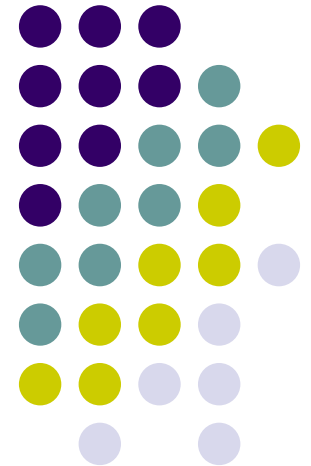


# Boosting: Combining Classifiers

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2009-02-26



The most material of this part come from:  
<http://sifaka.cs.uiuc.edu/taotao/stat/chap10.ppt>

# Boosting



- **INTUITION (三个臭皮匠，顶个诸葛亮)**
  - *Combining Predictions of an ensemble is more accurate than a single classifier*
- **Reasons**
  - Easy to find quite correct “rules of thumb” however **hard to find single highly accurate prediction rule.**
  - If the training examples are few and the hypothesis space is large then there are several equally accurate classifiers.
  - **Hypothesis space does not contain the true function**, but it has several good approximations.
  - **Exhaustive global search** in the hypothesis space **is expensive** so we can combine the predictions of several locally accurate classifiers.

# Cross Validation (交叉检验)



- **K-fold** Cross Validation
  - Divide the data set into  $k$  sub samples
  - Use  $k-1$  sub samples as the training data and one sub sample as the test data.
  - Repeat the second step by choosing different sub samples as the testing set.
- **Leave one out** Cross validation
  - Used when the training data set is small.
  - Learn several classifiers each one with one data sample left out
  - The final prediction is the aggregate of the predictions of the individual classifiers.

# Bagging



- Generate a random sample from training set
- Repeat this sampling procedure, getting a sequence of  $K$  independent training sets
- A corresponding sequence of classifiers  $C_1, C_2, \dots, C_k$  is constructed for each of these training sets, by using the same classification algorithm
- To classify an unknown sample  $X$ , let each classifier predict.
- The Bagged Classifier  $C^*$  then **combines** the predictions of the individual classifiers to generate the final outcome.  
(sometimes combination is simple voting)

# Boosting(Algorithm)



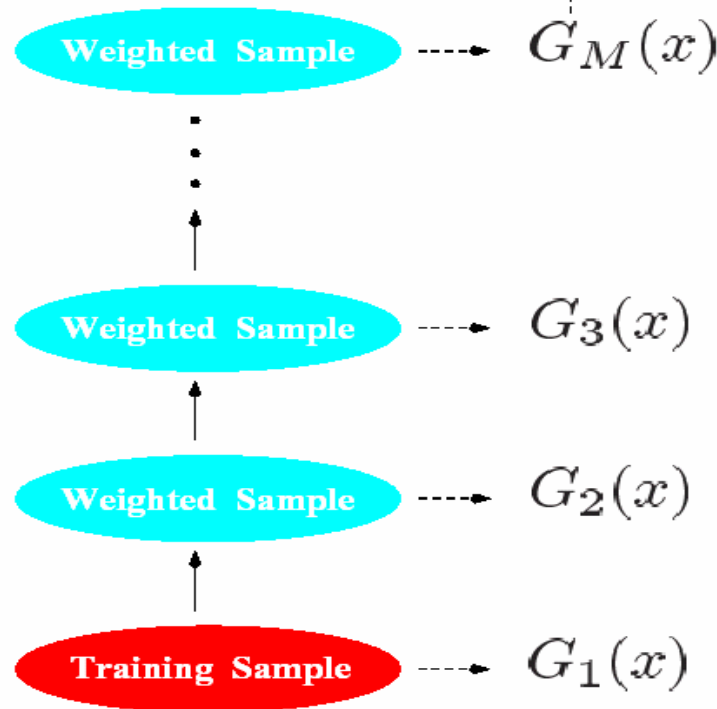
- $W(x)$  is the distribution of weights over the  $N$  training points  
 $\sum W(x_i)=1$
- Initially assign uniform weights  $W_0(x) = 1/N$  for all  $x$ , *step*  $k=0$
- At each iteration  $k$  :
  - Find best weak classifier  $C_k(x)$  using weights  $W_k(x)$
  - With error rate  $\epsilon_k$  and based on a **loss function**:
    - weight  $\alpha_k$  the classifier  $C_k$ 's weight in the final hypothesis
    - For each  $x_i$ , update weights based on  $\epsilon_k$  to get  $W_{k+1}(x_i)$
- $C_{FINAL}(x) = \text{sign} [ \sum \alpha_i C_i(x) ]$

