original mesh is corresponding to a face in the dual mesh and they have the same valence. It is well-known that to seamlessly cover the $\Re^2$ parameter plane without overlap by regular *n*-polygon, the choice can only be 3, 4 and 6. In the square case, since each vertex is valence 4, the dual is still a quad mesh. In the regular triangle case, the dual of the infinite covering mesh is a honeycomb mesh, and applying the dual operator twice will make it come back to a triangular mesh. This is the base viewpoint of our algorithm. In honeycomb meshes, the valence 6 vertex is called *regular* vertex, otherwise it is an *irregular* vertex.

## 2 Honeycomb subdivision

For simplicity, assume the given arbitrary topological mesh has no boundary and has only one connected branch, for the multi-branch case it can be viewed as a combination of independent subdivision operations on each branch. Distinguished from triangular mesh, the faces of the honeycomb mesh can not guarantee to be coplanar. So we introduce the central control vertex, denoted CCV, into our algorithm to improve the shape control and rendering abilities. In the later sections, we will discuss the importance of CCV.

### 2.1 Topological subdivision rules and their properties

The topological subdivision rules (see Fig. 1) are:

1) In the original mesh, on every face, create a new edge vertex for each edge, denoted by NEV, which is a linear combination of two end point of the edge and the CCV of the face.
2) In the original mesh, on every face, create a new face, denoted by NFF, by connecting the corresponding NEVs of its edges in turn.
3) In the original mesh, for each edge, create a new face, denoted by NVF, which is composed of the NEVs of all faces around it.
4) In the new mesh, for each face, create a new central control vertex, denoted NCCV. We denote NVCCV and NFCCV for the CCV of NVF and NFF respectively.

By applying the topological rules, we note that each face and vertex in the old mesh generates a new face, and the NCCVs(new central control vertices) are produced from the vertices and the CCVs of the old mesh.

For a given closed mesh $M(E,V,F)$ , let $V$ be the vertex set, let $E$ be the edge set and $F$ be the face set. The Euler's formula asserts that

$$V - E + F = C, \tag{1}$$

where $C$ is a constant determined by the topological properties of the given mesh. The table 1 lists the topological parameters changes after applying one step operation of several typical subdivision schemes. From this table, we can find out that our scheme is same as other schemes. That is the topological properties are invariant during the subdivision operation though new geometric elements are added for creating the new mesh. It is a necessary characteristic in practical applications. On the other hand, the increasing speed of the geometric elements in our scheme is slower than others. It makes the user more easily manipulate the processing meshes and improves the ability of detail control.

**Table 1** Subdivision schemes and topologic parameters

| Scheme | Vertex | Edge | Face | Topological constant |
|---|---|---|---|---|
| Honeycomb | $2\|E\|$ | $3\|E\|$ | $\|V\|+\|F\|$ | $C$ |
| Catmull-Clark | $\|V\|+\|E\|+\|F\|$ | $4\|E\|$ | $2\|E\|$ | $C$ |
| Doo-Sabin | $2\|E\|$ | $4\|E\|$ | $\|V\|+\|E\|+\|F\|$ | $C$ |
| Loop/Butterfly | $\|V\|+\|E\|$ | $2\|E\|+3\|F\|$ | $4\|F\|$ | $C$ |

Moreover, after one step of our scheme, all control vertices, definitely been new edge vertices, are valence 3, for it is only connected with the other new edge vertex of the corresponding edge, and the two neighbor new edge vertices of the corresponding same old face. The valence of NVF(new vertex face) is twice of the valence of the corresponding vertex, and the valence of NFF(new face face) is fixed. Hence, after two steps, the most face of mesh are hexagonal, the number of non-hexagonal faces is the sum of the number of irregular vertex and the number of non-hexagonal faces, and it will be fixed in the next subdivision steps.

