Naïve Bayes Classification

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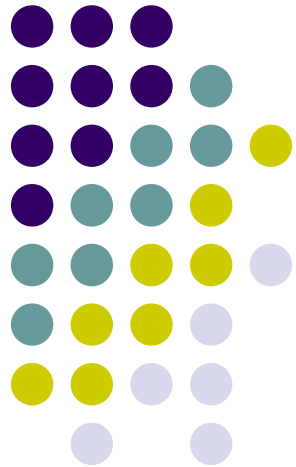
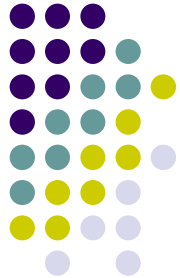


Outline

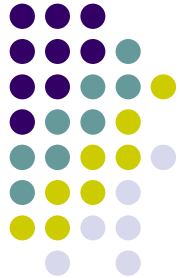
- Supervised learning:
  - classification & regression
- Bayesian Spam Filtering: an example
- Naïve Bayes classification

Supervised learning



- **Given:** training examples (*instances*) for which *attribute* values and target are known
  - E.g. age, weight, etc., for a patient and whether that patient has diabetes or not
  - **Instance:** $< \mathbf{x}_i, t_i >$
- **Goal:** function that predicts target variable accurately given new attribute values
  - **Learn:** mapping from $\mathbf{x}$ to $t(\mathbf{x})$.
- Accuracy is measured on *fresh* data

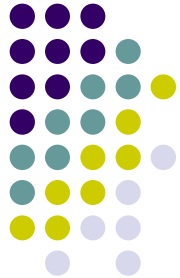
Regression vs. Classification



- Regression: predicting a *numeric* target variable given some attributes
  - E.g. predicting auto price given make, horse power, weight, etc.

- Classification: predicting a *nominal* target variable given some attributes
  - E.g. predicting type of car based on job and wealth of owner, type of best friend's car, etc.

Bayesian Spam Filtering: An example



- The materials of this part are mainly from
  - http://www.cs.mun.ca/~donald/slug/2005-04-07/slugspam.pdf
- Resources:
  - <http://www.cs.mun.ca/~donald/buryspam/>
    - extensive documenation on Donald’s buryspam.rb script.
  - <http://www.paulgraham.com/spam/>:
    - Paul Graham's spam proposal.
  - <http://www.mozilla.org/>:
    - Free Mozilla Thunderbird e-mail client.
  - <http://spambayes.sourceforge.net/>:
    - Plugins for other e-mail clients.
  - <http://spamassassin.sourceforge.net/doc/sa-learn.html>:
    - SpamAssassin's Bayesian classifier

You've Got Mail! ☺ or ☹

From uunet.uu.net!national-alliance.org!Crusader Sat Sep 30 17:26:42 1995

Received: from asso.nis.garr.it (@asso.nis.garr.it [192.12.192.10]) by

garfield.cs.mun.ca with SMTP id <102189>; Sat, 30 Sep 1995 17:26:31 -0230

Received: by asso.nis.garr.it (4.1/1.34/ABB950929)

id AA18937; Sat, 30 Sep 95 19:03:23 +0100

Received: by mercury.sfsu.edu (5.0/SMI-SVR4)