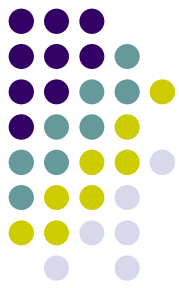


# Boosting (Algorithm)

FINAL CLASSIFIER

$$G(x) = \text{sign} \left[ \sum_{m=1}^M \alpha_m G_m(x) \right]$$





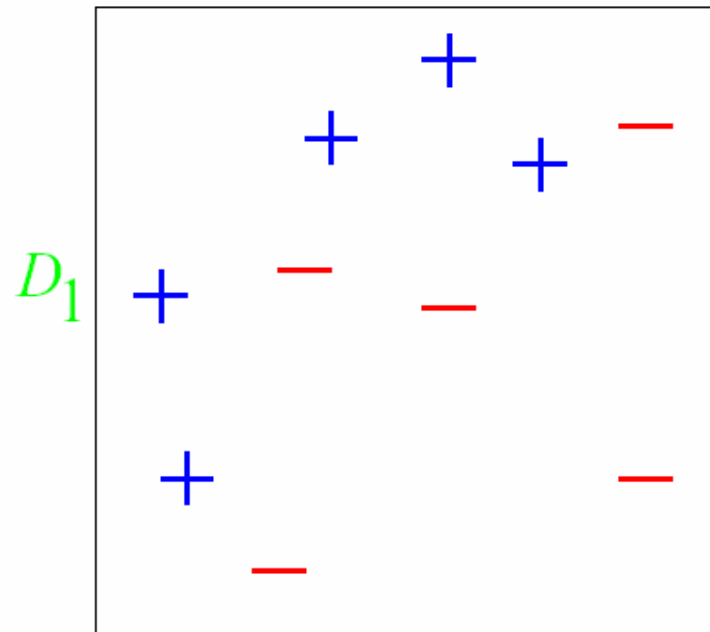
# AdaBoost(Algorithm)

- $W(x)$  is the distribution of weights over the  $N$  training points  
 $\sum W(x_i)=1$
- Initially assign uniform weights  $W_0(x) = 1/N$  for all  $x$ .
- At each iteration  $k$  :
  - Find best weak classifier  $C_k(x)$  using weights  $W_k(x)$
  - Compute  $\epsilon_k$  the error rate as  
 $\epsilon_k = [ \sum W(x_i) \cdot I(y_i \neq C_k(x_i)) ] / [ \sum W(x_i) ]$
  - weight  $\alpha_k$  the classifier  $C_k$ 's weight in the final hypothesis Set  
 $\alpha_k = \log ((1 - \epsilon_k) / \epsilon_k)$
  - For each  $x_i$ ,  $W_{k+1}(x_i) = W_k(x_i) \cdot \exp[\alpha_k \cdot I(y_i \neq C_k(x_i))]$
- $C_{FINAL}(x) = \text{sign} [ \sum \alpha_i C_i(x) ]$

$L(y, f(x)) = \exp(-y \cdot f(x))$  - the exponential loss function



# AdaBoost(Example)



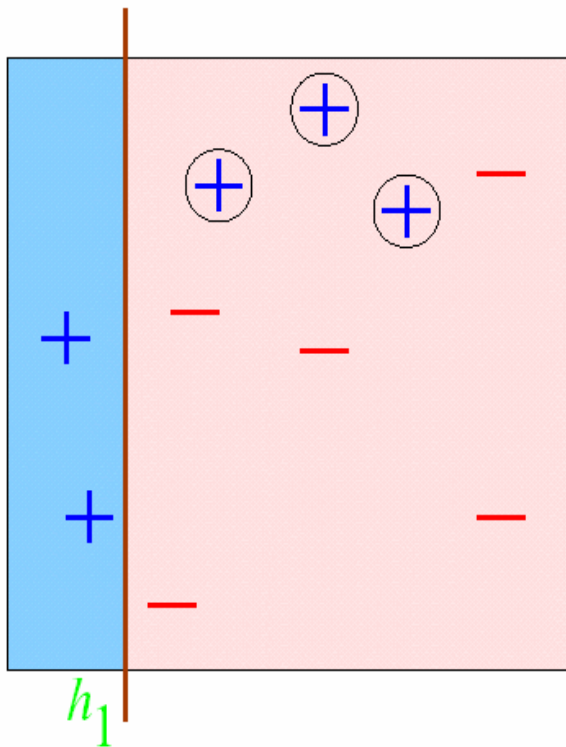
Original Training set : Equal Weights to all training samples