### 2.2 Geometric refinement rules

To design new schemes, the geometric refinement rules are much important. Popularly, the scheme must has the appropriate support, i.e., the influence area of vertices shall be small. Furthermore, the scheme shall have some kind of symmetry which will take effects to the limit surface of subdivision. Let $P[q;p_1,\ldots,p_n]$ be a polygonal face $P$ with valence $n$ on the original mesh, where $q$ is the central control vertex and $p_1,p_2,\ldots,p_n$ are the vertices of $P$. Applying one step of subdivision, we calculate the new edge vertex NEV of $P$ by

$$p_i' = (1-b_n)q + b_n\frac{p_i+p_{i+1}}{2}; \tag{2}$$

and the new face central vertex NFCV of $P$ is

$$q' = (1-a_n)q + a_n\overline{p}, \tag{3}$$

here the weights $a_n,b_n\in[0,1]$, the $\overline{p}$ is the barycenter of $P$, i.e., $\overline{p}=\sum_{i=1}^{n}p_i/n$ (See Fig. 2). Let $v$ be a vertex with valence $n$ in the original mesh, then the new central control vertex of corresponding NVF is computed by

$$\tilde{q} = (1-c_n)v + c_n\left[\sum_{i=1}^{n}\alpha_i p_i + \sum_{i=1}^{n}\beta_i q_i\right], \tag{4}$$

with $c_n,\alpha_n,\beta_n\in[0,1]$ and $\sum_{i=1}^{n}(\alpha_i+\beta_i)=1$, here the $p_i$ is the $i$th vertex connected to $v$, and $q_i$ is the CCV of $F_i$ who contains $v$ (see Fig. 3).

## 3 Limit properties of honeycomb subdivision schemes

### 3.1 Eigenvalue analysis of subdivision matrix

From the last section, we can see the faces on the mesh shrinking after subdivision, and the old vertices correspond to new created faces. Additionally, the NEVs and NFCCV in an NFF are only determined by the control vertices in the corresponding old face. Thus to investigate the local limit properties, we only consider the influence to a given n-sided face $P$ for our subdivision scheme. Let $q$ be $P$'s CCV, $p_1,p_2,\ldots,p_n$ be its vertices, and we

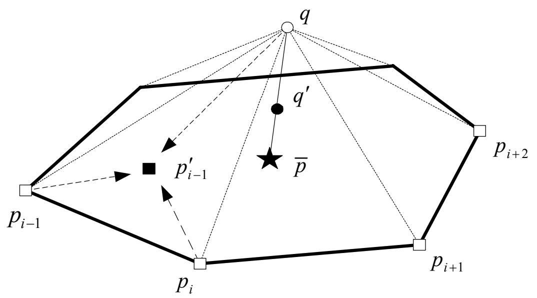



Fig. 2 New edge vertex (NEV) and new face central control vertex (NFCCV)

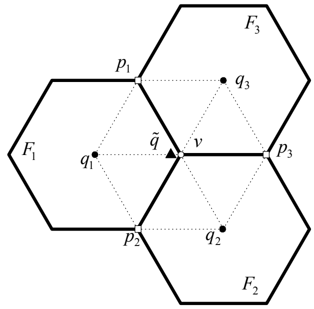



Fig3. New vertex central control vertex (NVCCV)

still denote $P=[q,p_1,p_2,\ldots,p_n]^T$ to be a column vector represented the control vertex series without confusion. Then we can write out following formula

$$P'=SP. \tag{5}$$

where $P'=[q',p_1',p_2',\ldots,p_n']^T$ is the control vertex series of the new face $P'$ and

$$S=\begin{pmatrix} 1-a_n & a_n/n & a_n/n & a_n/n & \cdots & a_n/n \\ 1-b_n & b_n/2 & b_n/2 & 0 & \cdots & 0 \\ 1-b_n & 0 & b_n/2 & b_n/2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 1-b_n & b_n/2 & 0 & 0 & \cdots & b_n/2 \end{pmatrix} \tag{6}$$

is the subdivision matrix. From the results of Reif[9], we only need to analysis the eigenvalues of $S$, because of the local invariance of honeycomb subdivision. For convenience, we introduce the symbol of circulant matrix

$$Cir(a_1,a_2,\ldots,a_n):=\begin{pmatrix} a_1 & a_2 & \cdots & a_n \\ a_n & a_1 & \cdots & a_{n-1} \\ \vdots & \ddots & \ddots & \vdots \\ a_2 & a_3 & \cdots & a_1 \end{pmatrix}. \tag{7}$$

