Computer Graphics 2015

4. Primitive Attributes

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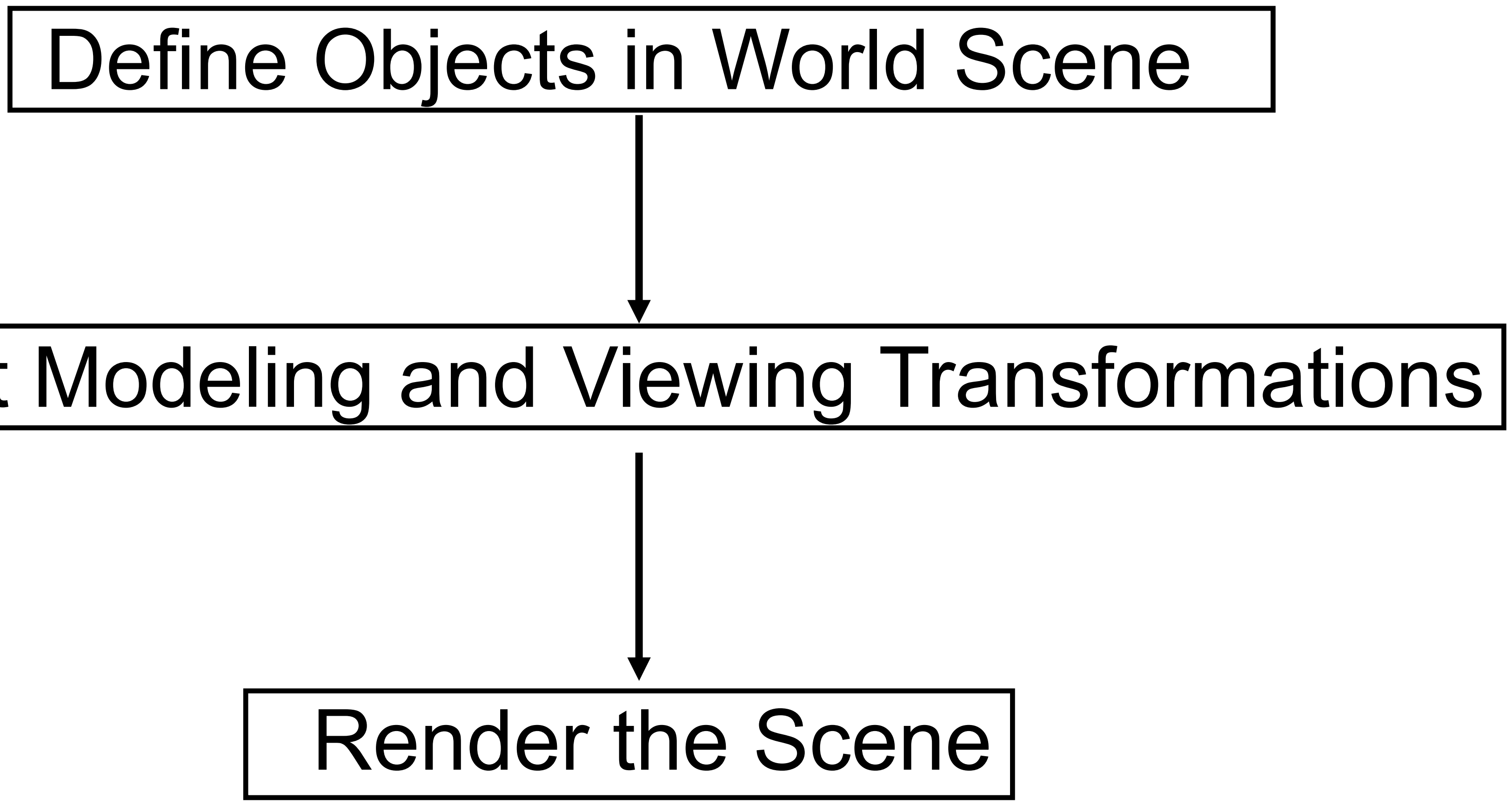
Previous lessons

- Rasterization
  - line
  - circle /ellipse ? => homework
- OpenGL and its rendering pipeline

3 Stages in OpenGL


Define Objects in World Scene
Set Modeling and Viewing Transformations
Render the Scene

Example Code

```
int main(int argc, char **argv)
{
    glutInit(&argc, argv);
    glutInitDisplayMode (
    GLUT_SINGLE | GLUT_RGB | GLUT_DEPTH);

    glutInitWindowPosition(100,100);
    glutInitWindowSize(300,300);
    glutCreateWindow ("square");

    glClearColor(0.0, 0.0, 0.0, 0.0);
    glMatrixMode(GL_PROJECTION);
    glLoadIdentity();
    glOrtho(0.0, 10.0, 0.0, 10.0, -1.0, 1.0);

    glutDisplayFunc(display);
    glutMainLoop();
    return 0;
}
```

```
void display(void)
{
    glClear( GL_COLOR_BUFFER_BIT);
    glColor3f(0.0, 1.0, 0.0);
    glBegin(GL_POLYGON);
        glVertex3f(2.0, 4.0, 0.0);
        glVertex3f(8.0, 4.0, 0.0);
        glVertex3f(8.0, 6.0, 0.0);
        glVertex3f(2.0, 6.0, 0.0);
    glEnd();
    glFlush();
}
```

Attribute parameters

- How to generate different display effects?
  - per primitive (C++)
  - system owns states (OpenGL)

- **OpenGL is a state machine!**

State parameters of OpenGL

- Attributes are assigned by OpenGL state functions:
  - color, matrix mode, buffer positions, Light ...
  - on state paras in this lesson

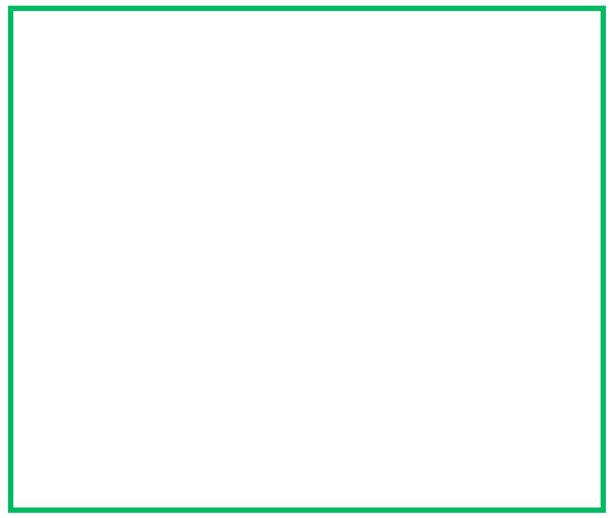
OpenGL Primitives

- GL_POINTS
- GL_LINES
- GL_LINE_STRIP
- GL_LINE_LOOP
- GL_TRIANGLES
- GL_QUADS
- GL_POLYGON
- GL_TRIANGLE_STRIP
- GL_TRIANGLE_FAN
- GL_QUAD_STRIP

1. GL_POLYGON and GL_TRIANGLE are the only ones in common usage
2. valid OpenGL polygons are closed, convex, co-planar and non-intersecting, which is always true for triangles!