# AdaBoost(Example)

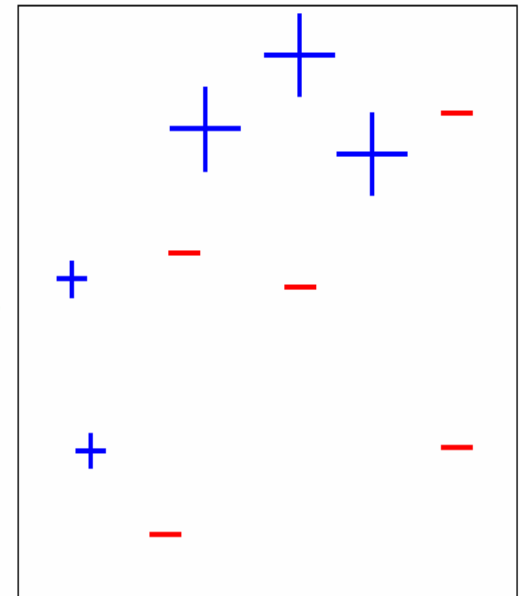
ROUND 1



$\epsilon_1 = 0.30$   
 $\alpha_1 = 0.42$



$D_2$

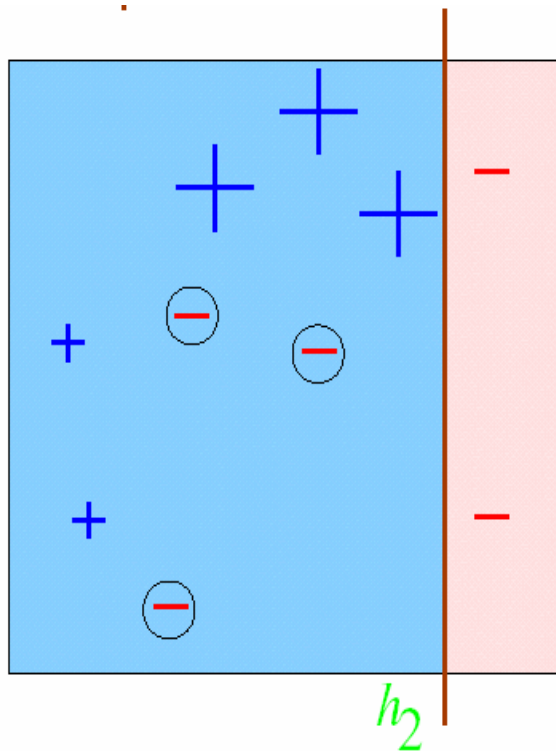


$$\alpha_k = \log((1 - \epsilon_k) / \epsilon_k)$$

# AdaBoost(Example)

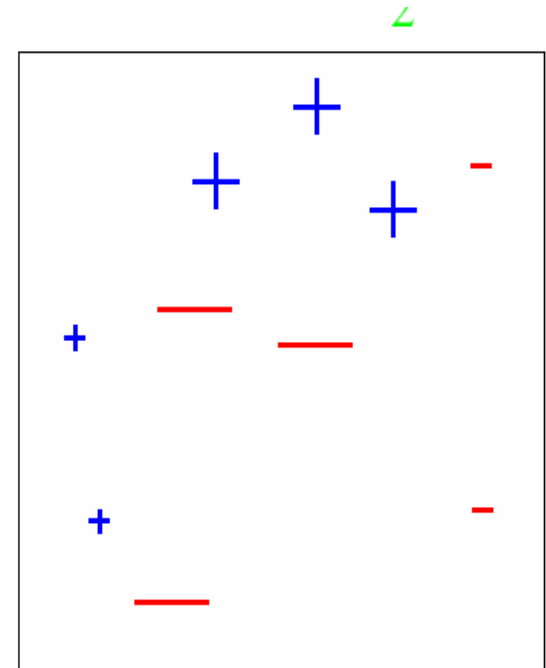


ROUND 2



$$\begin{aligned}\epsilon_2 &= 0.21 \\ \alpha_2 &= 0.65\end{aligned}$$

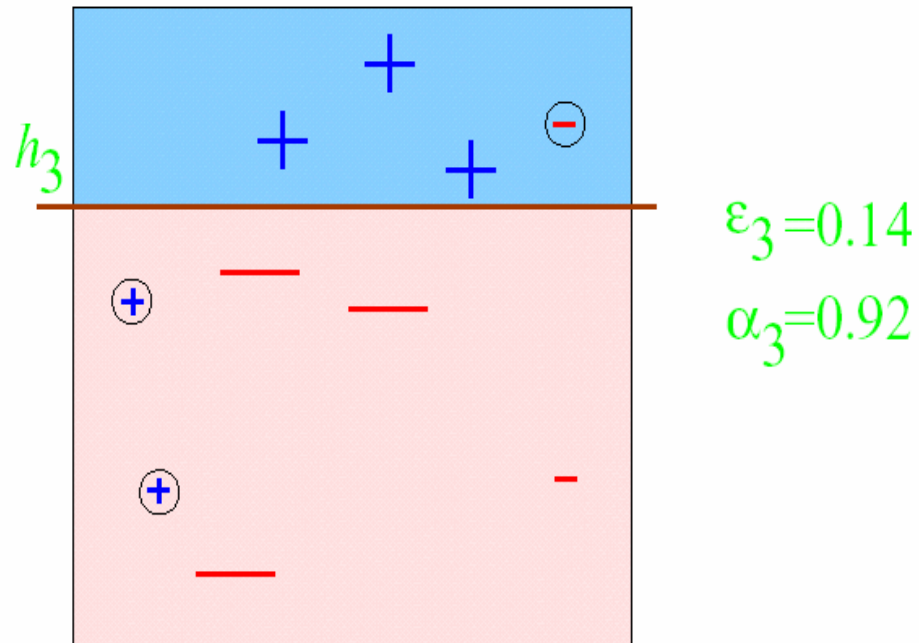
$D_3$





# AdaBoost(Example)

ROUND 3



# AdaBoost(Example)



$$H_{\text{final}} = \text{sign} \left( 0.42 \begin{array}{|c|} \hline \text{blue} \\ \hline \text{red} \\ \hline \end{array} + 0.65 \begin{array}{|c|} \hline \text{blue} \\ \hline \text{red} \\ \hline \end{array} + 0.92 \begin{array}{|c|} \hline \text{blue} \\ \hline \text{red} \\ \hline \end{array} \right)$$

The diagram illustrates the final AdaBoost hypothesis  $H_{\text{final}}$  as a weighted sum of three weak classifiers. Each classifier is represented by a square with a vertical brown line. The first classifier has a weight of 0.42 and a vertical line on the left. The second classifier has a weight of 0.65 and a vertical line on the right. The third classifier has a weight of 0.92 and a horizontal brown line. The regions to the left of the vertical lines and above the horizontal line are shaded blue, while the regions to the right of the vertical lines and below the horizontal line are shaded red. A large green bracket on the left side of the equation indicates the sign function applied to the sum.

Show demo ...



