Polygonization of Implicit Surfaces

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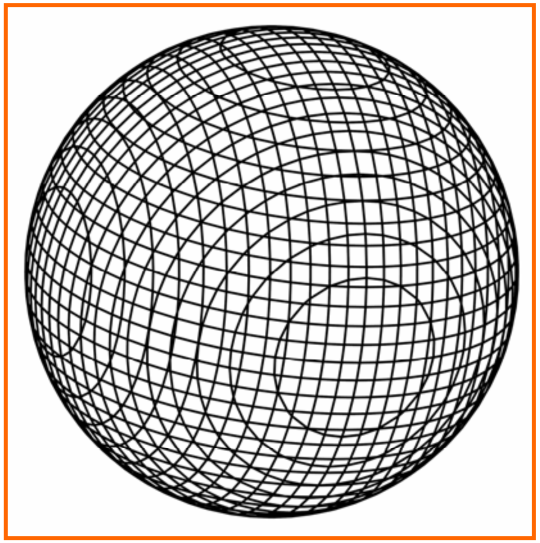
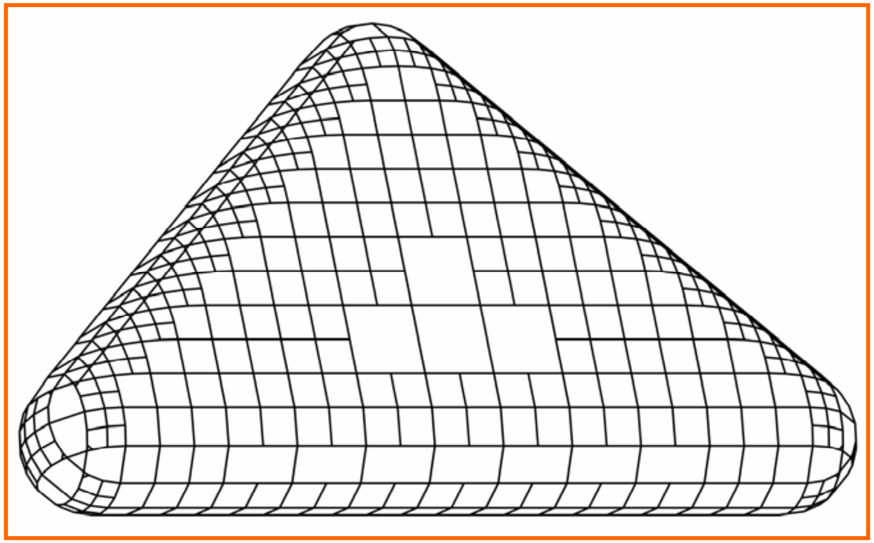
Polygonization of Implicit Surfaces

- Polygonization: conversion of implicit surface to polygonal mesh



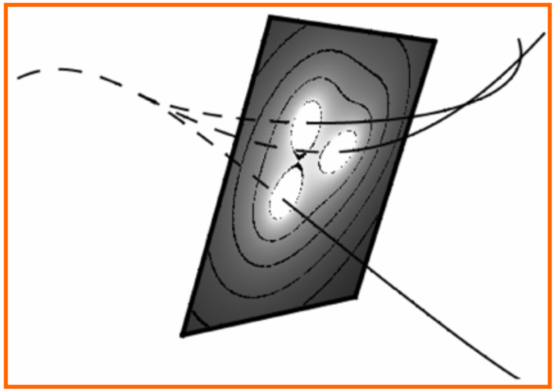
Uniform polygonization of a sphere



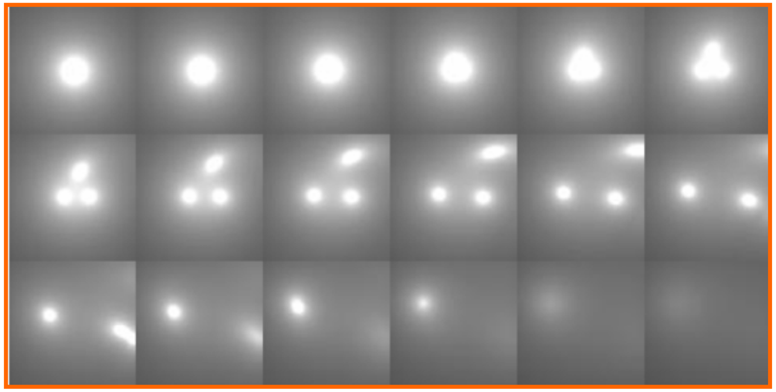
Non-uniform polygonization of an offset surface

Polygonization of Implicit Surfaces

- Purpose: design and rendering
  - Displaying implicit surface using polygons
  - Real-time approximate visualization method
    - Expensive voxel enumeration method: $O(n^3)$



Various contour levels are shown for distance surface to three space curves



A series of slices, in reading order, constitutes a fixed grid sampling.

Polygonization of Implicit Surfaces

- Two steps
  1. Partition space into cells
     - Uniform Partitioning
       - Converged Partitioning
       - Tracked Partitioning
     - Adaptive Partitioning
  2. Fit a polygon to surface in each cell
     - For Uniform Partitioning
     - For Adaptive Partitioning
- Ambiguity in Polygonization

Partition Space into Cells

- How do we know a cell contains the surface? $f(\mathbf{x})=0$
  - Straddling Cells
    - At least one vertex inside and outside surface
    - Non-straddling cells can still contain surface