Examples

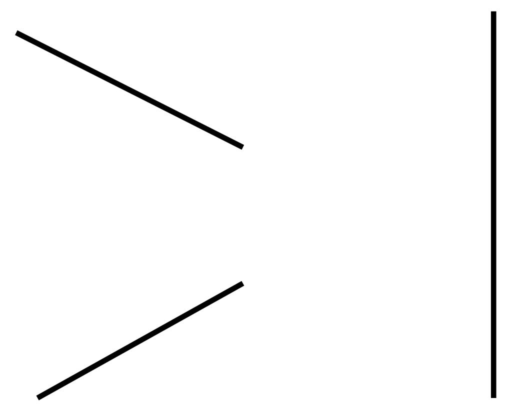
```
glBegin(GL_POLYGON);
        glVertex2i(0,0);
        glVertex2i(0,1);
        glVertex2i(1,1);
        glVertex2i(1,0);
glEnd() ;
```

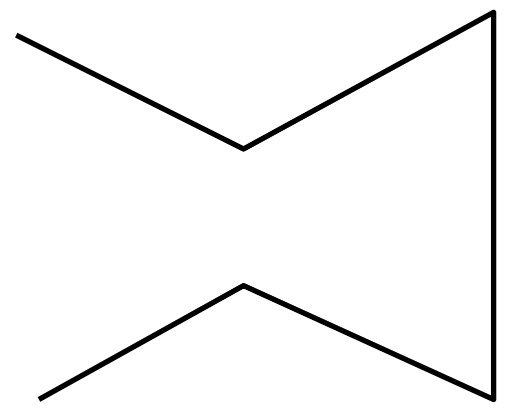


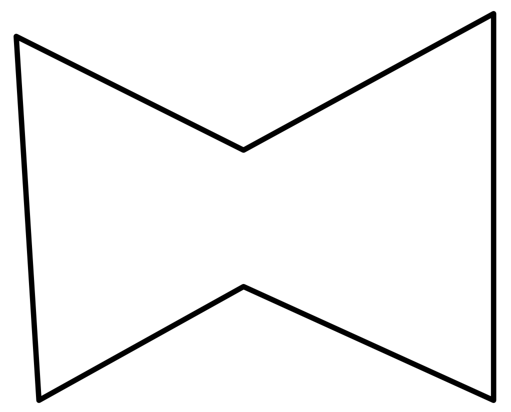
```
glBegin(GL_POINTS);
      glVertex2i(0,0);
      glVertex2i(0,1);
      glVertex2i(1,1);
      glVertex2i(1,0);
glEnd() ;
```

Examples

```
GLfloat list[6][2] ;

    glBegin(GL_LINES)
   for (int i = 0 ; i < 6 ;i++)
          glVertex2v(list[i]);
              glEnd() ;

  glBegin(GL_LINE_STRIP)
    for (int i = 0 ; i < 6 ;i++)
          glVertex2v(list[i]);
              glEnd() ;

  glBegin(GL_LINE_LOOP)
    for (int i = 0 ; i < 6 ;i++)
          glVertex2v(list[i]);
              glEnd() ;
```

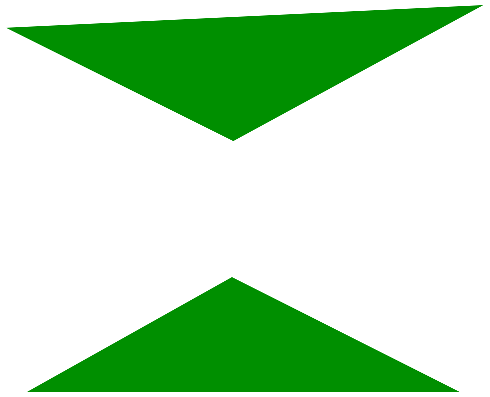




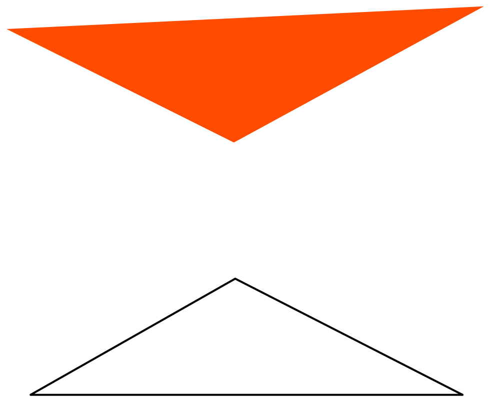


Examples

```
GLfloat list[6][2] ;

glColor3f(0.0, 1.0, 0.0);
glBegin(GL_TRIANGLES)
    for (int i = 0 ; i < 6 ;i++)
        glVertex2v(list[i]);
glEnd() ;
```

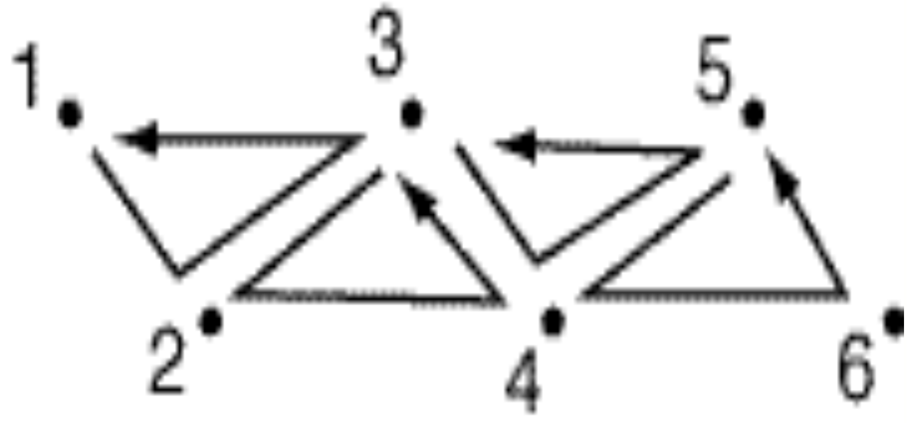


```
glBegin(GL_TRIANGLES)
    glColor3f(1.0, 0.0, 0.0);
    for ( i = 0 ; i < 3 ;i++)
        glVertex2v(list[i]);
    glColor3f(1.0, 1.0, 1.0);
    for ( i = 3 ; i < 6 ;i++)
        glVertex2v(list[i]);
glEnd() ;
```

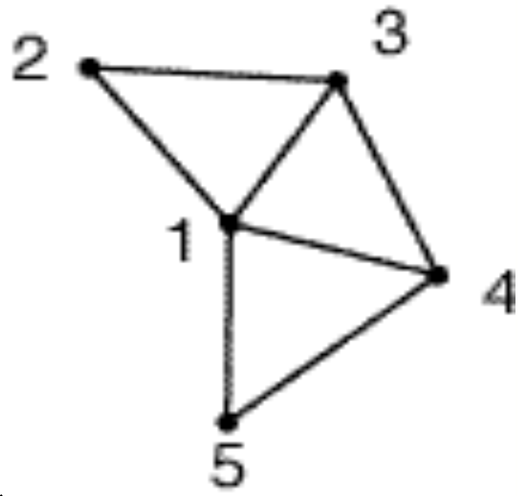


Examples

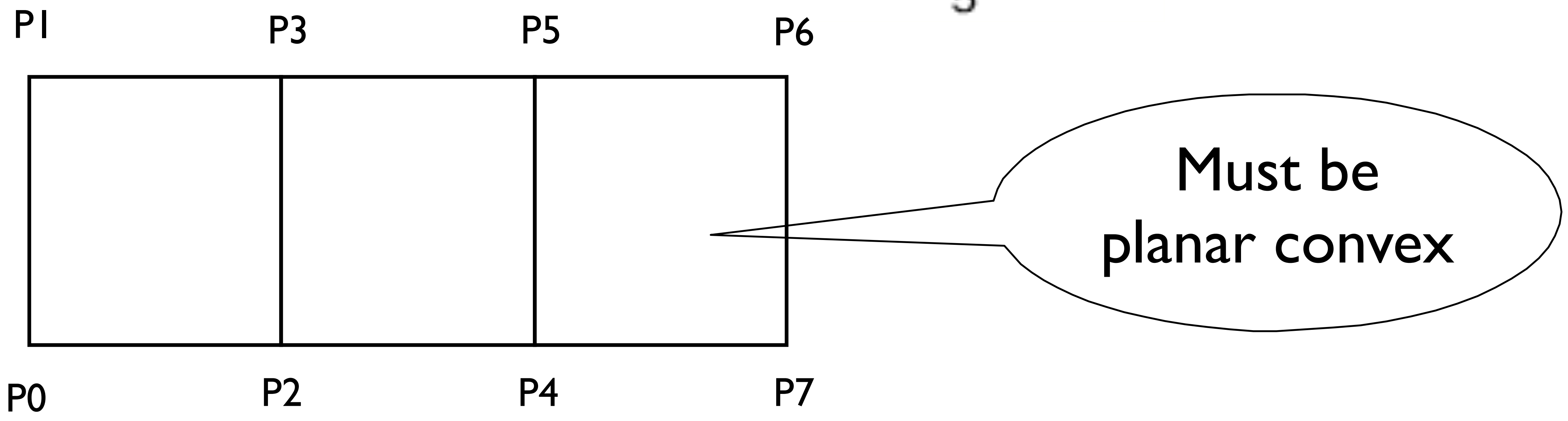
GL_TRIANGLE_STRIP



GL_TRIANGLE_FAN



GL_QUAD_STRIP



OpenGL Command Syntax

- All command names begin with gl
    - Ex.: glVertex3f(0.0, 1.0, 1.0);
- Constant names are in all uppercase
    - Ex.: GL_COLOR_BUFFER_BIT
- Data types begin with GL
    - Ex.: GLfloat onevertex[3];
- Most commands end in two characters that determine the data type of expected arguments
    - Ex.: glVertex3f(…) => 3 GLfloat arguments