# Boosting

- The final prediction is a combination of the prediction of several predictors.
- **Differences** between Boosting and previous methods?
  - It is **iterative**
  - Boosting: Successive classifiers **depends upon its predecessors**.
    - Previous methods : Individual classifiers were independent.
  - Training examples may have **unequal weights**.
  - **Look at errors** from previous classifier step to decide how to focus on next iteration over data
  - Set weights to **focus** more **on 'hard' examples**. (the ones on which we committed mistakes in the previous iterations)

$$t(\mathbf{x}) \approx \hat{f}(x) = \sum_{i=1}^k w_i h_i$$

$$C(\mathbf{x}) = \sum_{i=1}^k \alpha_i C_i(\mathbf{x}; \mathbf{w}_i)$$



# AdaBoost Case Study:

- Rapid Object Detection using a Boosted Cascade of Simple Features (IEEE CVPR2001)

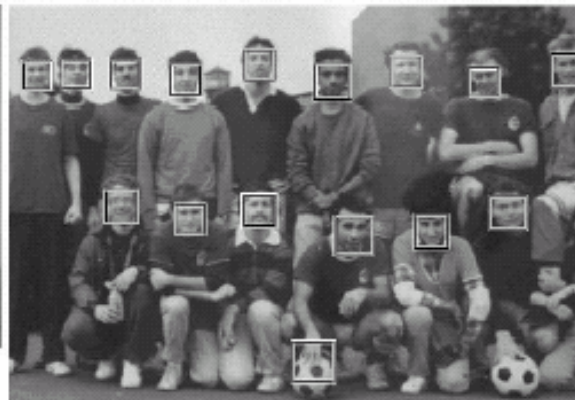
[Rapid object detection using a boosted cascade of simple features](#) - [所有 33 个版本»](#)

P Viola, M Jones - IEEE COMPUTER SOCIETY CONFERENCE ON COMPUTER VISION AND ..., 2001 - doi.ieeeecs.org

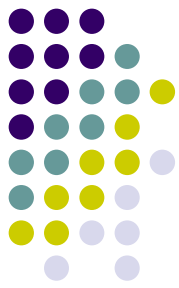
This paper describes a machine learning approach for visual object detection which is capable of processing images extremely rapidly and achieving high detection rates. This work is distinguished by three key contributions. The first is the introduction of a new image representation ...

[被引用次数: 537](#) - [网页搜索](#)

**1555 today!**



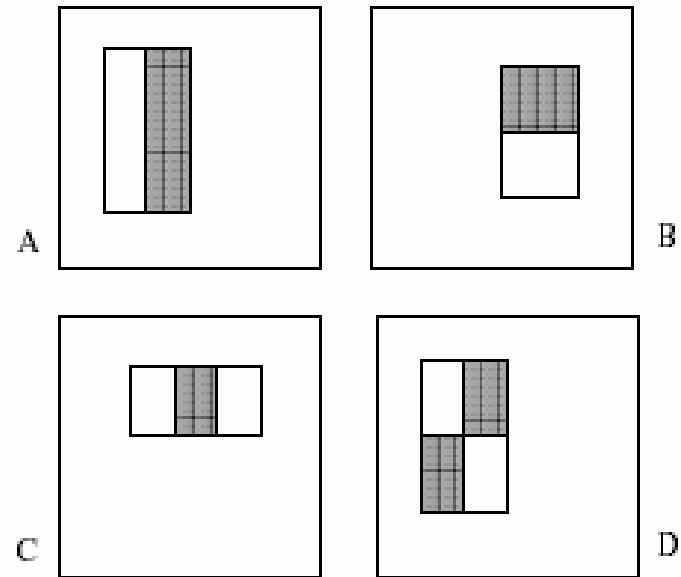
# Object detection using AdaBoost



- Object Detection
- Features
  - *two-rectangle*
  - *three-rectangle*
  - *four-rectangle*

