

Applied mathematics in computer science and technology

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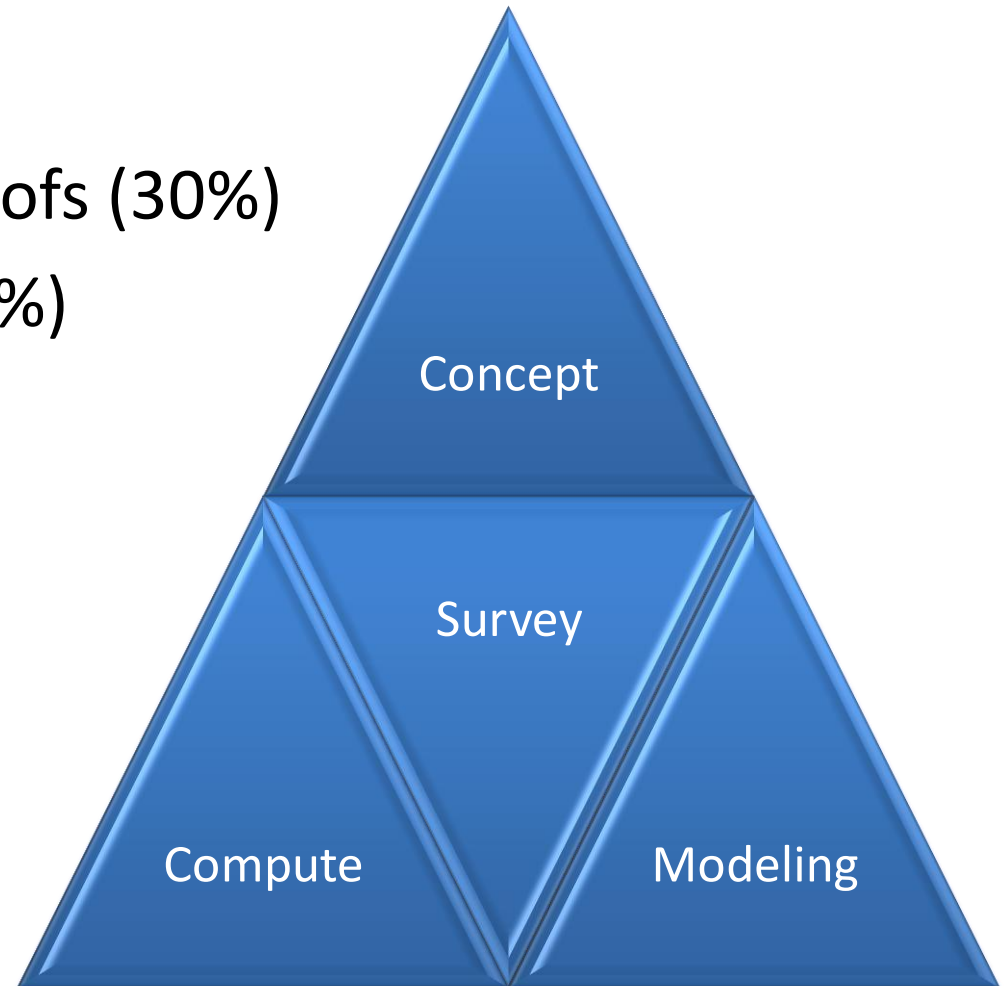
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Main purpose of the course

- To **improve** the mathematical theory of **self-cultivation**
 - Intuitively understand the mathematical thinking in the papers
- To **train skilled ability to express** mathematical concepts
 - How to formulate mathematical problems
 - How to abstract mathematical methods
- To **cultivate outstanding ability** to use mathematical models
 - The use of mathematical models to solve computer-related research issues

The final examination

- Question type
 - Concept (30%)
 - Calculation and proofs (30%)
 - Math modeling (30%)
 - Survey (10%)



孔子曰：智者乐山，仁者乐水

The final examination

- **Innovative Score System**

Type	Questions	Required	Discount of over-answered	Budget
Concept	6	3	50%	30
Computing	6	3	50%	30
Modeling	6	3	50%	30
Survey	3	1	50%	10
Total	21	10	---	100