id AA21676; Sat, 30 Sep 95 11:03:27 -0700

Date: Sat, 30 Sep 1995 15:33:27 -0230

From: Crusader@national-alliance.org

Message-Id: <913247217488@National-Alliance.org>

To: <donald@cs.mun.ca>

To: <XXXXXX@cs.mun.ca>

Subject: Piranhas

Apparently-To: Crusader@National-Alliance.org

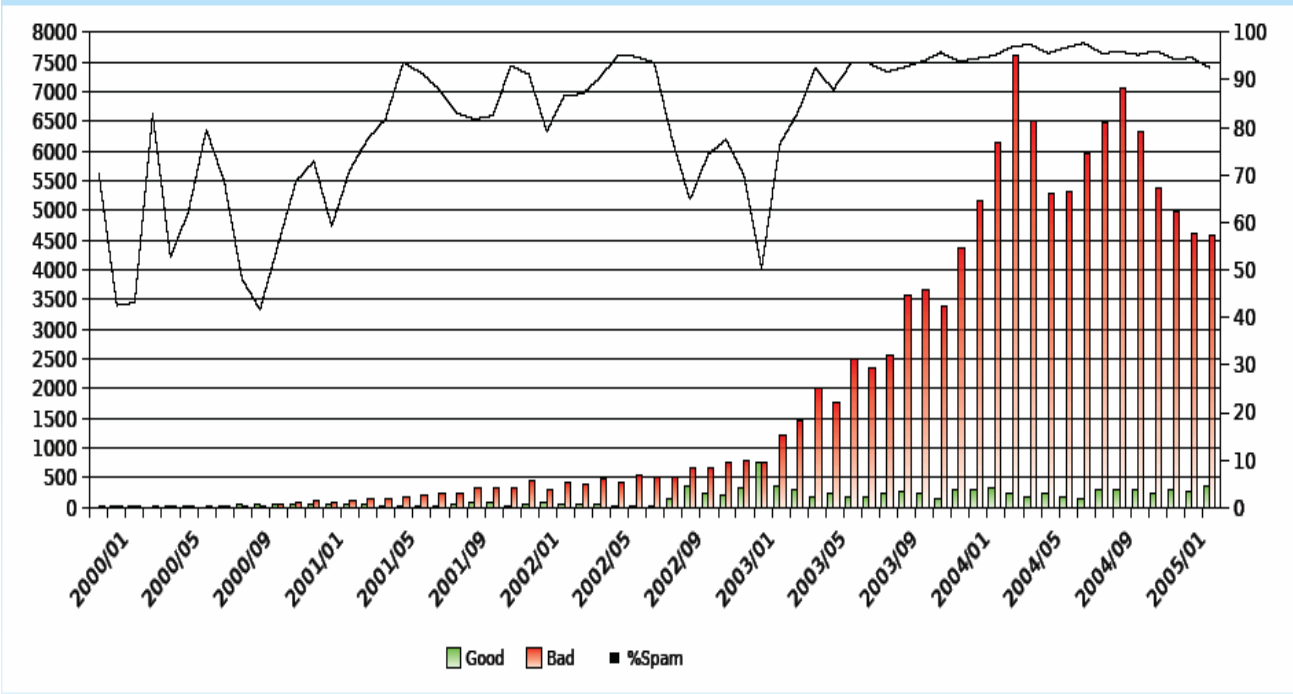
What would you say if a Liberal "social scientist" told you to jump into a pool filled with five hundred ravenous piranhas?

If you valued your life, you'd certainly refuse the invitation.

But what if the Liberal "social scientist" tried to convince you to go ahead and jump in, with the argument that "not all of the piranhas are aggressive. Some of them probably just want to make friends with you, and really aren't hungry either. To say that a

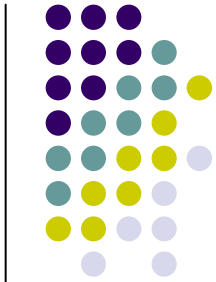
... blah, blah, blah...

Spam, Spam, and Spam!



- The volume of spam was increasing exponentially.
  - Currently, about 95% of my e-mail is spam.
  - I currently receive about 100 to 150 spam messages per day, over 5MB a week (down from about 200 to 250 per day).

Filtering Attempt: key words and regular expressions



- An early attempt to identify spam was to use ***procmail***.