glVertex

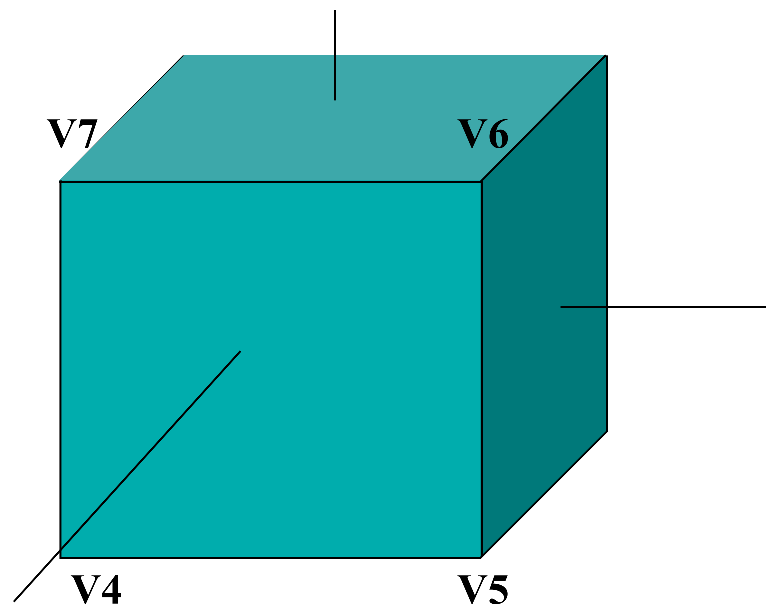
- All primitives are defined in terms of vertices
  - glVertex2f(x, y);
  - glVertex3f(x, y, z);
  - glVertex4f(x, y, z, w);
  - glVertex3fv(a); // with a[0], a[1], a[2]

Building Objects From Vertices

- Specify a primitive mode, and enclose a set of vertices in a glBegin / glEnd block
- glBegin(GL_POLYGON);
- glVertex3f(1.0, 2.0, 0.0);
- glVertex3f(0.0, 0.0, 0.0);
- glVertex3f(3.0, 0.0, 0.0);
- glVertex3f(3.0, 2.0, 0.0);
- glEnd();

OpenGL Example

```
void drawOneCubeface(size)
{
  static Glfloat v[8][3];
   v[0][0] = v[3][0] = v[4][0] = v[7][0] = -size/2.0;
   v[1][0] = v[2][0] = v[5][0] = v[6][0] =  size/2.0;
   v[0][1] = v[1][1] = v[4][1] = v[5][1] = -size/2.0;
   v[2][1] = v[3][1] = v[6][1] = v[7][1] =  size/2.0;
   v[0][2] = v[1][2] = v[2][2] = v[3][2] = -size/2.0;
   v[4][2] = v[5][2] = v[6][2] = v[7][2] =  size/2.0;

  glBegin(GL_POLYGON);
    glVertex3fv(v[0]);
    glVertex3fv(v[1]);
    glVertex3fv(v[2]);
    glVertex3fv(v[3]);
  glEnd();
}
```



Real examples in OpenGL|ES

```
float afVertices [] = {…};

glEnableVertexAttribArray(0);
glVertexAttribPointer(VERTEX_ARRAY,GL_FLOAT, GL_FALSE,afVertices);
glDrawArrays(GL_TRIANGLE_STRIP, 0, 4);
```

Note: there is no glBegin/glVertex/glEnd in OpenGL|ES

Colors

- OpenGL colors are typically defined as RGB components
    - each of which is a float in the range [0.0, 1.0]

- For the screen's background:
    - glClearColor(0.0, 0.0, 0.0); // black color
    - glClear(GL_COLOR_BUFFER_BIT);
- For objects:
    - glColor3f(1.0, 1.0, 1.0);   // white color

Other Commands in glBegin / glEnd blocks

- Not every OpenGL command can be located in such a block. Those that can include, among others:
    - glColor
    - glNormal (to define a normal vector)
    - glTexCoord (to define texture coordinates)
    - glMaterial (to set material properties)

Example

```
glBegin( GL_POLYGON );
    glColor3f( 1.0, 1.0, 0.0 ); glVertex3f( 0.0, 0.0, 0.0 );
    glColor3f( 0.0, 1.0, 1.0 ); glVertex3f( 5.0, 0.0, 0.0 );
    glColor3f( 1.0, 0.0, 1.0 ); glVertex3f( 0.0, 5.0, 0.0 );
glEnd();
```

Polygon Display Modes

- glPolygonMode(GLenum face, GLenum mode);
  - Faces: GL_FRONT, GL_BACK, GL_FRONT_AND_BACK
  - Modes: GL_FILL, GL_LINE, GL_POINT
  - By default, both the front and back face are drawn filled
- glFrontFace(GLenum mode);
  - Mode is either GL_CCW (default) or GL_CW
- glCullFace(Glenum mode);
  - Mode is either GL_FRONT, GL_BACK, GL_FRONT_AND_BACK;
- You must enable and disable culling with
  - glEnable(GL_CULL_FACE) or glDisable(GL_CULL_FACE);

Drawing Other Objects

- GLU contains calls to draw cylinders, cones and more complex surfaces called NURBS
- GLUT contains calls to draw spheres and cubes