Straddling Cells

Non-straddling Cells

Partition Space into Cells

- Guarantees
  - Interval analysis: numerical computation and error tracking
  - Lipschitz condition: extension of interval analysis

$$|f(\mathbf{x}) - f(\mathbf{y})| \leq \lambda ||\mathbf{x} - \mathbf{y}||$$

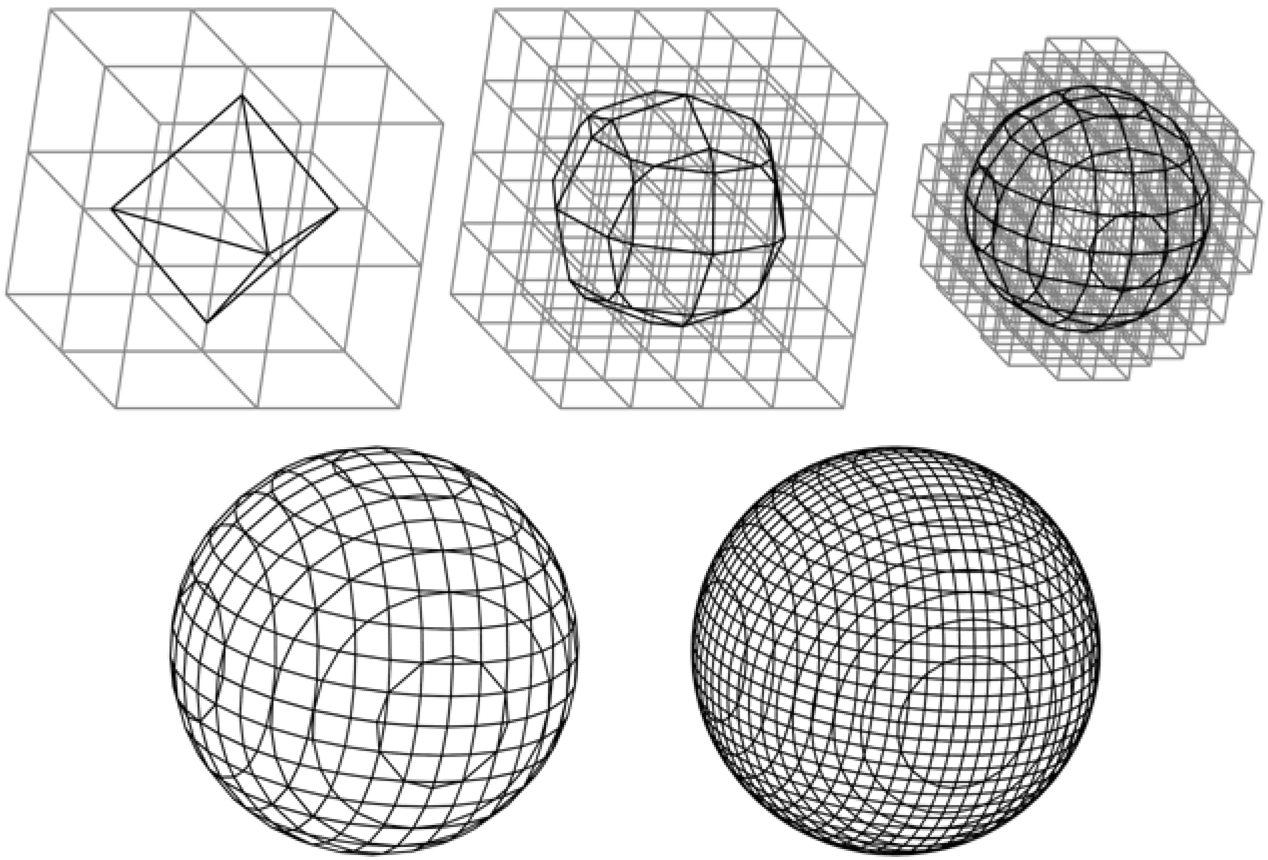
- If f is continuous, a root must exist along any edge connecting oppositely signed corners.

Converged Partitioning

1. Find a cube that bounds the surface
2. Subdivision of the cube: an octree
3. Recursive subdivision for each descendant cube
   - limiting depth
   - other criteria

- Accuracy: depth of octree

Example of Converged Partitioning

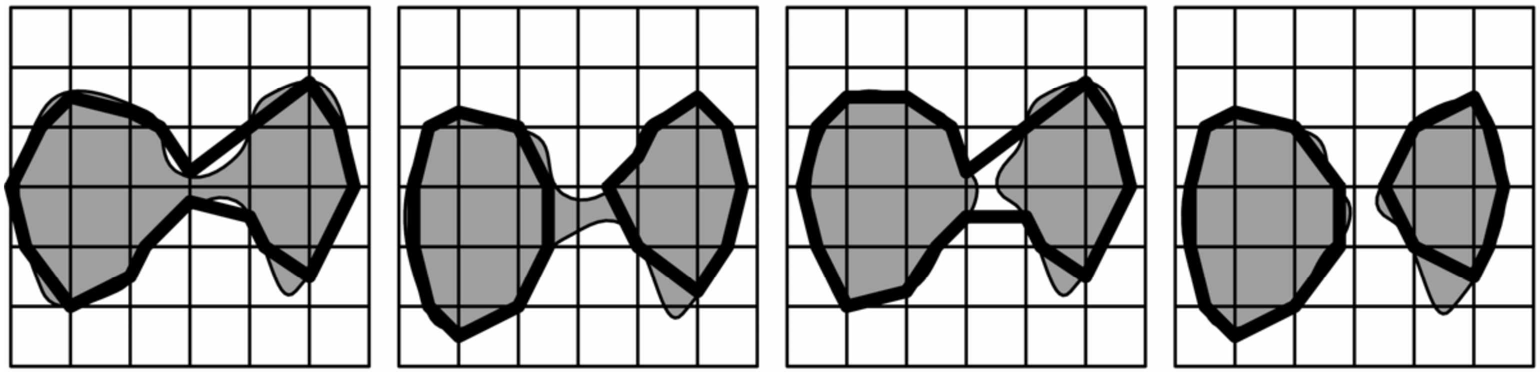


A sphere approximated from depths 1 through 5

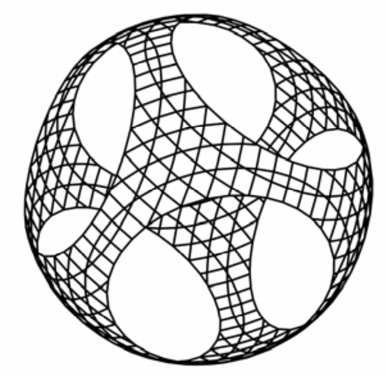
Converged Partitioning

- Advantage: simple
- Disadvantages
  - Capturing the detail?
  - Initial bounding box?
  - High computational cost

Converged Partitioning



(a) The topology of a connected implicit surface is correctly polygonized; (b) but a translated instance is not. (c) Two disjoint components are polygonized as a single component, (d) but a translated instance is polygonized properly.



