Applied mathematics in computer science and technology

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Main purpose of the course

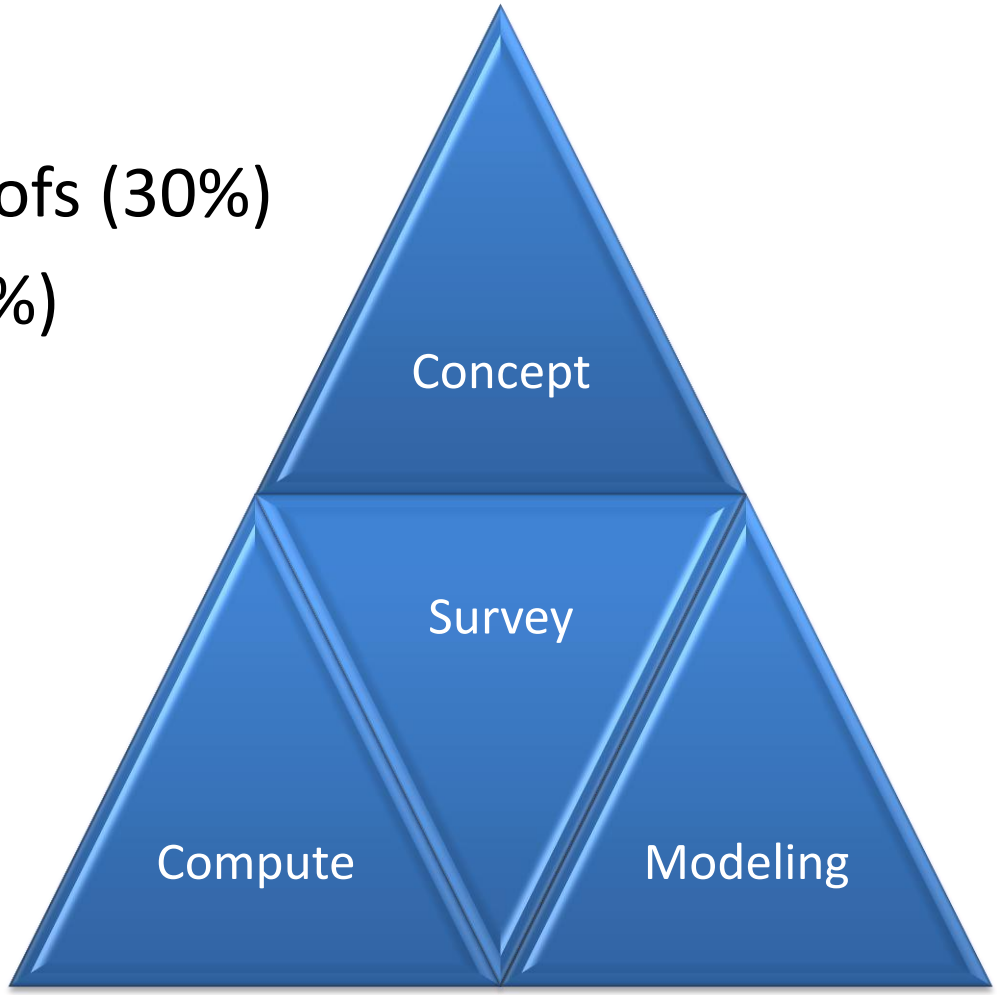
- To improve the mathematical theory of self-cultivation
  - Intuitively understand the mathematical thinking in the papers
- To train skilled ability to express mathematical concepts
  - How to formulate mathematical problems
  - How to abstract mathematical methods
- To cultivate outstanding ability to use mathematical models
  - The use of mathematical models to solve computer-related research issues

Lear to think in mathematical way

MAIN PURPOSE OF THE COURSE

The final examination

- Question type
  - Concept (30%)
  - Calculation and proofs (30%)
  - Math modeling (30%)
  - Survey (10%)



孔子曰：智者乐山，仁者乐水

The final examination

- **I**nnovative **S**core **S**ystem

Type	Questions	Required	Discount of over-answered	Budget
Concept	6	3	50%	30
Computing	6	3	50%	30
Modeling	6	3	50%	30
Survey	3	1	50%	10
Total	21	10	---	100