- This involved storing regular expressions in one's .procmailrc which would match phrases that occurred frequently in spam messages. e.g.

```
:0B:
* (mailto:|subject=(3D)?)(abuse|nosale|(un)?sub|delete|remov(e|al)|exclude|discontinue|nomore|
(additional|more)[-_.]info)|annuler=|(reply|yes|subscribe|remove)["']? (yourself|instructions|
.*from .*mailing|.*(in|as).*subject)|To remove.*(e-?mail|message)|to.* be .*(r.?e.?m.?o.?v.?e.?d?|
taken (from|off)|extracted)|for removal|removal instructions|remove:mailto|to (be taken out|
get off|opt out|remove from|unsubscribe)|remove (requests|me)|(do not|don.t) respond|(global|
no need to) remove|remove[a-z0-9]*@|cancel subscription|remove(me|you|\.(php|html?))|opt.?out|
perman(ent|tently) remov(ed?|al)|remov(ed?|al) (notification|requests|administrator)|$removal:|
remove in the|if you wish to receive|(/|mail)-?(refuse|remove|unsu(b|s))|["']remove["']|off.?list|
reject.?mail
junk/junk.a
```

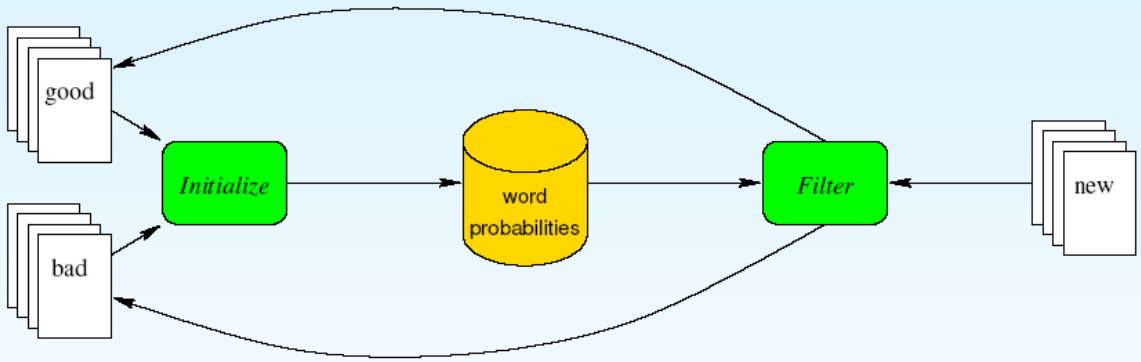
- Problems
  - Its binary nature could very easily generate false positives.
  - Spam kept changing too rapidly for this strategy to be effective.
  - As a result, the **.procmailrc** le became very large and cumbersome to change.

A Better Solution

- In August 2002, Paul Graham advocated Bayesian filtering as an effective means of combating the spam problem. He didn't actually discover the technique, but he did help popularize it.

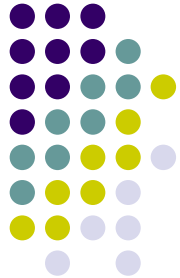
- Accuracy rate was claimed to be in excess of 99%.
- The technique has two distinct phases:

A collection of good and bad messages are examined and statistical probabilities for all words is dervied based upon the relative frequencies of each word in the two collection of messages.



The filter uses these statistical probabilities during its analysis of incoming messages to classify them as spam or non-spam.

The Initializing Phase



If we know:

msg_b = total number of bad messages,
msg_g = total number of good messages,
w_b = occurrences of word w in bad messages and
w_g = occurrences of word w in good messages,

then we can assign a probability to each word w based upon the relative frequencies that w occurred in all the bad and good messages.

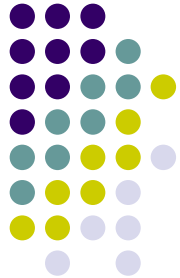
$$P(w) = \max\left(\min\left(\frac{r_b}{r_g + r_b}, 0.99\right), 0.01\right)$$

where:

$$r_b = \min\left(\frac{w_b}{msg_b}, 1.0\right), \qquad r_g = \min\left(\frac{2w_g}{msg_g}, 1.0\right)$$

The further away $P(w)$ is from 0.5 (neutral), the more useful the word w is in determining whether or not the message is spam.