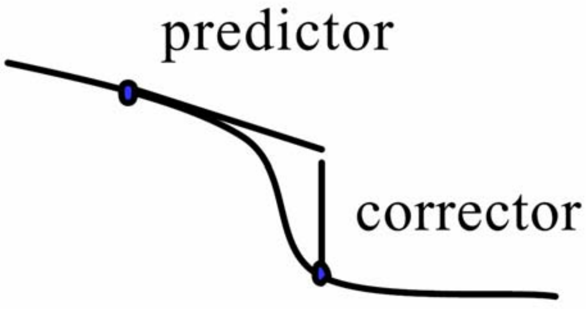
A sphere truncated by the root cube

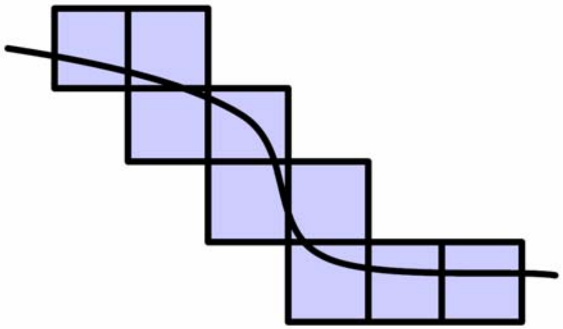
Tracked Partitioning

- Octree approach
  - An initial "seed cell" is selected by user or iteration
    - A seed cube for each disjoint object
  - New cells propagate along seed cell edges that intersect the surface.
  - The cells are stored as an adjacency graph