After one step subdivision, the polygonal faces will rotate while shrinking. It naturally forms the edge-vertex correspondence. For easily analysis, same as the $\sqrt{3}$ -subdivision[5], we consider the double step subdivision matrix and adjust its rows to rebuild vertex-vertex correspondence. So we select

$$\tilde{S}=RSS=\begin{pmatrix} d_n & e_n & \cdots & e_n \\ f_n & & & \\ \vdots & & Cir(h_n,g_n,l_n,\ldots,l_n,g_n) & \\ f_n & & & \end{pmatrix}, \tag{8}$$

where

$$\begin{cases} d_n=(1-a_n)^2+a_n(1-b_n), \\ e_n=(1-a_n+b_n)a_n/n, \\ f_n=(1-a_n+b_n)(1-b_n), \\ h_n=a_n(1-b_n)/n+b_n^2/4, \\ g_n=a_n(1-b_n)/n+b_n^2/2, \\ l_n=a_n(1-b_n)/n, \end{cases} \qquad R=\begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 1 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{pmatrix}.$$

By the properties of circulant matrix, it is easy to know that the eigenvalues are

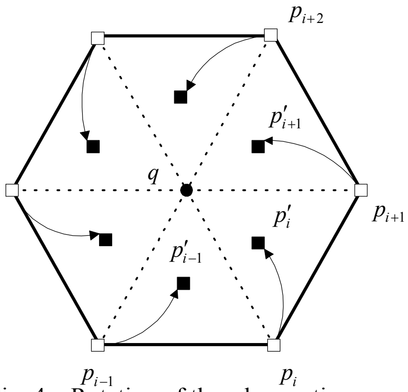


Fig. 4 Rotation of the edge vertices

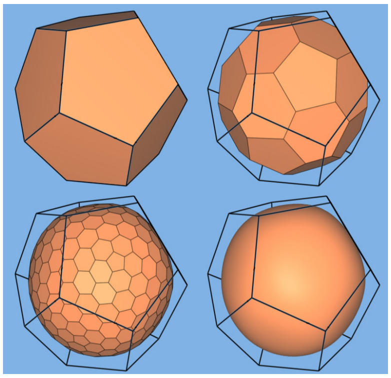


Fig. 5 Applying honeycomb subdivision on dodecagon

$$\left\{1,(a_n-b_n)^2,\frac{1}{2}b_n^2\left[1+\cos(2\pi\frac{1}{n})\right],\frac{1}{2}b_n^2\left[1+\cos(2\pi\frac{2}{n})\right],\ldots,\frac{1}{2}b_n^2\left[1+\cos(2\pi\frac{n-1}{n})\right]\right\}, \tag{9}$$

Imitating the discussions of Reif[9], the limit surface depends on the eigen-structure of $\widetilde{S}$ .To reach the tangent plane continuity, the eigenvalues must satisfy

$$\lambda_0=1>\lambda_1=\lambda_2>\lambda_3\ldots>\lambda_n, \tag{10}$$

Now we analysis the selections of weights $a_n$ and $b_n$ for a given $n$ . Firstly, we always assume that $0\le a_n,b_n\le 1$ to guarantee that all cases are convex linear combinations. Then the largest eigenvalue of $\widetilde{S}$ is 1. Let $\varphi_n=2\pi/n$, and note $\cos(j\varphi_n)=\cos((n-j)\varphi_n)$, so the second largest eigenvalue must be $b_n^2[1+\cos\varphi_n]/2$ . It follows

$$(a_n-b_n)^2<\tfrac{1}{2}b_n[1+\cos\varphi_n], \tag{11}$$

i.e.,

$$\frac{\cos(\varphi_n/2)}{1+\cos(\varphi_n/2)}b_n<a_n<\frac{\cos(\varphi_n/2)}{1-\cos(\varphi_n/2)}b_n. \tag{12}$$

Thus we can pick some special sets of weight coefficients for practice applications:

- To obtain a symmetric mask, we pick $b_n\equiv 2/3$ and the new edge vertex will be the barycenter of the triangle composed by two old edge vertices and the central control vertex. Then pick $a_n=(4-\cos\varphi_n)/9$ to be similar to $\sqrt{3}$ -subdivision. Fig. 5 shows a demo by applying honeycomb subdivision on dodecagon in this set of weights.
- Simply choose $a_n=b_n\equiv const$ .