Initializing Examples



$msg_b = 69,449 \qquad msg_g = 9,580$

$w = \text{buy}:$

$w_b = 4,434 \qquad w_g = 171$

$$r_b = \frac{4,434}{69,449} = 0.063845 \qquad r_g = \frac{2\times171}{9,580} = 0.035699 \qquad P(w) = \frac{0.063845}{0.035699+0.063845} = 0.641374$$

$w = \text{university}:$

$w_b = 198 \qquad w_g = 1,243$

$$r_b = \frac{198}{69,449} = 0.002851 \qquad r_g = \frac{2\times1243}{9,580} = 0.259499 \qquad P(w) = \frac{0.002851}{0.002851+0.259499} = 0.0108672$$

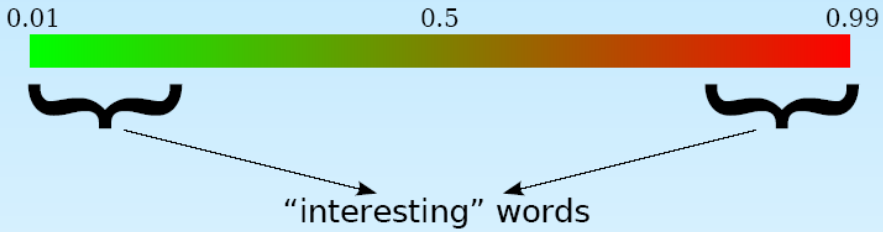
$w = \text{and}:$

$w_b = 158,729 \qquad w_g = 70,828$

$$r_b = \min\left(\frac{158,729}{69,449}, 1.0\right) = 1 \qquad r_g = \min\left(\frac{2\times70,828}{9,580}, 1.0\right) = 1 \qquad P(w) = \frac{1}{1+1} = 0.5$$

The Filtering Phase

Once we have generated a database of words and their corresponding probabilities, we can start to filter incoming messages. When we receive a message, we identify the n most *interesting* words, w_i (Graham's algorithm sets $n = 15$). These are the words whose probabilities are furthest away from neutral.



We then apply the following formula:

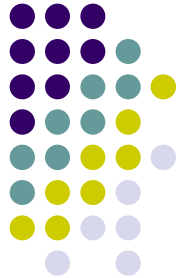
$$p = \frac{prod_1}{prod_1 + prod_2}$$

where:

$$prod_1 = \prod_{i=1}^{n} P(w_i), \qquad prod_2 = \prod_{i=1}^{n} (1 - P(w_i))$$

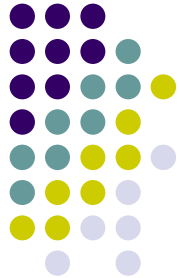
The closer that p is to one, the more likely the message is spam.

Filtering Example #1



```
From: Michael Rayment
To: Donald Craig, Geoff Holden
Organization: Memorial University
Subject: Abstract for talk

            What's with Python Anyway?
      A Look at Why Python is so Popular
                    by
              Michael Rayment

This talk will look at how the designers of Python
got it right when they designed the language and wrote
the Python interpreter.  A quick tour of the language
will reveal a symmetric object oriented data
structure model at the heart of the language along
with a rich set of program constructs that allow the
programmer to manipulate the data easily.  All data
are dynamically typed objects that can be assigned
to variables without worrying about type declarations.
Python "does the right thing" according to the context
in which the data is used.  The variable name is
simply a handle.  Among other things this talk will
describe some of the salient features of the language
such as list manipulation, creating classes, handling
exceptions, scope of variables and most importantly
the rich set of predefined module libraries.
```

Filtering Example #1 (Cont'd)