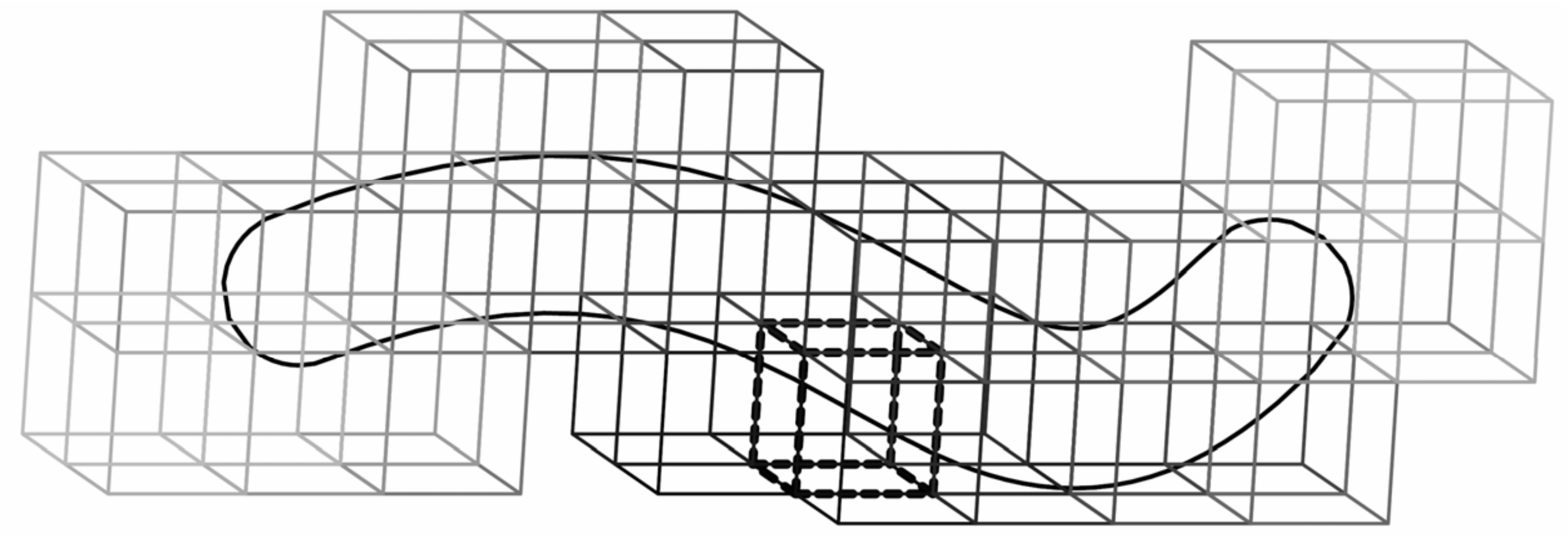
Tracked Partitioning

- Predictor-corrector
  - Extrapolate in tangent
  - Solve for surface location





Example of Tracked Partitioning

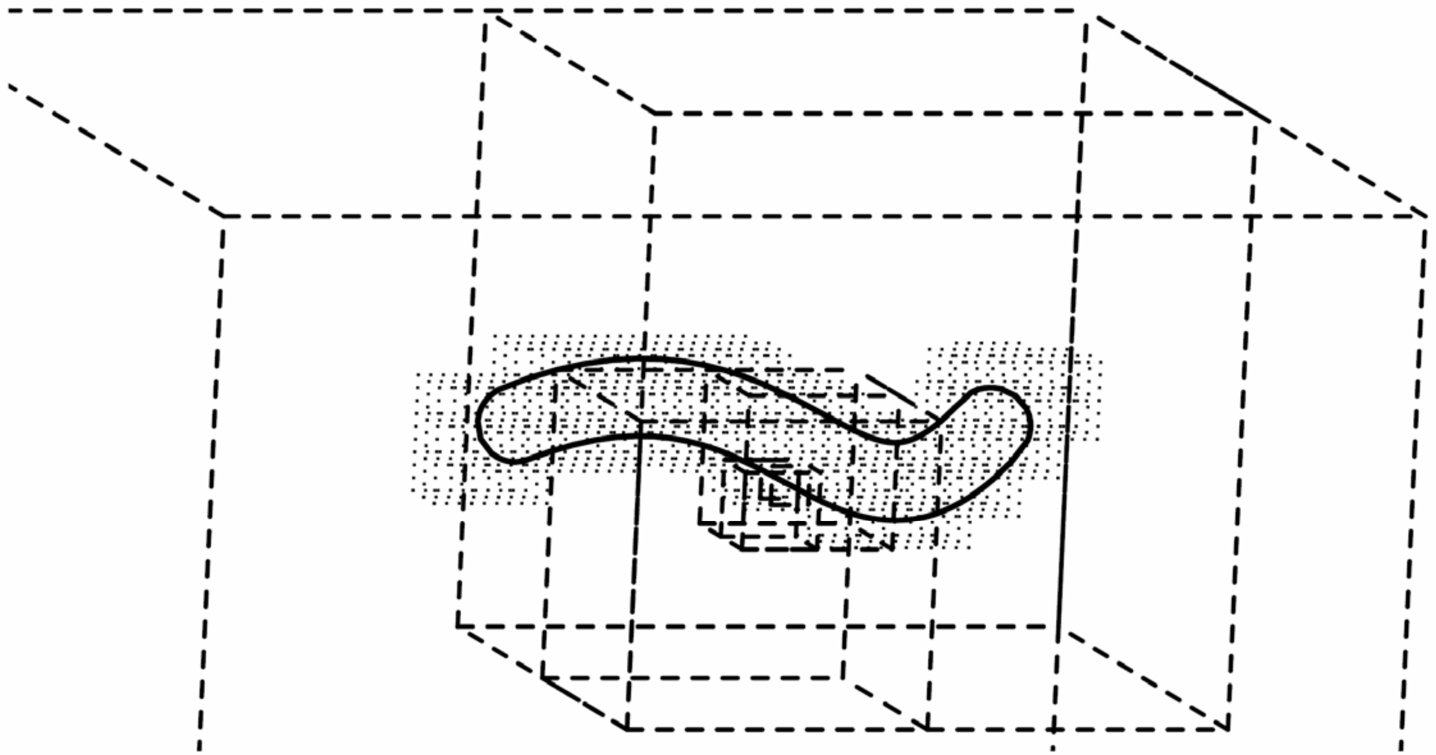


Surface tracking: darker cubes are older; the seed cube is dashed.

Tracked Partitioning

- Aggregated octree approach
  - As cells are added to the octree, new parent and root nodes are required
  - Octree neighbor finding techniques are used to prevent any redundant measuring or filling of space.

Example of Tracked Partitioning



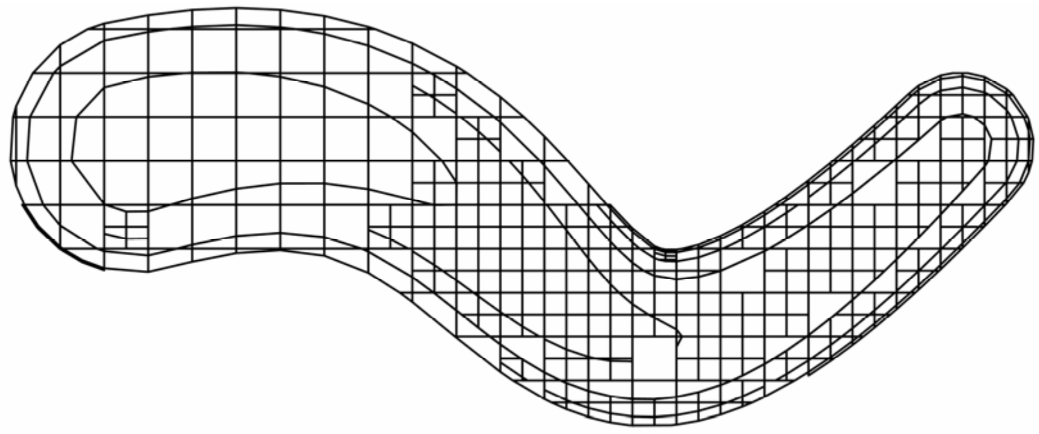
Creation of the aggregated octree; the expanding root nodes are dashed

Tracked Partitioning

- Advantage
  - Computational efficiency
- Disadvantage
  - How to choose seed cells? especially for the implicit surface with disjoint parts
  - How to capture detail?

Adaptive Partitioning

- Adaptively subdivide those cubes containing highly curved or intersecting surfaces



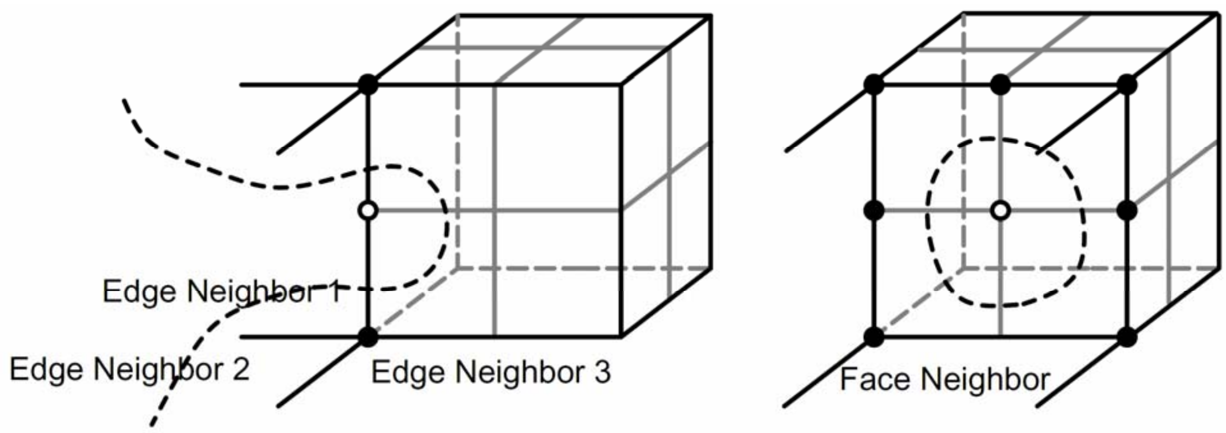
Adaptively subdivided generalized cylinder