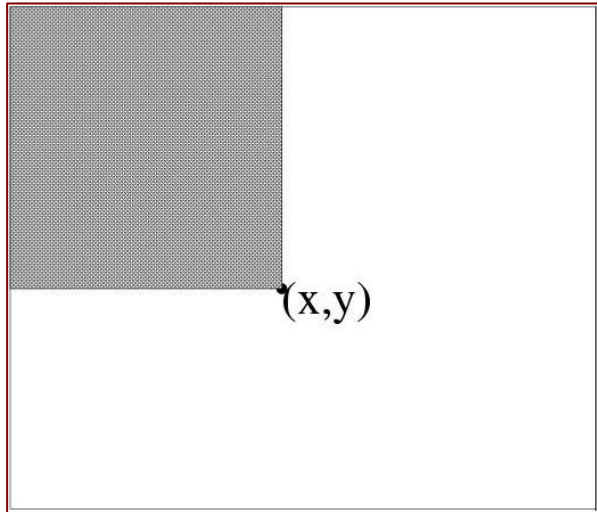
Size: 24x24

Feature: 180,000





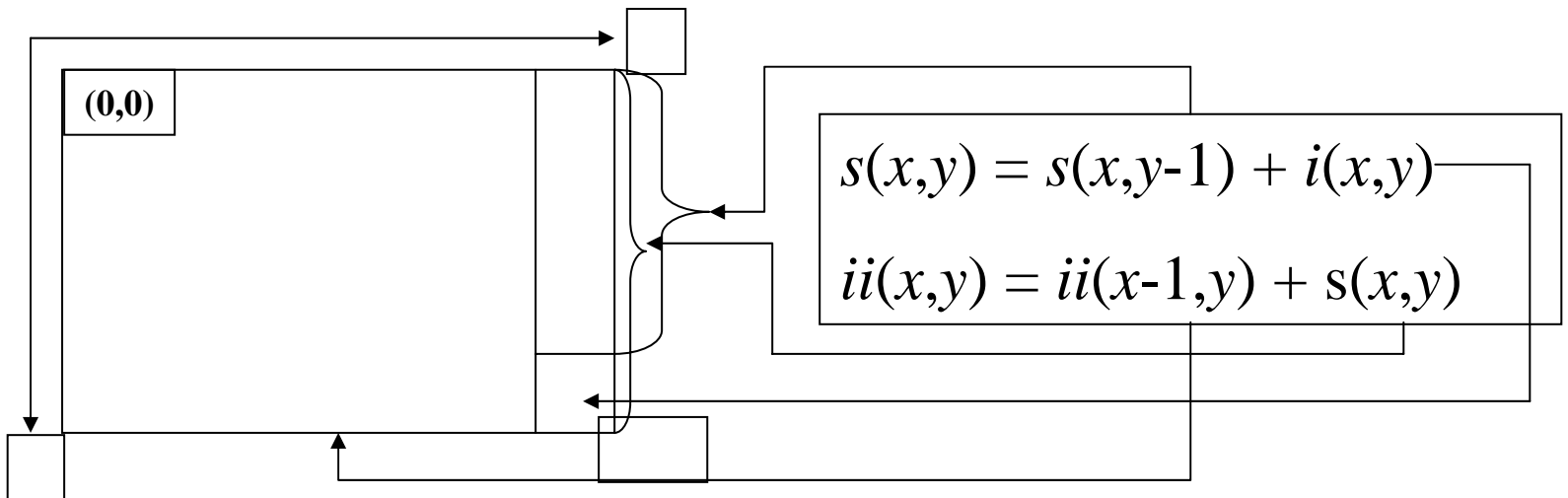
- Integral Image



Definition: The integral image at location  $(x,y)$  contains the sum of the pixels above and to the left of  $(x,y)$  , inclusive:

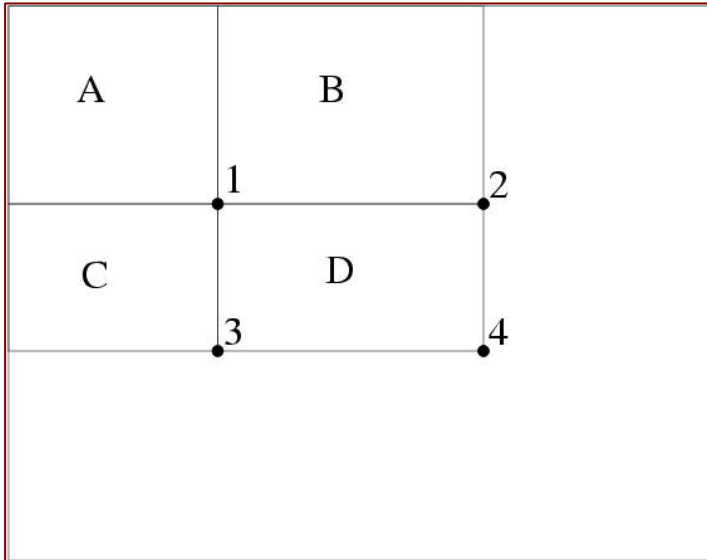
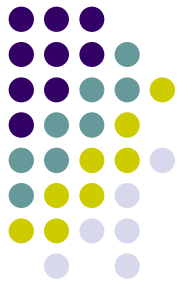
$$ii(x,y) = \sum_{x' \leq x, y' \leq y} i(x',y'),$$