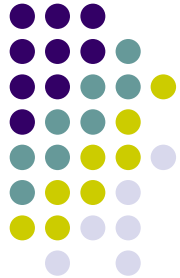


$$
\begin{aligned}
P(\text{variables}) &= 0.01 \\
P(\text{Memorial}) &= 0.01 \\
P(\text{declarations}) &= 0.01 \\
P(\text{Craig}) &= 0.01 \\
P(\text{wrote}) &= 0.01 \\
P(\text{Geoff}) &= 0.01 \\
P(\text{Abstract}) &= 0.01 \\
P(\text{Python}) &= 0.01 \\
P(\text{dynamically}) &= 0.0104355365 \\
P(\text{University}) &= 0.0108397943 \\
P(\text{Rayment}) &= 0.0108484296 \\
P(\text{Rayment}) &= 0.0108484296 \\
P(\text{exceptions}) &= 0.0132622019 \\
P(\text{typed}) &= 0.0139836777 \\
P(\text{Anyway}) &= 0.0198030382
\end{aligned}
$$

$$
\begin{aligned}
prod_1 &= 4.88921 \times 10^{-30} \\
prod_2 &= 0.842786 \\
p &= \frac{4.88921 \times 10^{-30}}{4.88921 \times 10^{-30} + 0.842786} \\
&= 5.80125 \times 10^{-30}
\end{aligned}
$$

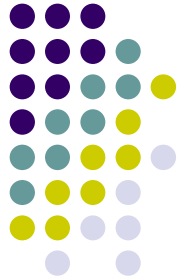
Message is not spam!

Filtering Example #2



```
From cubitus@realtywebsites.com  Tue Mar 22 02:16:29 2005
Return-Path: <cubitus@realtywebsites.com>
Received: from realtywebsites.com (unknown [218.85.167.135])
        by mercury.cs.mun.ca (Postfix) with SMTP id 0D3DB18B6E3
        for <donald@garfield.cs.mun.ca>; Tue, 22 Mar 2005 02:16:18 -0330 (NST)
From: <cubitus@realtywebsites.com>
To: <donald@cs.mun.ca>
Subject: At your service, online pharm
Date: Tue, 22 Mar 2005 13:50:20 -0500
Mime-Version: 1.0
Content-Type: text/plain; charset=us-ascii
Message-Id: <20050322054619.0D3DB18B6E3@mercury.cs.mun.ca>
X-Virus-Scanned: by amavisd-new at cs.mun.ca

If you need high quality medication and
would love to save on outrageous retail pricing,
then OnlinePharmacy is for you.

Shopping online for your drug needs saves you the hassle of going to the
doctor,  answering embarassing questions and waiting in line to receive
good treatment?  OnlinePharmacy has been dedicated to serving its online
clientel since 2001.

Try us to find out why ordering your medications online has never been easier.

http://www.fastmail336.com/rx/?76
```

Filtering Example #2 (cont'd)



$P(\text{OnlinePharmacy}) = 0.99$
$P([218) = 0.99$
$P(\text{pharm}) = 0.0420049316$
$P(\text{answering}) = 0.0459504311$
$P(\text{unknown}) = 0.9534104082$
$P(\text{retail}) = 0.9132323275$
$P(\text{medications}) = 0.9128298221$
$P(\text{medication}) = 0.9104607169$
$P(\text{would}) = 0.0919725162$
$P(\text{doctor}) = 0.9036781616$
$P(\text{questions}) = 0.1016689722$
$P(\text{serving}) = 0.1121766169$
$P(\text{dedicated}) = 0.1185694551$
$P(\text{easier}) = 0.1433214124$
$P(\text{going}) = 0.1536839685$