Criteria for Subdivision of a Cube

1. Whether any edge of the cube intersects the surface
2. Whether a maximum subdivision depth or a minimum cube size has been reached,
3. Whether more than one polygon results from the cube
4. The planarity of the polygon
5. The divergence of vertex normals from the normal at the polygon center

Cube Edge Intersect Surface

- If the edge of a parent cube connects two equally signed corners and the midpoint is differently signed, then the three neighbors along that edge should be subdivided.
- For each face of a parent cube, if the four child corners that are midpoints of the four edges of the face all agree in sign but disagree with the center of the face, then the face neighbor should be subdivided. (otherwise, a hole will appear)



Planarity of Polygon

- The polygon vertices P_i, their unit normal $\boldsymbol{N_i}$, the unit normal of polygon center $\boldsymbol{N}$.
- The planarity of polygon is estimated by

$$\mathrm{Max}(V_i, \boldsymbol{N})$$

where $i \in [1, nPolygon]$, V_i is normalized vector P_iP_{i+1}

Divergence of Vertex Normals

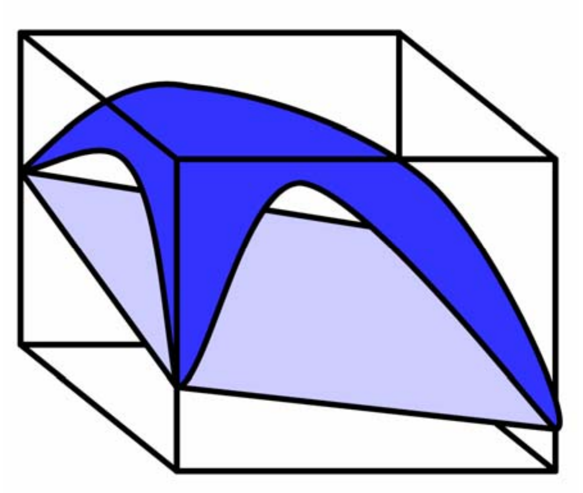
- The polygon vertices P_i, their unit normal $\boldsymbol{N_i}$, the unit normal of polygon center $\boldsymbol{N}$.
- The divergence of vertex normals is estimated by

$$\mathrm{Min}(\boldsymbol{N_i}, \boldsymbol{N})$$

where $i \in [1, nPolygon]$

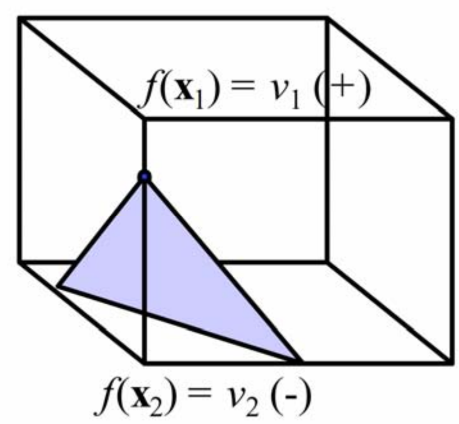
Fit a polygon to surface in each cell

- Cell contain surface: Piecewise-linear approximation (polygon)
- Intersections between surface and cell edges



Fit a polygon to surface in each cell

- Determine intersection on cell edges
  - Linear interpolation function values



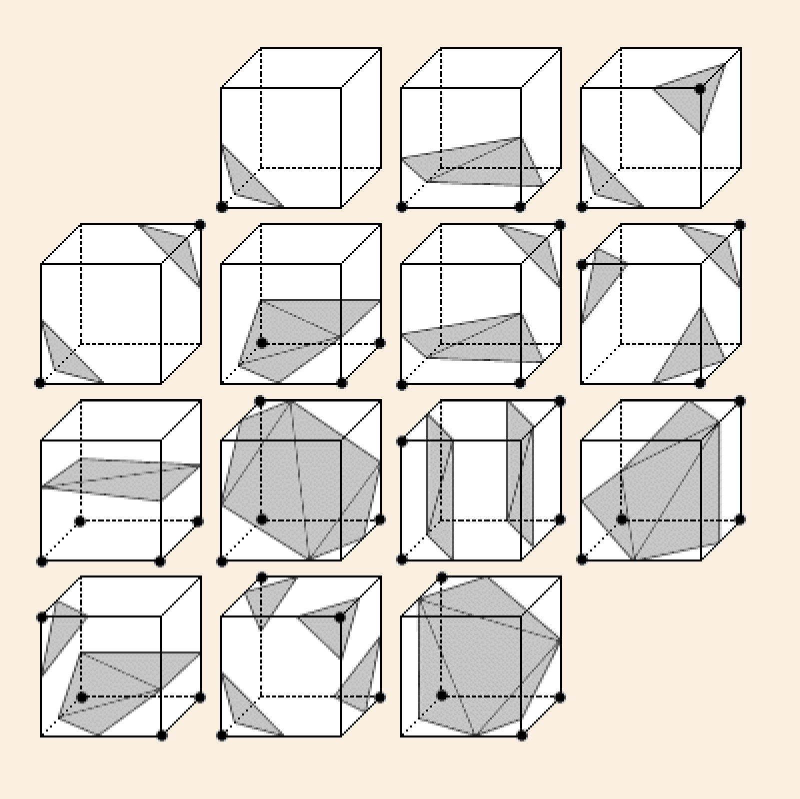
$$\mathbf{x} = \frac{v_1}{v_1 + v_2}\mathbf{x}_1 + \frac{v_2}{v_1 + v_2}\mathbf{x}_2$$

  - Find root of $f(\mathbf{r}(t))=0$ where $\mathbf{r}(t)=\mathbf{x}_1+t(\mathbf{x}_2-\mathbf{x}_1)$

Uniform Cells – Marching Cube (1)

- For each corner of cube, either + or – (256 cases)
  - Rotation around 3 axis
  - Mirroring about 3 axis
  - Inverting the state of all corners (flipping the normals)