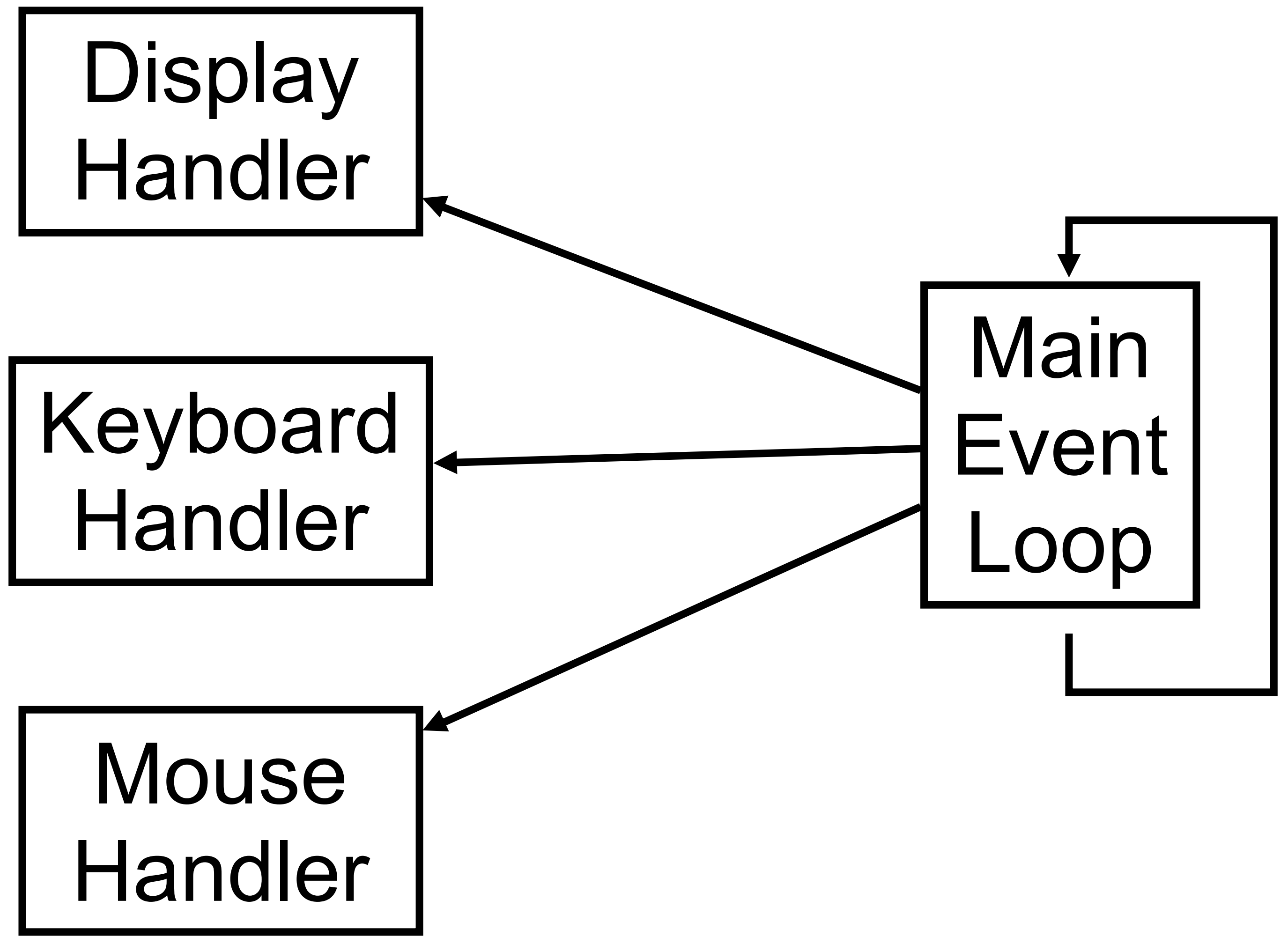
Compiling OpenGL Programs

- To use GLUT :
  - #include <GL/glut.h>
  - This takes care of every other include you need
  - Make sure that glut.lib (or glut32.lib) is in your compiler's library directory, and that the object module or DLL is also available
- See *OpenGL Game Programming* or online tutorials for details

Structure of GLUT-Assisted Programs

- GLUT relies on user-defined callback functions, which it calls whenever some event occurs
    - Function to display the screen
    - Function to resize the viewport
    - Functions to handle keyboard and mouse events

Event Driven Programming



Simple GLUT Example

Displaying a square

```
int main (int argc,  char *argv[])
{
   glutInit(&argc, argv);
   glutInitDisplayMode(GLUT_RGBA | GLUT_DOUBLE);

   int windowHandle
       = glutCreateWindow("Simple GLUT App");

   glutDisplayFunc(redraw);
   glutMainLoop();

   return 0;
}
```

Display Callback

Called when window is redrawn

```
void redraw()
{
  glClear(GL_COLOR_BUFFER_BIT);

  glBegin(GL_QUADS);
  glColor3f(1, 0, 0);
    glVertex3f(-0.5,  0.5, 0.5);
    glVertex3f( 0.5,  0.5, 0.5);
    glVertex3f( 0.5, -0.5, 0.5);
    glVertex3f(-0.5, -0.5, 0.5);
  glEnd(); // GL_QUADS

  glutSwapBuffers();
}
```

More GLUT

Additional GLUT functions

```
glutPositionWindow(int x,int y);
glutReshapeWindow(int w, int h);
```

Additional callback functions

```
glutReshapeFunction(reshape);
glutMouseFunction(mousebutton);
glutMotionFunction(motion);
glutKeyboardFunction(keyboardCB);
glutSpecialFunction(special);
glutIdleFunction(animate);
```

Reshape Callback

Called when the window is resized

```
void reshape(int w, int h)
{
   glViewport(0.0,0.0,w,h);

   glMatrixMode(GL_PROJECTION);
   glLoadIdentity();
   glOrtho(0.0,w,0.0,h, -1.0, 1.0);

   glMatrixMode(GL_MODELVIEW);
   glLoadIdentity();
}
```

Mouse Callbacks

Called when the mouse button is pressed

```
void mousebutton(int button, int state, int x, int y)
{
   if (button==GLUT_LEFT_BUTTON && state==GLUT_DOWN)
   {
      rx = x; ry = winHeight - y;
   }
}
```

Called when the mouse is moved with button down

```
void motion(int x, int y)
{
   rx = x; ry = winHeight - y;
}
```

Keyboard Callbacks

Called when a button is pressed

```
void keyboardCB(unsigned char key, int x, int y)
{
  switch(key)
  { case 'a': cout<<"a Pressed"<<endl; break; }
}
```

Called when a special button is pressed

```
void special(int key, int x, int y)
{
  switch(key)
  { case GLUT_F1_KEY:
      cout<<“F1 Pressed"<<endl; break; }
}
```

OpenGL – GLUT Example

```
#include <gl/glut.h>
#include <stdlib.h>
static GLfloat spin = 0.0;
void init( void )
{
  glClearColor( 0.0, 0.0, 0.0, 0.0 );
  glShadeModel( GL_FLAT );
}
```

```
void display( void )
{
glClear( GL_COLOR_BUFFER_BIT );
glPushMatrix();
glRotatef( spin, 0.0, 0.0, 1.0 );
glColor3f( 1.0, 1.0, 1.0 );
glRectf( -25.0, -25.0, 25.0, 25.0 );
glPopMatrix();
glutSwapBuffers();
}
```

OpenGL – GLUT Example

```
void spinDisplay( void )
{
    spin += 2.0;
    if( spin > 360.0 )
    spin -= 360.0;
    glutPostRedisplay();
}
```

```
void reshape( int w, int h )
{
    glViewport( 0, 0, (GLsizei) w, (GLsizei)
    h );
    glMatrixMode( GL_PROJECTION );
    glLoadIdentity();
    glOrtho( -50.0, 50.0, -50.0, 50.0, -1.0, 1.0 );
    glMatrixMode( GL_MODELVIEW );
    glLoadIdentity();
}
```

OpenGL – GLUT Example

```
void mouse( int button, int state, int x, int y )
{
      switch( button )
      {
       case GLUT_LEFT_BUTTON:
              if( state == GLUT_DOWN )
                     glutIdleFunc( spinDisplay );
              break;
       case GLUT_RIGHT_BUTTON:
              if( state == GLUT_DOWN )
                     glutIdleFunc( NULL );
              break;
       default:      break;
      }
}
```

OpenGL – GLUT Example

```
int main( int argc, char ** argv )
{
    glutInit( &argc, argv );
    glutInitDisplayMode( GLUT_DOUBLE | GLUT_RGB );
    glutInitWindowSize( 250, 250 );
    glutInitWindowPosition( 100, 100 );
    glutCreateWindow( argv[ 0 ] );

    init();
    glutDisplayFunc( display );
    glutReshapeFunc( reshape );
    glutMouseFunc( mouse );
    glutMainLoop();
    return 0;
}
```

