

# Applied mathematics in computer science and technology

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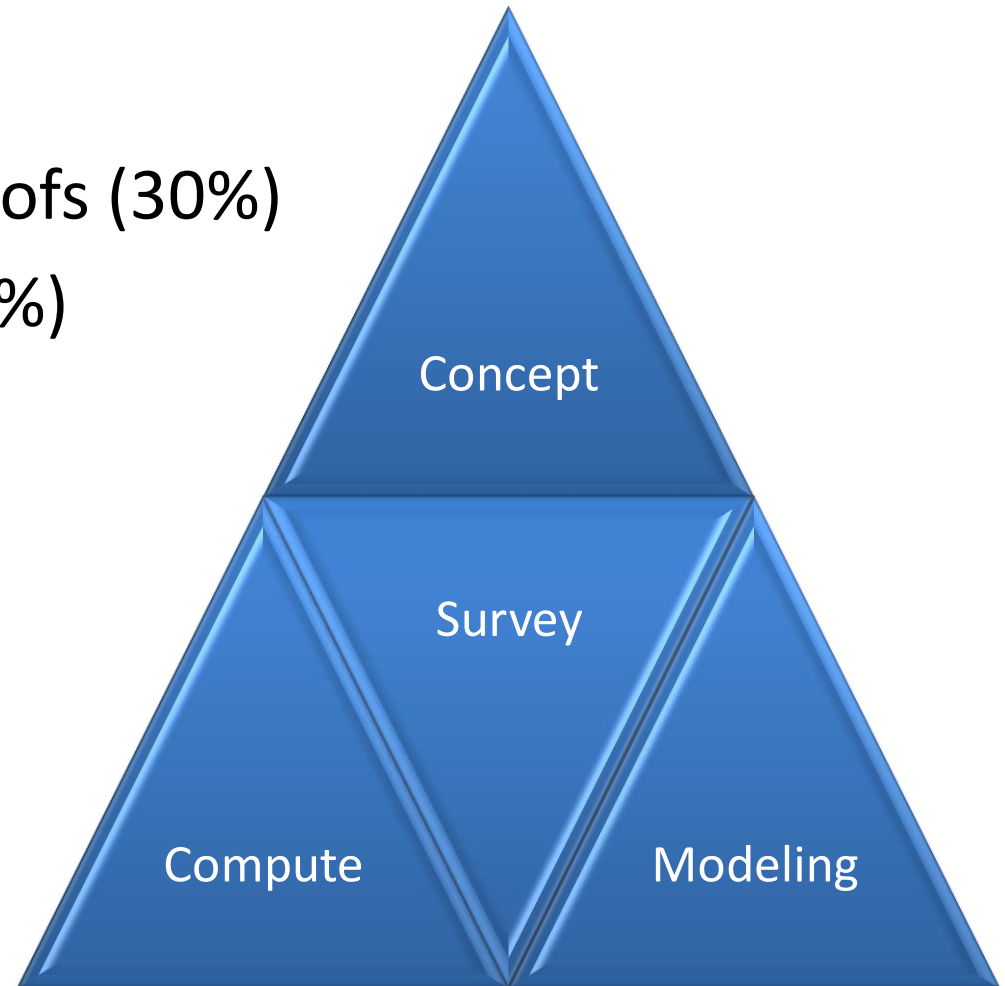
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# Main purpose of the course

- To **improve** the mathematical theory of **self-cultivation**
  - Intuitively understand the mathematical thinking in the papers
- To **train skilled ability to express** mathematical concepts
  - How to formulate mathematical problems
  - How to abstract mathematical methods
- To **cultivate outstanding ability** to use mathematical models
  - The use of mathematical models to solve computer-related research issues

# The final examination

- Question type
  - Concept (30%)
  - Calculation and proofs (30%)
  - Math modeling (30%)
  - Survey (10%)



**孔子曰：智者乐山，仁者乐水**

# The final examination

- **Innovative Score System**

Type	Questions	Required	Discount of over-answered	Budget
Concept	6	3	50%	30
Computing	6	3	50%	30
Modeling	6	3	50%	30
Survey	3	1	50%	10
Total	21	10	---	100