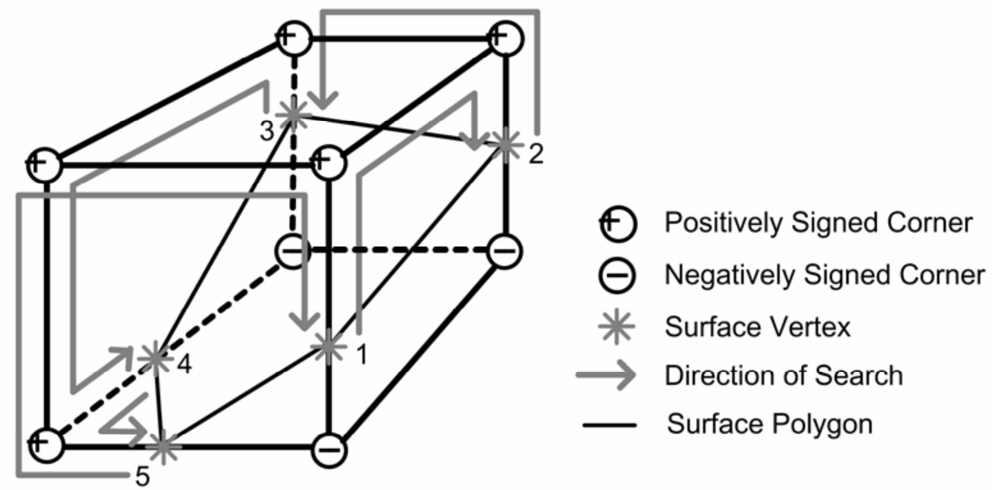
- Only 14 nontrivial cases left!



14 combinations

Uniform Cells – Method (2)

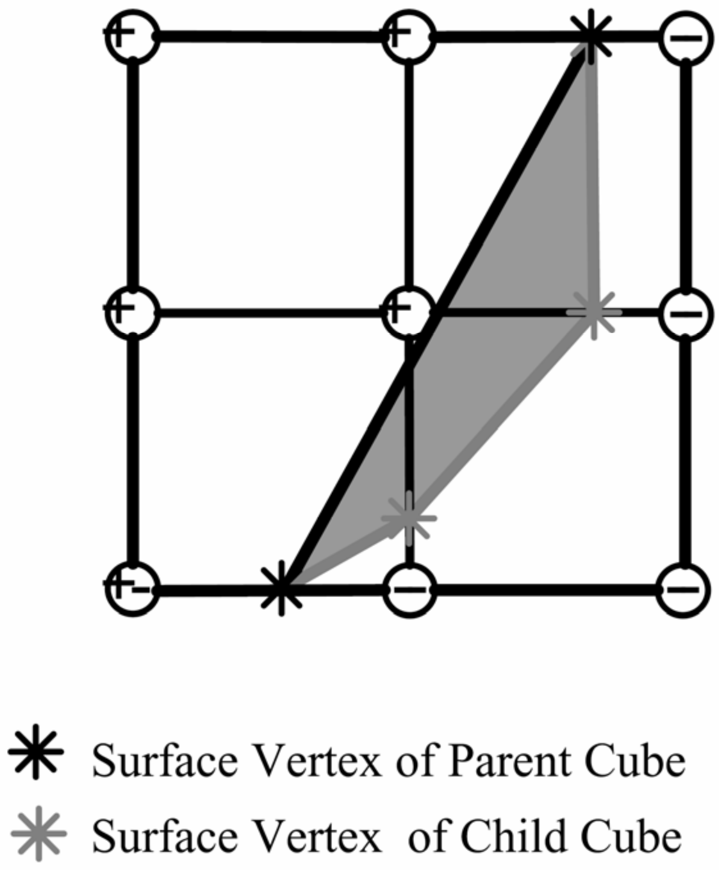
1. Beginning from any surface vertex
2. Going towards the positive corner
3. Clockwise until to the next surface vertex



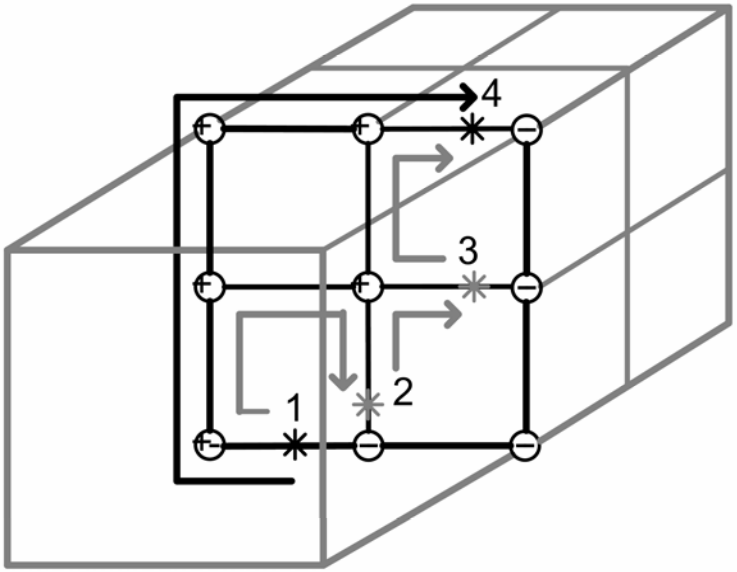
Algorithm to order vertices

Adaptive Cells

- A crack would occur without modification to the cube polygonization algorithm!