Web Resources

http://www.opengl.org

http://nehe.gamedev.net

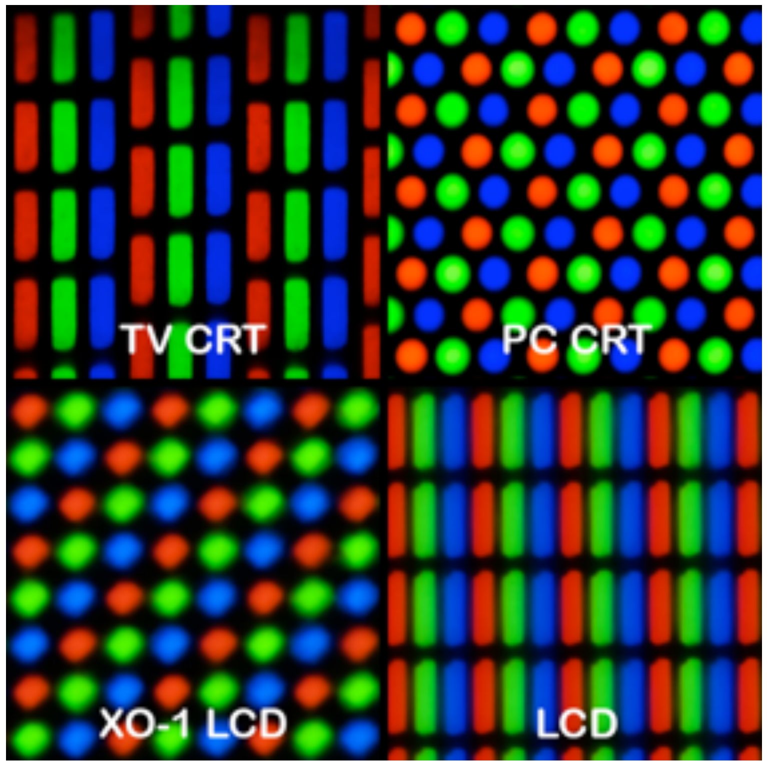
http://www.xmission.com/~nate/glut.html

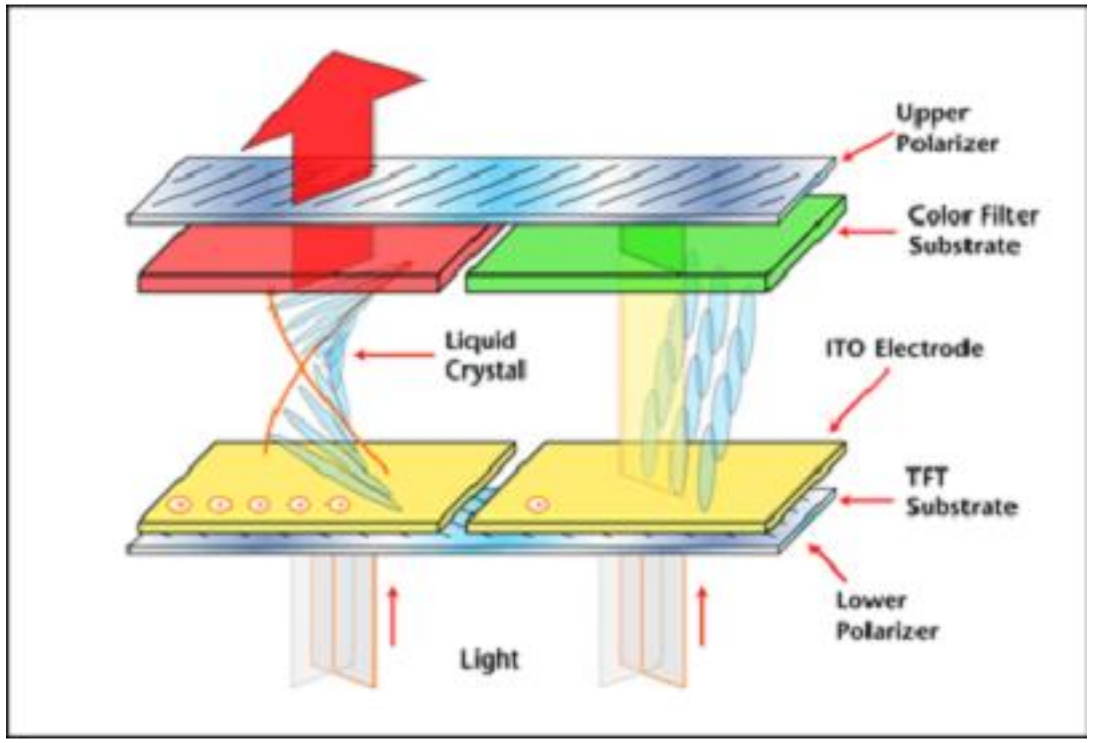
Color and greyscale

- Color is a fundamental primitive attribute

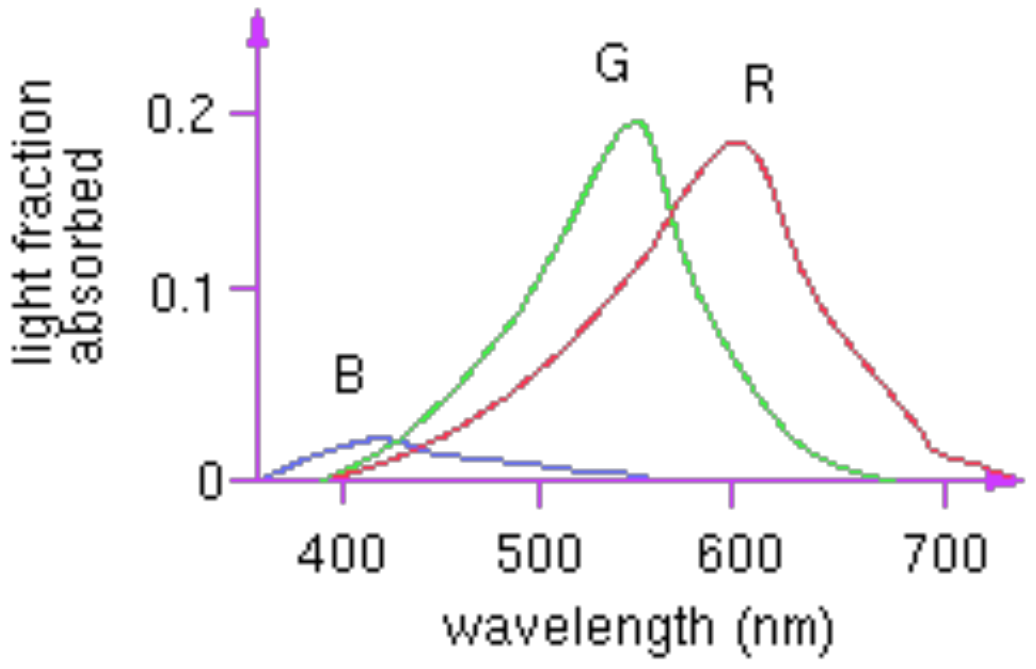
- RGB color model
- Color lookup table / Color map
- Greyscale

Why RGB?

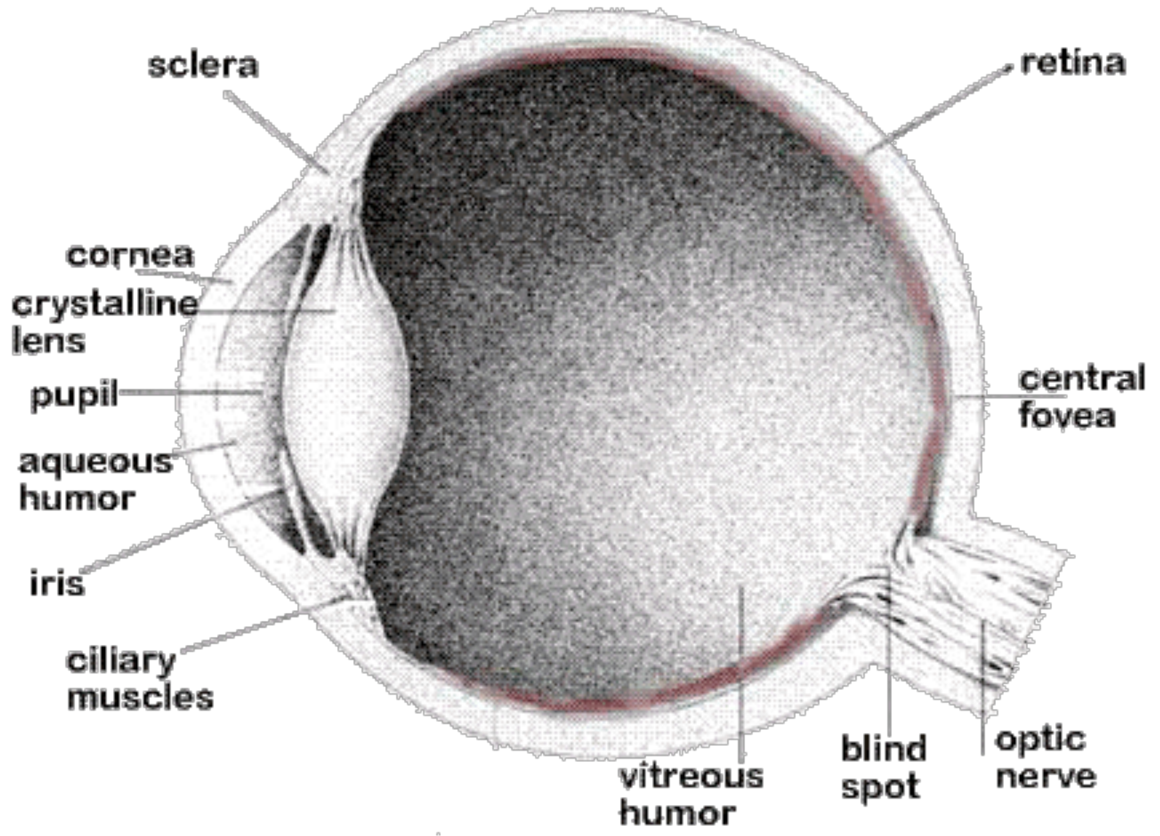

TV CRT
PC CRT
XO-1 LCD
LCD


Upper Polarizer
Color Filter Substrate
Liquid Crystal
ITO Electrode
TFT Substrate
Lower Polarizer
Light