# The final examination

- Example of ISS: **normal**

Type	Answered	Required	Correct	Gain
Concept	3	3	3	30
Computing	3	3	2	20
Modeling	3	3	3	30
Survey	1	1	1	10
Total	11	10	9	90

# The final examination

- Example of ISS: **love computing**

Type	Answered	Required	Correct	Gain
Concept	2	3	2	20
Computing	4	3	4=3+1	35
Modeling	3	3	3	30
Survey	1	1	1	10
Total	10	10	10	95

# The final examination

- Example of ISS: **love concept**

Type	Answered	Required	Correct	Gain
Concept	6	3	$5=(3+2)$	40
Computing	1	3	1	10
Modeling	2	3	2	20
Survey	2	1	$2\sim(2-1)$	10
Total	11	10	9	80



# Final Review

道可道，非常道

名可名，非常名

《道德经》开篇语

# Similar course at top universities

- Princeton:
  - Mathematical methods in Computer Science
  - <http://www.cs.princeton.edu/~boaz/methods2003/>
  - 讲授图论、拓扑初步、线性规划、矩阵论、统计初步等
- Cambridge:
  - Mathematical methods for Computer Science
  - <http://www.cl.cam.ac.uk/teaching/0809/CST/node38.html>
  - 讲授傅立叶方法、小波分析、不等式与极限理论、Markov链等统计理论
- 均以计算机图形学、计算机视觉、图像处理、人工智能、人机交互等计算机科学中的问题为背景进行讲解

# Our course

- Fundamentals of 4 math topics:
  - Statistical learning
  - Variational methods
  - Partial differential equations
  - Optimization methods

# Concepts in Statistical learning

- What is machine learning?
- The categories of learning methods
  - Supervised learning
  - Unsupervised learning
- Fundamental statistical concept
  - Prior, likelihood, Posterior
  - Markov chain

# Computing methods in learning

- Point estimations
  - Bayesian formula
  - Binary distribution, Gaussian distribution
- Classification
  - Naïve Bayesian classification, decision tree
  - SVM, boosting
- Clustering
  - K-means, MOG, spectral clustering
- Time variance data
  - Hidden Markov Chain
  - Karman filter

# Data modeling

- Geometric description:
  - Dimensional reduction
  - Kernel methods
- Algebra description:
  - Classification v.s. regression
  - How to overcome over-fitting?

# Concepts in variational methods

- variational problems:
  - 两点间的最短连线问题
  - 最速降线(brachistochrone)问题
  - 测地线(geodesic line)问题

# 变分法中的符号

- 给定函数  $y(x)$ 
  - 宗量:  $x$
  - 函数:  $y(x)$
  - 宗量的增量:  $\Delta x$
  - 函数的增量:
    - $\Delta y = y(x + \Delta x) - y(x)$
  - 当两点无限接近:
    - $\Delta x \rightarrow dx, \Delta y \rightarrow dy$
  - 略去高阶微量:
    - $dy = y'(x)dx$
  - 当在  $x$  处取得函数极值
    - $dy = 0$
- 给定泛函  $\Pi(y)$ 
  - 宗量:  $y$
  - 泛函:  $\Pi(y)$
  - 函数的变分:  $\delta y$
  - 泛函的变分:  $\delta \Pi$ 
    - $\delta \Pi = \Pi(y + \delta y) - \Pi(y)$
  - 在计算  $\delta \Pi$  时可以展开  $\Pi(y + \delta y)$  中的被积函数只保留线性项
  - 当在  $y$  处取得泛函极值
    - $\delta \Pi = 0$

函数  $y(x)$  在定义域内与  $y(x) + \delta y(x)$  处处无限接近



# Partial differential equation

- Different types of PDE:
  - Can you distinguish them?
  - Laplacian equation, Poisson equation
- Basic concepts:
  - Curve/surface representation
    - Parametric or implicit definition
  - Tangent, normal, curvature
  - Gradient, Divergence

# Computing in PDE

- Laplacian operator
- Discrete operators used in PDE
- How to numerically solve special PDEs
  
- Applications of PDE

# Optimization methods

- Linear methods
- Non-linear methods

# How to prepare the final examination

读

Read

读书

Read book

读数学书

Read math book

很认真地读数学书

Read math book seriously