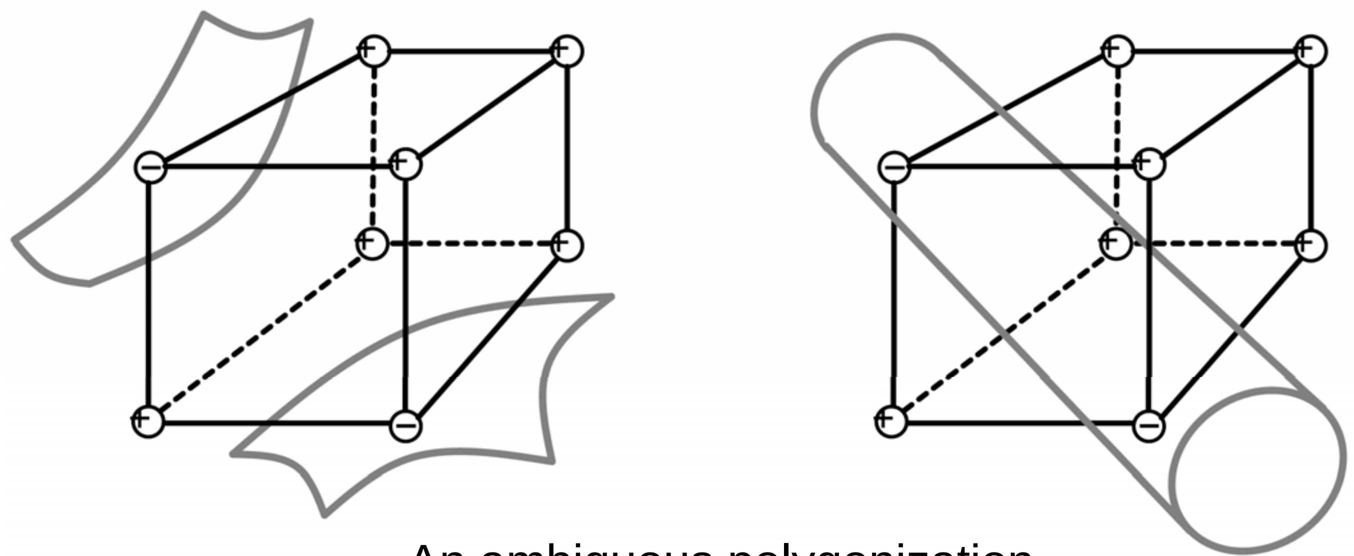


Adaptive Cells: modification



When proceeding around a face, if the face neighbor is more highly divided, proceed about the neighbor's face, reversing direction at every surface vertex, until reaching the next surface vertex located on an edge belonging to the original face.

Ambiguities in Polygonization



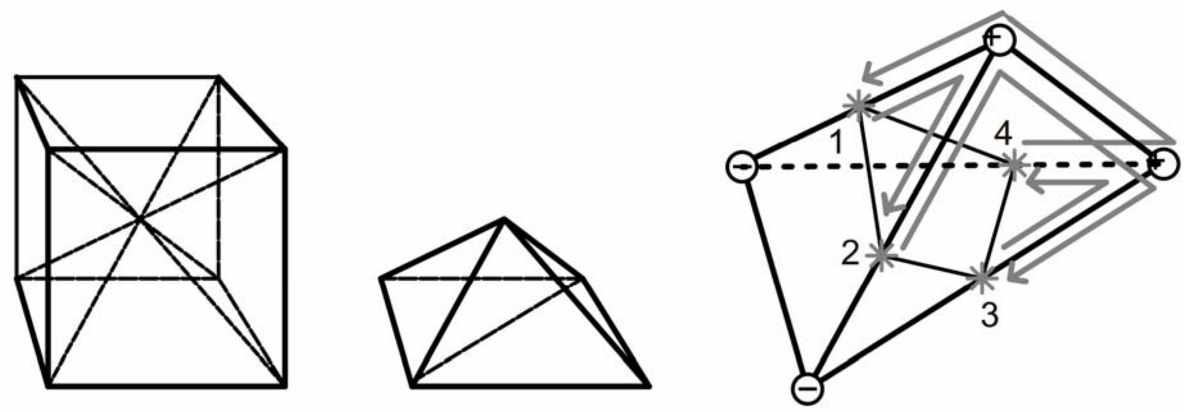
An ambiguous polygonization

Which one is right?

- For CT or MRI data application: both of them are right!
- For implicit surface application: only one is right!

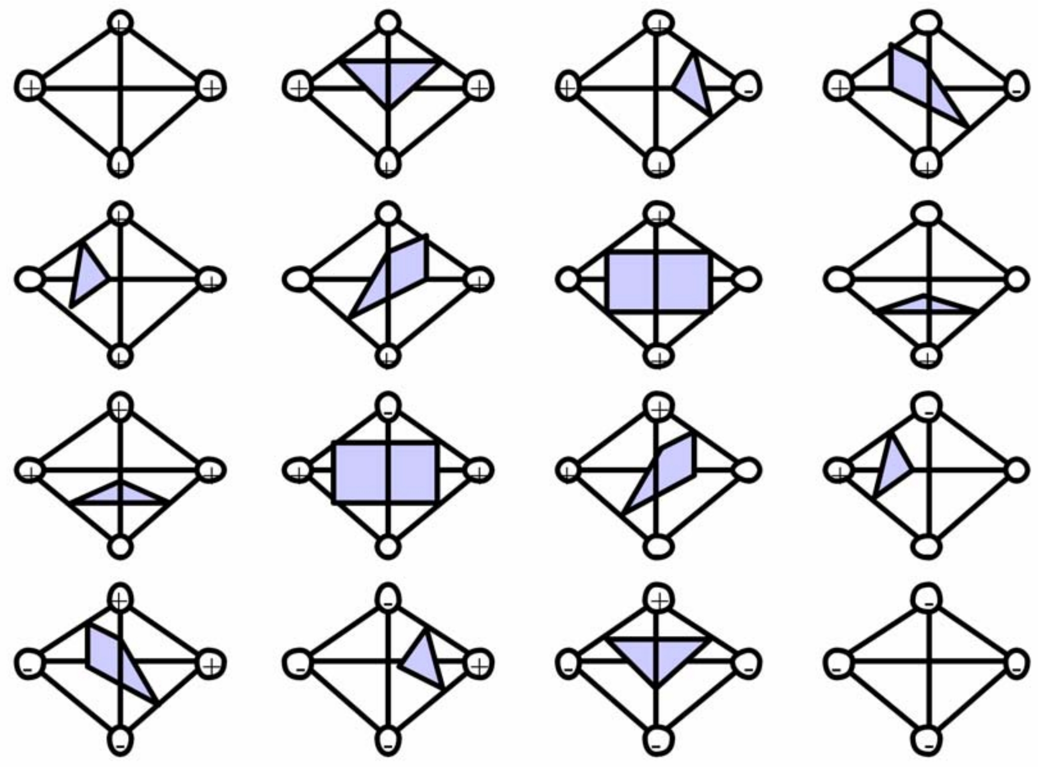
Ambiguities in Polygonization

- Solution: decomposing the cube into to 12 tetrahedra
- Fit a polygon to surface in each tetrahedra: as before