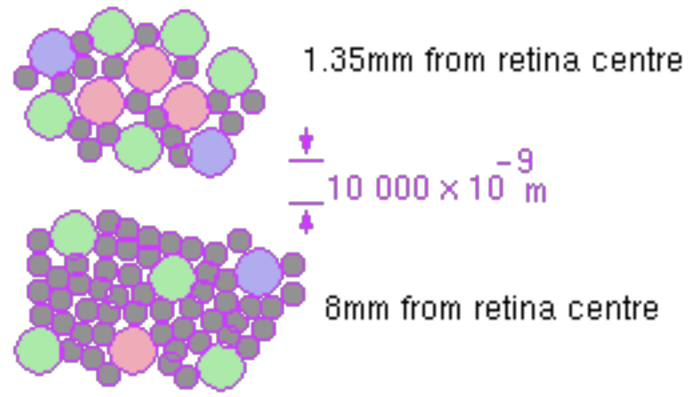
Color Model



light fraction absorbed
0.2
0.1
0
G
R
B
400
500
600
700
wavelength (nm)



sclera
retina
cornea
crystalline lens
pupil
aqueous humor
iris
ciliary muscles
central fovea
vitreous humor
blind spot
optic nerve



1.35mm from retina centre
$10\ 000 \times 10^{-9}$ m
8mm from retina centre

Color perception

- Three types of cones:

S	M	L	
Blue	Green	Red	roughly approximate
430nm	560nm	610nm	peak sensitivities

- Colorblindness results from a deficiency of one cone type.

OpenGL Color function

- GLUT_RGB and
- GLUT_RGBA with alpha channel

- glColor3f (1.0, 1.0, 1.0);
- glColor3i (0, 255, 255);
- glColor3fv (colorArray);

OpenGL Color function

- Color index mode
  - glIndexi (196);
- Color blending function
  - glEnable (GL_BLEND);
  - glDisable (GL_BLEND);
  - glBlendFunc (sFactor, dFactor);

OpenGL Color Array

- Defined in the latest OpenGL standard
  - glEnableClientState (GL_COLOR_ARRAY);
  - glColorPointer (...);

  - glEnableClientState (GL_VERTEX_ARRAY);
  - glVertexPointer (...);

Attributes of

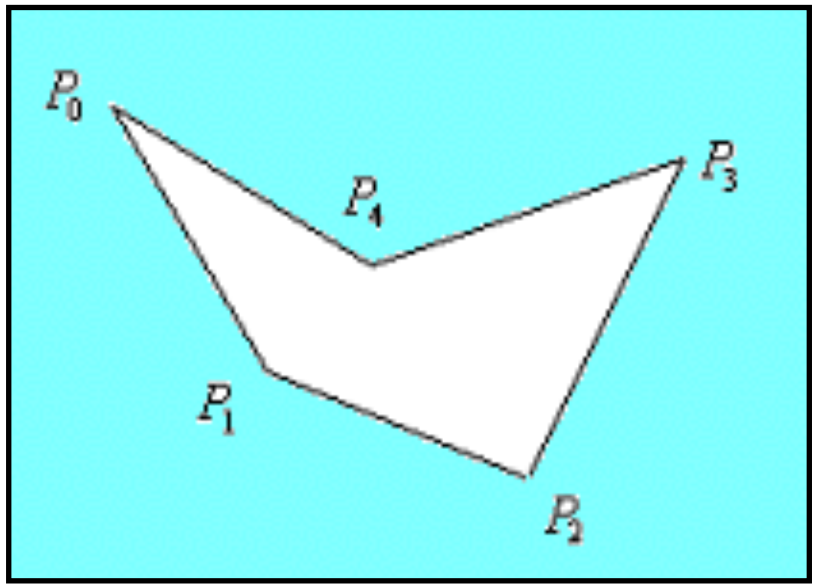
- Point
    - Size and Color
- Line
    - line width
    - line style
    - brush

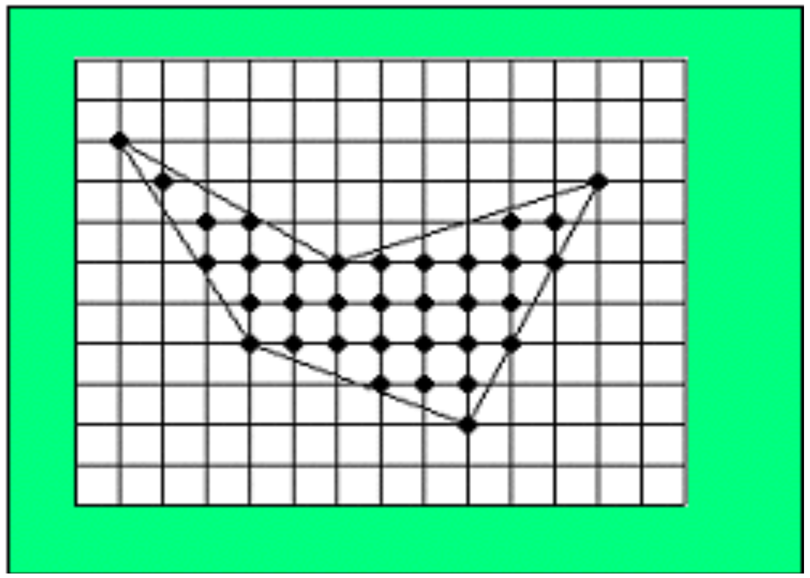
Region attributes

- defined by a planar polygon
  - filling style:
    - wireframe,
    - fill,
    - tiling pattern

Polygon filling

- Polygon representation

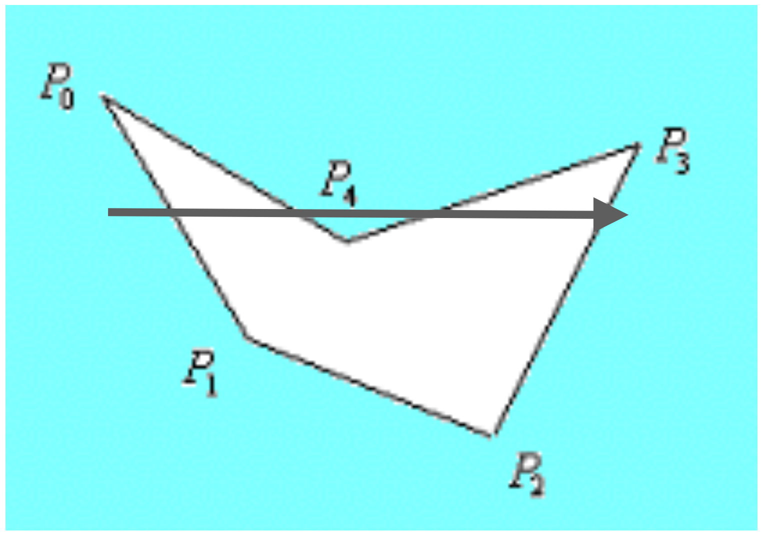




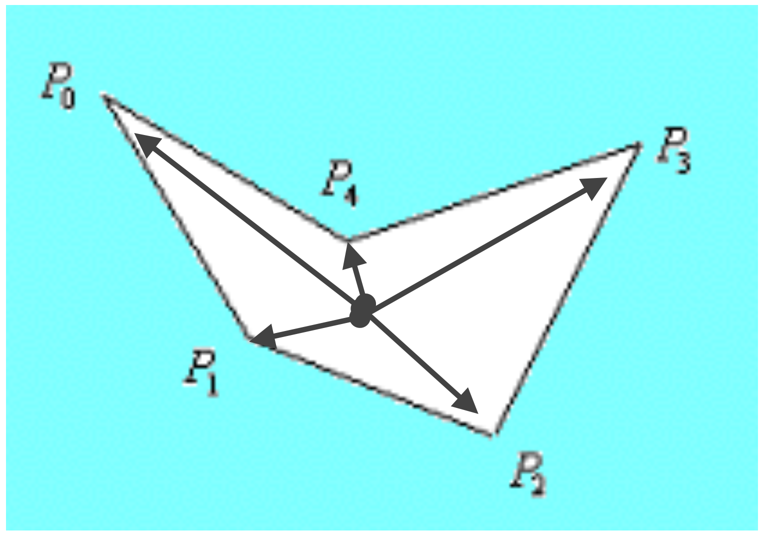
-
- By vertex By lattice
- Polygon filling:
- vertex representation vs lattice representation