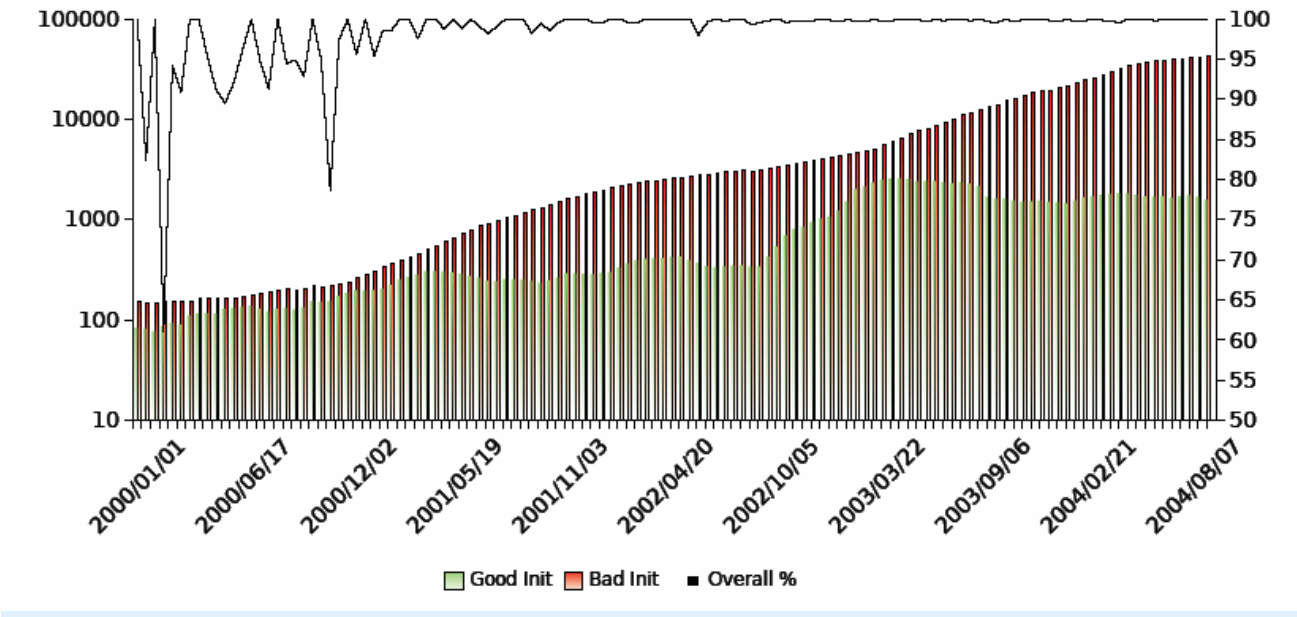
$prod_1 = 3.38881 \times 10^{-9}$

$prod_2 = 1.28555 \times 10^{-10}$

$p = \frac{3.38881 \times 10^{-9}}{3.38881 \times 10^{-9} + 1.28555 \times 10^{-10}} = 0.963451$

Message is spam!

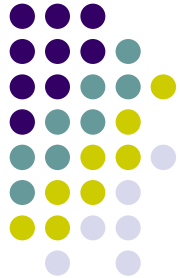
Performance



- Initially, the filter performs very poorly, sometimes misclassifying up to one- third of all messages.

- After being initialized with about 2000 bad messages and 200 good messages, the filter hits very close to 100% effectiveness.

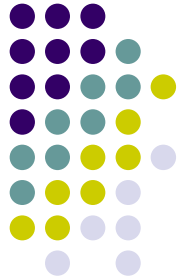
Bayesian filtering: Advantages & Disadvantages



- Advantages
  - Conceptually very easy to understand.
  - Very effective (filters more than 99.691%)
  - Everyone's filter is essentially customized, making it very difficult for spammers to defeat everyone's filter with a single message.
  - Many e-mail clients now either directly or indirectly support Bayesian filtering.

- Disadvantages
  - You need to have a corpus of good and bad messages to initialize the filter.
  - Initialization is a bit time consuming (but this can be made quicker by caching word counts for each mail folder).
  - On each message, a user-specific database of word probabilities has to be consulted. This makes Bayesian filtering somewhat resource intensive and probably not ideal for sites with large user bases.
  - False positives do happen (but are rare).