Fit a Polygon to Surface in each Tetrahedra

- Marching tetrahedra

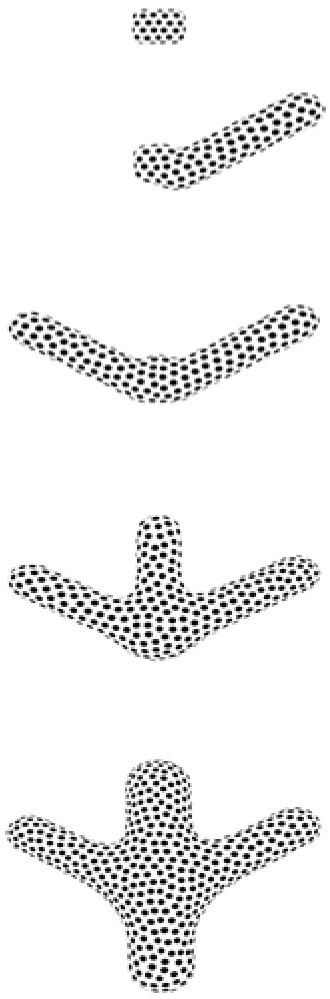


Other Methods for Displaying Implicit Surfaces

- Particle System
- Mesh Particles
- Ray Tracing

Particle System: By Heckbert94

- Constrain particle system to implicit surface
- Particles exert repulsion forces onto each other to spread out across surface
- Particles subdivide to fill open gaps
- Particles commit suicide if overcrowded
- Display particle as oriented disk

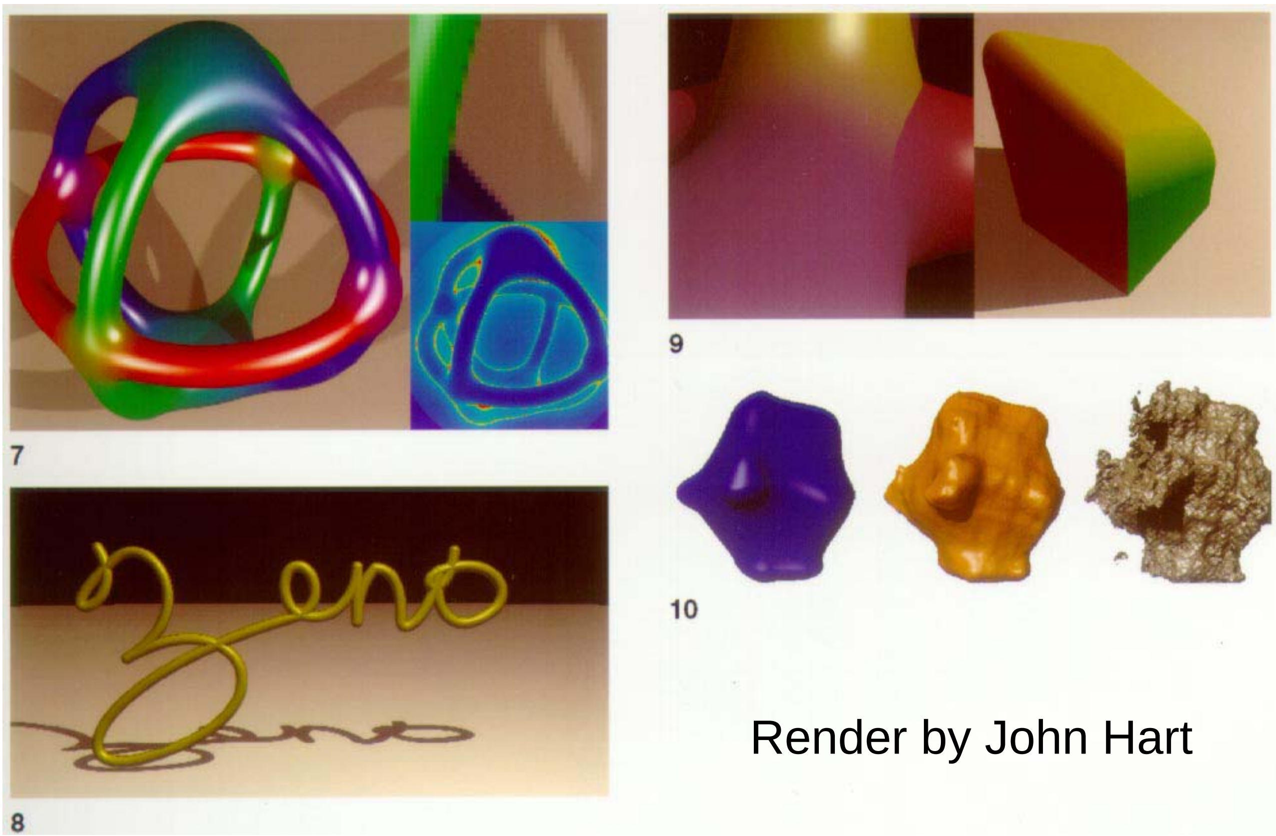


Mesh Particles: By Hart97

- Use particles as vertices
- Connect vertices into mesh
- Problems:
  - Which vertices should be connected?
  - How should vertices be reconnected when surface moves?
- Solution: Morse theory
- Track/find critical points of function to detect changes in topology of implicit surface

Ray Tracing

- Problem: Find root of $f(\mathbf{r}(t))=0$ robustly!



Render by John Hart