Polygon filling

- fill a polygonal area → test every pixel in the raster to see if it lies inside the polygon.



even-odd test



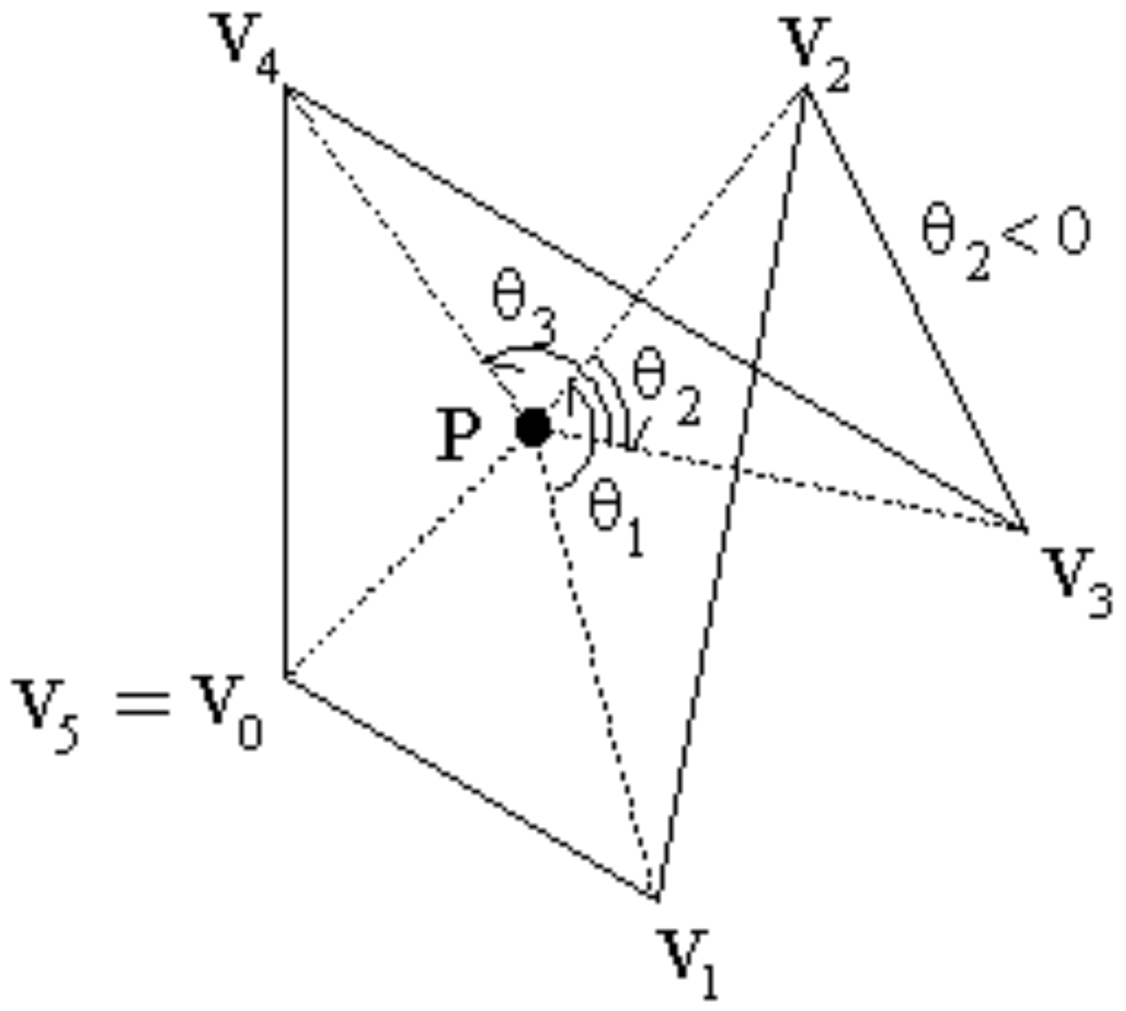
winding number test

Question5: How to Judge…?

Inside check

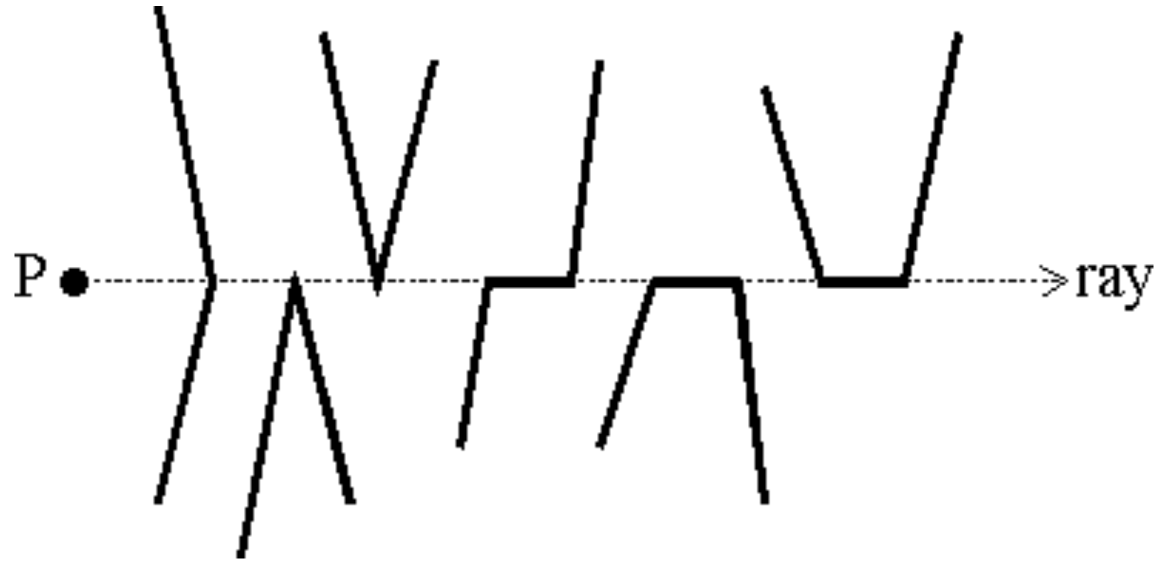
$$\mathbf{wn} = \frac{1}{2\pi}\sum_{i=0}^{n-1}\theta_i$$

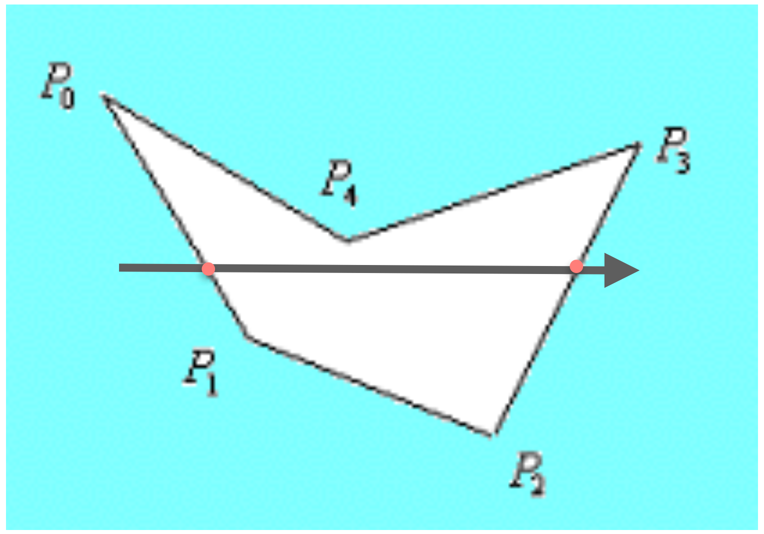
$$= \frac{1}{2\pi}\sum_{i=0}^{n-1}\arccos\left(\frac{\mathbf{PV}_i \cdot \mathbf{PV}_{i+1}}{|\mathbf{PV}_i|\,|\mathbf{PV}_{i+1}|}\right)$$



