Concept Learning

Hongxin Zhang
zhx@cad.zju.edu.cn

State Key Lab of CAD&CG, ZJU
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Overview

- Learning from examples
- General-to-specific ordering of hypotheses
- Version spaces and candidate elimination algorithm
- Inductive bias

Introduction

- What is concept learning?
  - Induce Boolean function from a sample of positive/negative training examples.
  - Infer the general definition of some concept, given examples labeled as members or nonmembers of the concept.
- Concept learning in daily life
  - 根据人证物证判断犯罪嫌疑人是否有罪
  - 根据笔试面试决定是否录用
  - And more …

A Demo Task – *Enjoy Sport*

- **Concept:**
  - days on which my friend Tom enjoys his favorite water sports
- **Task:**
  - predict the value of ”Enjoy Sport” for an arbitrary day based on the values of the other attributes

Training Data

Attributes

ID	Sky	Temp	Humidity	Wind	Water	Forecast	Enjoy
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

Example

Representing Hypothesis

- Hypothesis *h* is described as a conjunction of constraints on attributes

- Each constraint can be:
  - A specific value : e.g. Water=Warm
  - A don’t care value : e.g. Water=?
  - No value allowed (null hypothesis): e.g. Water=Ø

- Example: hypothesis *h*

	Sky	Temp	Humid	Wind	Water	Forecast	
<	Sunny	?	?	Strong	?	Same	>

A Demo Task – *Enjoy Sport*

- **Given:**
  - Instance Space ***X***: Possible days described by the attributes=[Sky, Temp, Humidity, Wind, Water, Forecast]
  - Target function ***c***: Enjoy Sport X → {0,1}
  - Hypotheses Space ***H***:
    - conjunction of literals e.g. < Sunny ? ? Strong ? Same >
  - Training examples ***D***:
    - positive and negative examples of the target function, <x1,c(x1)>,… , <xn,c(xn)>
- **Determine:**
  - A hypothesis *h* in ***H*** such that *h*(***x***)=*c*(***x***) for all ***x*** in ***X***.

The Inductive (归纳) Learning Hypothesis

- Any hypothesis found to approximate the target function well over a sufficiently large set of training examples will also approximate the target function well over other unobserved examples.

➢ Find the hypothesis that best fits the training data

- 根据已知推断未知，假定已知满足某种规律

Number of Instances, Concepts, Hypotheses

- Sky: sunny, cloudy, rainy
- Air Temp: warm, cold
- Humidity: normal, high
- Wind: strong, weak
- Water: warm, cold
- Forecast: same, change

#distinct instances : 3*2*2*2*2*2 = 96
#distinct concepts : 2^{96}
#syntactically distinct hypotheses : 5*4*4*4*4*4=5120
#semantically distinct hypotheses : 1+4*3*3*3*3*3=973

Perspective

- Concept learning can be formulated as a **searching:**
  - *through a predefined space of potential hypotheses for the hypothesis that* ***best fits*** *the training examples.*

- *General-to-specific* ordering

$$h_j \geq_g h_k \longleftrightarrow (\forall x \in X)[(h_k(x)=1) \rightarrow (h_j(x)=1)]$$

  - Example : <Sunny,?,?,?,?,?> >= <Sunny,?,?,Strong,?,?>
  - Introduce a **hierarchy** structure into hypotheses space, which leads to efficient searching strategy.

Algorithms

Algorithm	Order	Strategy	N/P
FIND-S	Specific-to-general	Top-down	Positive
LIST-THEN-ELIMINATE	General-to-Specific	Bottom-up	Negative
CANDIDATE-ELIMINATION	Bi-directional	Bi-directional	Both

FIND-S

- h0 = <0, 0, 0, 0, 0, 0>
- h1 = <Sunny, Warm, Normal, Strong, Warm, Same>
- h2 = <Sunny, Warm, ?, Strong, Warm, Same>
- h3 = <Sunny, Warm, ?, Strong, Warm, Same>
- h4 = <Sunny, Warm, ?, Strong, ?, ?>

Training examples:

1. <Sunny, Warm, Normal, Strong, Warm, Same>, Enjoy Sport = ***Yes***
2. <Sunny, Warm, High, Strong, Warm, Same>, Enjoy Sport = ***Yes***
3. <Rainy, Cold, High, Strong, Warm, Change>, Enjoy Sport = ***No***
4. <Sunny, Warm, High, Strong, Cool, Change>, Enjoy Sport = ***Yes***

Report the ***most specific*** hypothesis

LIST-THEN-ELIMINATE

- h0 = <?, ?, ?, ?, ?, ? >
- h1 = <Sunny, ?, ?, ?, ?, ? > or
  <?, Warm, ?, ?, ?, ? > or
  <?, ?, Normal, ?, ?, ? > or
  <?, ?, ?, Weak, ?, ? > or
  <?, ?, ?, ?, Cold, ? > or
  <?, ?, ?, ?, ?, Same >