### 3.2 Limit points of honeycomb subdivision scheme

From last subsection, we find out that the position of the limit point corresponded to the original central vertex is a limit value of the linear combinations of the original face's barycenter and the central vertex. So by the new vertex generation formulas, we have

$$\begin{pmatrix}S(q)\\S(\overline{p})\end{pmatrix}=\begin{pmatrix}1-a_n & a_n\\1-b_n & b_n\end{pmatrix}\begin{pmatrix}p\\\overline{q}\end{pmatrix}, \tag{13}$$

and

$$\begin{pmatrix} S^m(q) \\ S^m(\overline{p}) \end{pmatrix} = \begin{pmatrix} \dfrac{(1-b_n)+a_n(b_n-a_n)^m}{1+a_n-b_n} & \dfrac{a_n(1-(b_n-a_n)^m)}{1+a_n-b_n} \\ \dfrac{(1-b_n)(1-(b_n-a_n)^m)}{1+a_n-b_n} & \dfrac{a_n+(1-b_n)(b_n-a_n)^m}{1+a_n-b_n} \end{pmatrix} \begin{pmatrix} p \\ \overline{q} \end{pmatrix}, \tag{14}$$

As $|a_n - b_n| < 1$, it follows

$$S^\infty = \frac{1-b_n}{1+a_n-b_n} q + \frac{a_n}{1+a_n-b_n} \overline{p}, \tag{15}$$

Thus, the limit point is lying on the line segment of the original face's barycenter and the central vertex. It is beneficial to user interface design.

## 4 Implementation details and discussions

### 4.1 Data structure

In our experimental system, we exploit the extended semi-edge structure as the base storage data structure, which is frequently used in CAD and computer animation system. The semi-edge structure, which stores lots of relationship information of vertices, edges and faces, is efficient in edge-vertex, vertex-face and edge-face search processing. It makes the 1-neighborhood search subroutines much fast in our algorithm implementation. Due to the dual property of the data structure, we can fast create the dual mesh. Furthermore, the data importing and exporting are implemented in our system for data exchanging with other CAD or computer animation systems.

### 4.2 Boundaries

In practical animation and industrial applications, it is usually necessary to be able to process control meshes with well-defined boundary which should result in surfaces with user satisfied boundary curves. But the 1-neighborhood of boundary vertex is incomplete, or in other words, the information is deficient. So we must define special boundary subdivision rules. In our implementation, for a given boundary vertex, if the valence is great than 2, then adding a new edge vertex into the new vertex face, with the formula

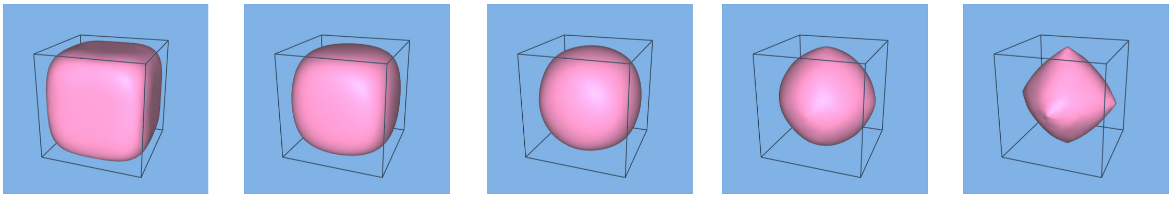
$$v_b = (1-s_n)v + s_n \frac{v_1+v_2}{2}, \tag{15}$$

where the weight $s_n \in [0,1]$ is a function of $n$ and $v_1, v_2$ are two connected boundary vertices with $v$. In practice, it produces satisfied visual results.

### 4.3 Selections of NCV

The central control vertex is vital in our subdivision scheme, since it is a key factor of the limit surface shapes. For the traditional original polygonal meshes have no central control vertex, it gives our scheme additional freedoms to choose central control vertices. If we choose the CCV of a given face on the original mesh to be the barycenter of it, then the limit surface will interpolate the vertex due to the limit point property derived from last section and will result the expectable shapes. Oppositely if we choose the CCV to be far from the barycenter of the given face, then the limit shape will seem much strange and artistic.

The new vertex central control vertex NVCCV generated from subdivision process is also a freely adjustable factor. Especially when the original mesh is relatively simple, it will highly affect the surface shapes. Refer to formula (4), if we select $c_n = 0$, i.e., the NVCCV is superposed with the original vertex, then the final shape will more approximate to the original mesh. And if we select $c_n = 1$, i.e., the NVCCV is the convex linear combination of the vertices and the CCVs in the 1-neighborhood, then the final shape will relatively contract more. Through the



$c_n = 0.00$ $c_n = 0.25$ $c_n = 0.50$ $c_n = 0.75$ $c_n = 1.00$

Fig. 6 Shape changes with the $c_n$ switching from 0 to 1.