Using the following pair of recurrences:





- Features Computation



Using the integral image any rectangular sum can be computed in four array references

$$\mathbf{ii(4) + ii(1) - ii(2) - ii(3)}$$



- Given example images  $(x_1, y_1), \dots, (x_n, y_n)$  where  $y_i = 0, 1$  for negative and positive examples respectively.
- Initialize weights  $w_{1,i} = \frac{1}{2m}, \frac{1}{2l}$  for  $y_i = 0, 1$  respectively, where  $m$  and  $l$  are the number of negatives and positives respectively.
- For  $t = 1, \dots, T$ :

1. Normalize the weights,

$$w_{t,i} \leftarrow \frac{w_{t,i}}{\sum_{j=1}^n w_{t,j}}$$

so that  $w_t$  is a probability distribution.

2. For each feature,  $j$ , train a classifier  $h_j$  which is restricted to using a single feature. The error is evaluated with respect to  $w_t$ ,  $\epsilon_j = \sum_i w_i |h_j(x_i) - y_i|$ .
3. Choose the classifier,  $h_t$ , with the lowest error  $\epsilon_t$ .
4. Update the weights:

$$w_{t+1,i} = w_{t,i} \beta_t^{1-e_i}$$

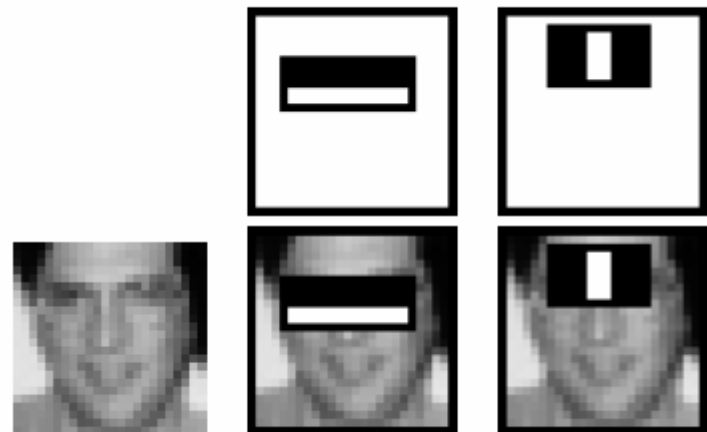
where  $e_i = 0$  if example  $x_i$  is classified correctly,  $e_i = 1$  otherwise, and  $\beta_t = \frac{\epsilon_t}{1-\epsilon_t}$ .

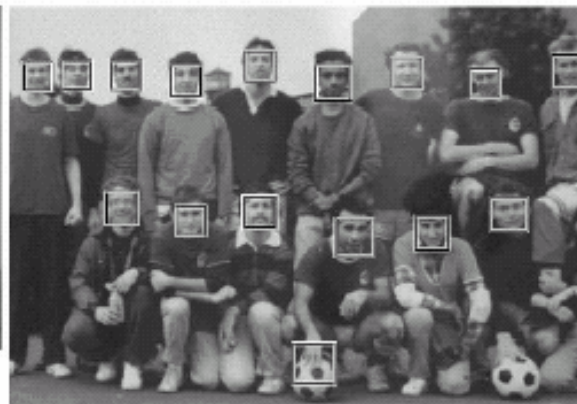
- The final strong classifier is:

$$h(x) = \begin{cases} 1 & \sum_{t=1}^T \alpha_t h_t(x) \geq \frac{1}{2} \sum_{t=1}^T \alpha_t \\ 0 & \text{otherwise} \end{cases}$$

where  $\alpha_t = \log \frac{1}{\beta_t}$

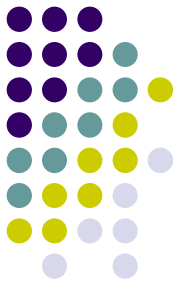
# AdaBoost algorithm for classifier learning





# Homework

- Implement this CVPR paper.
  - Hint: You can use OpenCV.



# Thank you