Regression and Classification



- Function Approximation
  - Learn f: $X \rightarrow Y$
  - Learn $P(\ Y \mid X\ )$

- For classification: find best Y
  - from regression to classification

$$Y_{\max} = \arg\max_{Y} P(Y \mid X)$$

Bayes Rule

$$P(Y \mid X) = \frac{P(X \mid Y)P(Y)}{P(X)}$$

$$(\forall i, j)P(Y = y_i \mid X = x_j) = \frac{P(X = x_j \mid Y = y_i)P(Y = y_i)}{P(X = x_j)}$$

How will we represent $P(X \mid Y)$, $P(Y)$?

How many parameters must we estimate?

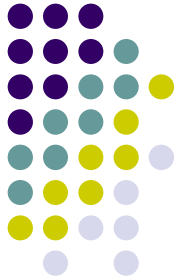
Naïve Bayes

- Suppose $X=<X_1, X_2, \ldots, X_n>$
- Naïve Bayes assumption:

$$P(X \mid Y) = \prod_i P(X_i \mid Y)$$

i.e., that X_i and X_j are conditionally independent, given Y

Naïve Bayes Classifier



Assume target function $f : X \to V$, where each instance x described by attributes $\langle a_1, a_2 \ldots a_n \rangle$. Most probable value of $f(x)$ is:

$$v_{MAP} = \underset{v_j \in V}{\operatorname{argmax}} P(v_j | a_1, a_2 \ldots a_n)$$

$$v_{MAP} = \underset{v_j \in V}{\operatorname{argmax}} \frac{P(a_1, a_2 \ldots a_n | v_j) P(v_j)}{P(a_1, a_2 \ldots a_n)}$$

$$= \underset{v_j \in V}{\operatorname{argmax}} P(a_1, a_2 \ldots a_n | v_j) P(v_j)$$

Naive Bayes assumption:

$$P(a_1, a_2 \ldots a_n | v_j) = \prod_i P(a_i | v_j)$$

which gives

Naive Bayes classifier: $v_{NB} = \underset{v_j \in V}{\operatorname{argmax}} P(v_j) \prod_i P(a_i | v_j)$

Naïve Bayes Algorithm

Naive_Bayes_Learn($examples$) 训练分类器

For each target value v_j

$\hat{P}(v_j) \leftarrow$ estimate $P(v_j)$ 通过计算频率来估算概率

For each attribute value a_i of each attribute a

$\hat{P}(a_i|v_j) \leftarrow$ estimate $P(a_i|v_j)$

Classify_New_Instance(x) 使用分类器

$$v_{NB} = \underset{v_j \in V}{\operatorname{argmax}} \hat{P}(v_j) \prod_{a_i \in x} \hat{P}(a_i|v_j)$$

Naïve Bayes: a simple example

- Play tennis again

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Naïve Bayes: a simple example

$\langle Outlk = sun, Temp = cool, Humid = high, Wind = strc$

Want to compute:

$$v_{NB} = \underset{v_j \in V}{\operatorname{argmax}} P(v_j) \prod_i P(a_i|v_j)$$

$P(y) = 9/14, P(n) = 5/14$

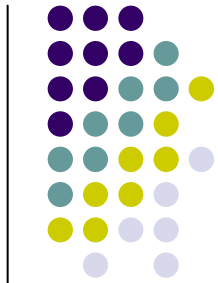
$P(sun/y) = 2/9 \quad P(cool/y) = 3/9 \quad P(high/y) = 3/9 \quad P(strong/y) = 3/9$

$$P(y)\ P(sun|y)\ P(cool|y)\ P(high|y)\ P(strong|y) = .005$$

$$P(n)\ P(sun|n)\ P(cool|n)\ P(high|n)\ P(strong|n) = .021$$

$$\rightarrow v_{NB} = n$$

What you should know:



- Learning (generative) classifiers based on Bayes rule
- Conditional independence
- Naïve Bayes assumption and its consequences
- Naïve Bayes with discrete inputs, continuous inputs

Homework

- What if we have continuous x_i

$$p(x \mid \mu, \delta) \sim \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\delta^2}}$$