The final examination

- Example of ISS: normal

Type	Answered	Required	Correct	Gain
Concept	3	3	3	30
Computing	3	3	2	20
Modeling	3	3	3	30
Survey	1	1	1	10
Total	11	10	9	90

The final examination

- Example of ISS: love computing

Type	Answered	Required	Correct	Gain
Concept	2	3	2	20
Computing	4	3	4=3+1	35
Modeling	3	3	3	30
Survey	1	1	1	10
Total	10	10	10	95

The final examination

- Example of ISS: love concept

Type	Answered	Required	Correct	Gain
Concept	6	3	5=(3+2)	40
Computing	1	3	1	10
Modeling	2	3	2	20
Survey	2	1	2~(2-1)	10
Total	11	10	9	80

Final Review

道可道，非常道

名可名，非常名

《道德经》开篇语

Similar course at top universities

- Princeton:
  - Mathematical methods in Computer Science
  - http://www.cs.princeton.edu/~boaz/methods2003/
  - 讲授图论、拓扑初步、线性规划、矩阵论、统计初步等
- Cambridge:
  - Mathematical methods for Computer Science
  - http://www.cl.cam.ac.uk/teaching/0809/CST/node38.html
  - 讲授傅立叶方法、小波分析、不等式与极限理论、Markov链等统计理论
- 均以计算机图形学、计算机视觉、图像处理、人工智能、人机交互等计算机科学中的问题为背景进行讲解

Our course

- Fundamentals of 4 math topics:
  - Statistical learning
  - Variational methods
  - Partial differential equations
  - Optimization methods

Concepts in Statistical learning

- What is machine learning?
- The categories of learning methods
  - Supervised learning
  - Unsupervised learning

- Fundamental statistical concept
  - Prior, likelihood, Posterior
  - Markov chain

Computing methods in learning

- Point estimations
  - Bayesian formula
  - Binary distribution, Gaussian distribution
- Classification
  - Naïve Bayesian classification, decision tree
  - SVM, boosting
- Clustering
  - K-means, MOG, spectral clustering
- Time variance data
  - Hidden Markov Chain
  - Karman filter

Data modeling

- Geometric description:
  - Dimensional reduction
  - Kernel methods
- Algebra description:
  - Classification v.s. regression
  - How to overcome over-fitting?

Concepts in variational methods

- varational problems:
  - 两点间的最短连线问题
  - 最速降线(brachistochrone)问题
  - 测地线(geodesic line)问题

变分法中的符号

- 给定**函数**$y(x)$
  - 宗量：x
  - 函数：$y(x)$
  - 宗量的增量：Δx
  - 函数的增量：
    - $\Delta y = y(x+\Delta x) - y(x)$
  - 当两点无限接近：
    - $\Delta x \to dx$，$\Delta y \to dy$
  - 略去高阶微量：
    - $dy = y'(x)dx$
  - 当在x处取得函数极值
    - $dy = 0$

- 给定**泛函**$\Pi(y)$
  - 宗量：y
  - 泛函：$\Pi(y)$
  - 函数的变分：δy
  - 泛函的变分：
    - $\delta\Pi = \Pi(y+\delta y) - \Pi(y)$
  - 在计算$\delta\Pi$时可以展开$\Pi(y+\delta y)$中的被积函数只保留线性项
  - 当在y处取得泛函极值
    - $\delta\Pi = 0$

函数$y(x)$在定义域内与$y(x)+\delta y(x)$处处无限接近

Partial differential equation

- Different types of PDE:
  - Can you distinguish them?
  - Laplacian equation, Poisson equation

- Basic concepts:
  - Curve/surface representation
    - Parametric or implicit definition
  - Tangent, normal, curvature
  - Gradience, Divergence

Computing in PDE

- Laplacian operator
- Discrete operators used in PDE
- How to numerically solve special PDEs

- Applications of PDE

Optimization methods

- Linear methods
- Non-linear methods

How to prepare the final examination

读	Read
读书	Read book
读数学书	Read math book
很认真地读数学书	Read math book seriously