<Rainy, Cold, High, Strong, Warm, Change>, Enjoy Sport = ***No***

Report the ***most general*** hypothesis

CANDIDATE-ELIMINATION (1)

S0: {<0, 0, 0, 0, 0, 0>}

***S1*: {**<Sunny, Warm, Normal, Strong, Warm, Same>}

S2: {<Sunny, Warm, ?, Strong, Warm, Same>}

G0, G1, G2: {<?, ?, ?, ?, ?, ? >}

Training examples:

1. <Sunny, Warm, Normal, Strong, Warm, Same>, Enjoy Sport = ***Yes***
2. <Sunny, Warm, High, Strong, Warm, Same>, Enjoy Sport = ***Yes***

遇到正例与Find-s方法类似

CANDIDATE-ELIMINATION (2)

S2,S3: {<Sunny, Warm, ?, Strong, Warm, Same>}

G3: {<Sunny,?,?,?,?,?> <?,Warm,?,?,?,?> <?,?,?,?,?,Same>}

G2: {<?,?,?,?,?,?>}

Training examples:

3. <Rainy, Cold, High, Strong, Warm, Change>, Enjoy Sport = ***No***

遇到反例与List-then-eliminate方法类似

CANDIDATE-ELIMINATION (3)

S3: {<Sunny, Warm, ?, Strong, Warm, Same>}

↓

S4: {<Sunny, Warm, ?, Strong, ?, ?>}

G4:{<Sunny,?,?,?,?,?> <?,Warm,?,?,?,?>}

↑

G3:{<Sunny,?,?,?,?,?> <?,Warm,?,?,?,?> <?,?,?,?,?,Same>

Training examples:

4. <Sunny, Warm, High, Strong, Cool, Change>, Enjoy Sport = ***Yes***

Final Version Space

S4: {<Sunny, Warm, ?, Strong, ?, ?>}

<Sunny,?,?,Strong,?,?> <Sunny,Warm,?,?,?,?> <?,Warm,?,Strong,?,?>

G4:{<Sunny,?,?,?,?,?> <?,Warm,?,?,?,?>}

Report the ***version space*** – all possible hypotheses

<Sunny,Warm,Normal,Strong,Cool,Change>

<Rainy,Cold,Normal,Light,Warm,Same>

<Sunny,Warm,Normal,Light,Warm,Same>

<Sunny,Cold,Normal,strong,Warm,Same>

Remarks

- **Convergence Condition**
  - Noise Free (No Errors)
  - The target concept **DOES** exist in the searching hypotheses space ***H***
- **What Training Example Should the Learner Request Next?**
  - Satisfy half the hypotheses in the current version space
  - Fastest Convergence, Least Sample Needed, Best Uncertainty Elimination
- **How Can Partially Learned Concepts Be Used?**
  - Absolutely Accept <Sunny,Warm,Normal,Strong,Cool,Change>
  - Absolutely Deny <Rainy,Cold,Normal,Light,Warm,Same>
  - Pending
    - <Sunny,Warm,Normal,Light,Warm,Same>
    - <Sunny,Cold,Normal,Strong,Warm,Same>

Inductive Bias (归纳的偏差)

- Bias Vs. Unbiase

- The **Futility** (无用性) of Bias-Free Learning
  - Too large searching space
  - The convergence is impossible
  - Rational inference is impossible

- Inductive bias of CANDIDATE-ELIMINATION algorithm
  - The target concept ***c*** is contained in the given hypothesis space ***H***.
  - Inductive System == Deductive System + ***Inductive Bias***

Summary

- Concept learning can be cast as ***Searching*** through predefined hypotheses space.
- The ***general-to-specific*** partial ordering of hypotheses leads to efficient searching strategy, such as CANDIDATE-ELIMINATION algorithm.
- A practical concept learning methods must employ ***inductive bias***. Otherwise, they can only classify the observed training examples.
- Version spaces and the CANDIDATE-ELIMINATION algorithm provide a useful ***conceptual framework*** for studying concept learning. However, their correctness rely on the noise-free training examples and the ability of provided hypotheses space to express the unknown target concepts.

EnjoySport revisit

- **Given:**
  - Instances ***X***: Possible days, each described by the attributes
    - *Sky* (*Sunny*, *Cloudy*, and *Rainy*)
    - *Temp* (*Warm* and *Cold*)
    - *Humidity* (*Normal* and *High*)
    - *Wind* (*Strong* and *Weak*)
    - *Water* (*Warm* and *Cool*)
    - *Forecast* (*Same* and *Change*)
  - Hypotheses ***H***: Each hypothesis is described by a ***conjunction*** of constraints. These constraints may be “?” (any value), “0” (no value), or a specific value.

设想编程调试时候的情景

- **Determine:**
  - A hypothesis *h* in ***H*** such that $h(x) = c(x)$ for all *x* in ***X***.

Thank you