The final examination

- Example of ISS: **normal**

Type	Answered	Required	Correct	Gain
Concept	3	3	3	30
Computing	3	3	2	20
Modeling	3	3	3	30
Survey	1	1	1	10
Total	11	10	9	90

The final examination

- Example of ISS: **love computing**

Type	Answered	Required	Correct	Gain
Concept	2	3	2	20
Computing	4	3	4=3+1	35
Modeling	3	3	3	30
Survey	1	1	1	10
Total	10	10	10	95

The final examination

- Example of ISS: **love concept**

Type	Answered	Required	Correct	Gain
Concept	6	3	$5=(3+2)$	40
Computing	1	3	1	10
Modeling	2	3	2	20
Survey	2	1	$2\sim(2-1)$	10
Total	11	10	9	80

Final Review

道可道，非常道

名可名，非常名

《道德经》开篇语

Similar course at top universities

- Princeton:
 - Mathematical methods in Computer Science
 - <http://www.cs.princeton.edu/~boaz/methods2003/>
 - 讲授图论、拓扑初步、线性规划、矩阵论、统计初步等
- Cambridge:
 - Mathematical methods for Computer Science
 - <http://www.cl.cam.ac.uk/teaching/0809/CST/node38.html>
 - 讲授傅立叶方法、小波分析、不等式与极限理论、Markov链等统计理论
- 均以计算机图形学、计算机视觉、图像处理、人工智能、人机交互等计算机科学中的问题为背景进行讲解

Our course

- Fundamentals of 4 math topics:
 - Statistical learning
 - Variational methods
 - Partial differential equations
 - Optimization methods

Concepts in Statistical learning

- What is machine learning?
- The categories of learning methods
 - Supervised learning
 - Unsupervised learning
- Fundamental statistical concept
 - Prior, likelihood, Posterior
 - Markov chain

Computing methods in learning

- Point estimations
 - Bayesian formula
 - Binary distribution, Gaussian distribution
- Classification
 - Naïve Bayesian classification, decision tree
 - SVM, boosting
- Clustering
 - K-means, MOG, spectral clustering
- Time variance data
 - Hidden Markov Chain
 - Karman filter

Data modeling

- Geometric description:
 - Dimensional reduction
 - Kernel methods
- Algebra description:
 - Classification v.s. regression
 - How to overcome over-fitting?

Concepts in variational methods

- variational problems:
 - 两点间的最短连线问题
 - 最速降线(brachistochrone)问题
 - 测地线(geodesic line)问题

变分法中的符号

- 给定函数 $y(x)$
 - 宗量: x
 - 函数: $y(x)$
 - 宗量的增量: Δx
 - 函数的增量:
 - $\Delta y = y(x + \Delta x) - y(x)$
 - 当两点无限接近:
 - $\Delta x \rightarrow dx, \Delta y \rightarrow dy$
 - 略去高阶微量:
 - $dy = y'(x)dx$
 - 当在 x 处取得函数极值
 - $dy = 0$
- 给定泛函 $\Pi(y)$
 - 宗量: y
 - 泛函: $\Pi(y)$
 - 函数的变分: δy
 - 泛函的变分: $\delta \Pi$
 - $\delta \Pi = \Pi(y + \delta y) - \Pi(y)$
 - 在计算 $\delta \Pi$ 时可以展开 $\Pi(y + \delta y)$ 中的被积函数只保留线性项
 - 当在 y 处取得泛函极值
 - $\delta \Pi = 0$

函数 $y(x)$ 在定义域内与 $y(x) + \delta y(x)$ 处处无限接近

Partial differential equation

- Different types of PDE:
 - Can you distinguish them?
 - Laplacian equation, Poisson equation
- Basic concepts:
 - Curve/surface representation
 - Parametric or implicit definition
 - Tangent, normal, curvature
 - Gradient, Divergence

Computing in PDE

- Laplacian operator
- Discrete operators used in PDE
- How to numerically solve special PDEs

- Applications of PDE

Optimization methods

- Linear methods
- Non-linear methods

How to prepare the final examination

读

Read

读书

Read book

读数学书

Read math book

很认真地读数学书

Read math book seriously