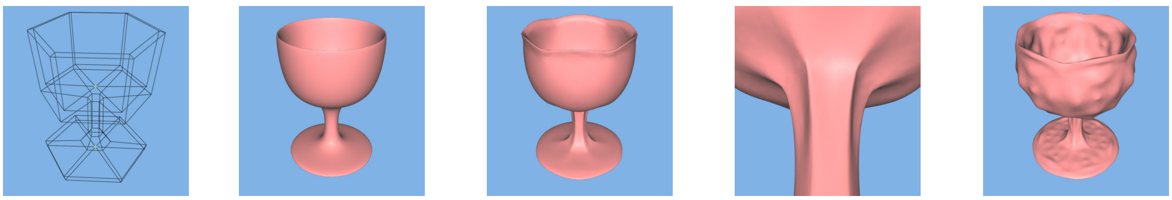
adjustment of $c_n$, user can properly control the fairness and approximation of the limit surfaces. Fig. 6 demonstrates the changes of the limit surface shapes of a unit cube by applying the honeycomb scheme, with the $c_n$ switching from 0 to 1.

## 5 Examples and comparison

Obey the scheme in this paper, we have done several experiments on our system, and it gives good results, see Fig. 7 and Fig. 8. Fig. 7 is a comparison between the honeycomb subdivision and the Catmull-Clark schemes. It is easy to find out that the honeycomb subdivision are susceptive to the local stretch and irregular cases, and it can produce some goffers as shown in Fig. 7(c)(d). Since it gives more lifelikeness results, it is suitable for biology modeling. Moreover, because of the strong control abilities of CCVs, we can append more level of details into the models by perturbing the CCVs and it produces particular dump effect, see Fig. 7(e).

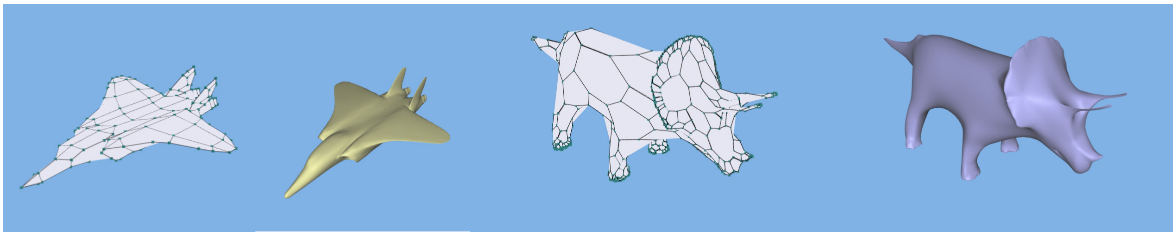
## 6 Conclusion

This paper presents a novel hexagonal mesh based subdivision scheme, named honeycomb subdivision. It has many advantages such as the flexible coefficients selection, the convenience of shape control, the slower increase of the mesh complexity, and the fargoing applicability. Moreover, it can produce the interpolatory effects by choosing special coefficients. Honeycomb subdivision can process on types of mesh very well. It extends the polygonal mesh based complicated surface modeling methods. The scheme is applicable for animation modeling and industrial prototype design. Due to its flexibility, future works should aim at the applications of it. We will further investigate the interpolatory algorithm to interpolate the given point set, the sophisticated boundary constraints and the accelerating rendering algorithms based on subdivision method.



(a) (b) (c) (d) (f)

Fig. 7 The comparison of subdivision schemes. (a) is the original mesh, (b) is the result of Catmull-Clark schemes, (c) is the result of Honeycomb schemes, (d) is a local zoom view of (c), and (e) is the result after effects.



(a) The fighter model (b) The Triceratops

Fig. 8 Honeycomb subdivision examples

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## 蜂窝细分

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**摘要**: 本文给出了一类新颖的基于六边形网格的细分方法．该方法被形象地称为蜂窝细分方法．该方法拓广了细分曲面的种类．通过中心控制点概念的引入，使得蜂窝细分具有参数选取灵活，形状控制容易，网格复杂性增长缓慢，适用范围广等优点．本文分析了蜂窝细分方法的极限性质以及参数选取规则，可保证细分曲面处处达到切平面连续，并在适当条件下具有插值能力．该方法适用于动画造型和工业造型设计．

**关键词**: 曲面造型； 细分曲面； 蜂窝细分； 网格对偶

**中图法分类号**: TP391 **文献标识码**: A