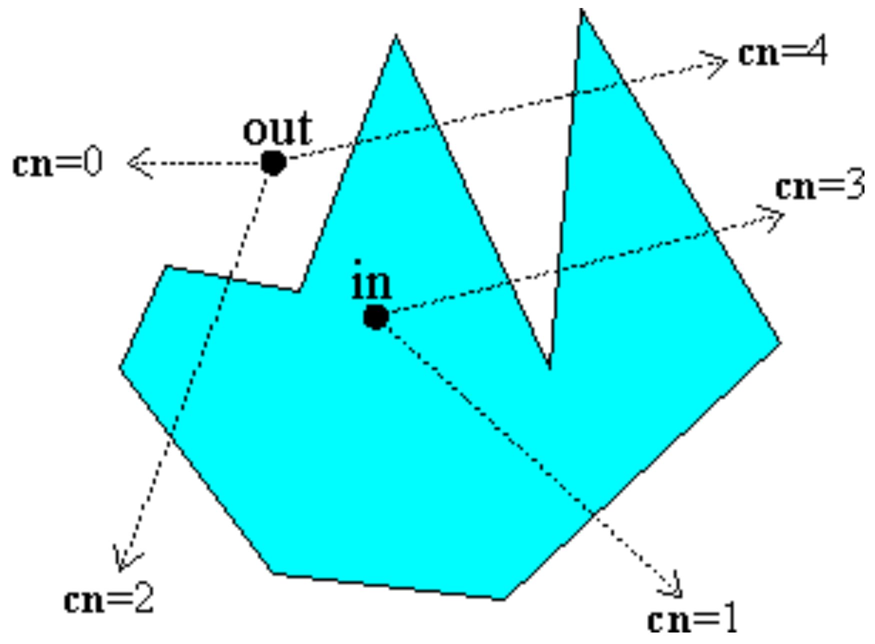
Question6: How to improve …?

Inside check





even-odd test

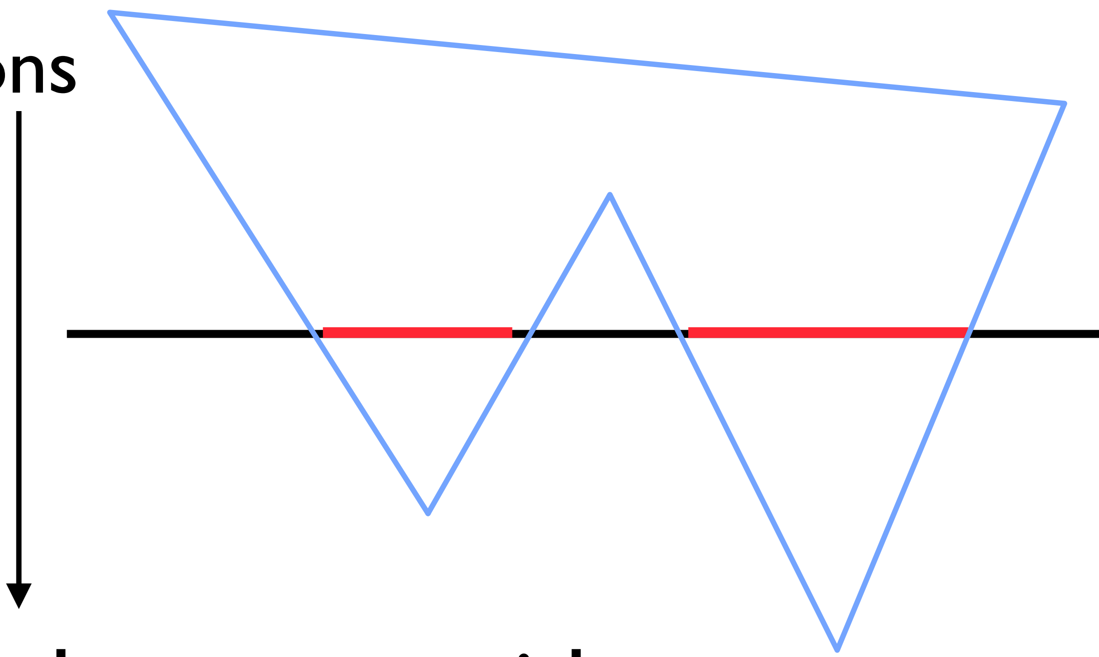


Scan Line Methods

- Makes use of the *coherence* properties
  - Spatial coherence : Except at the boundary edges, adjacent pixels are likely to have the same characteristics
  - Scan line coherence : Pixels in the adjacent scan lines are likely to have the same characteristics
- Uses intersections between area boundaries and scan lines to identify pixels that are inside the area

Scan Line Method

- Proceeding from left to right the intersections are paired and intervening pixels are set to the specified intensity
- Algorithm
    - Find the intersections of the scan line with all the edges in the polygon
    - Sort the intersections by increasing X-coordinates
    - Fill the pixels between pair of intersections



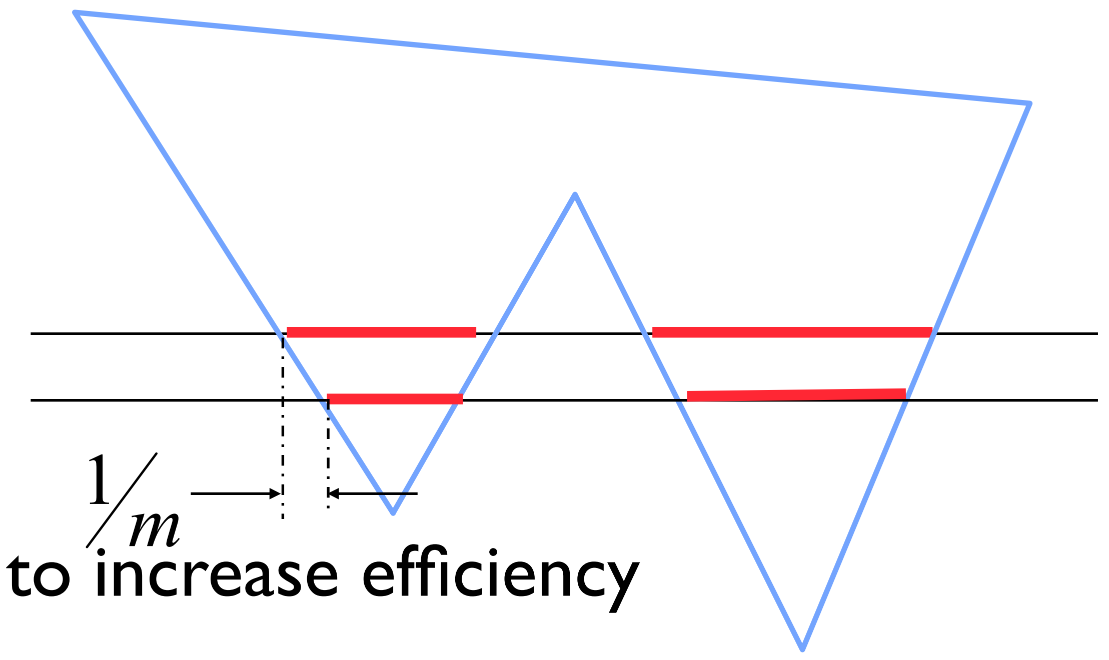
Discussion 5 : How to speed up, or how to avoid calculating intersection

Efficiency Issues in Scan Line Method

- Intersections could be found using edge coherence

  the X-intersection value x_{i+1} of the lower scan line can be computed from the X-intersection value x_i of the preceeding scanline as

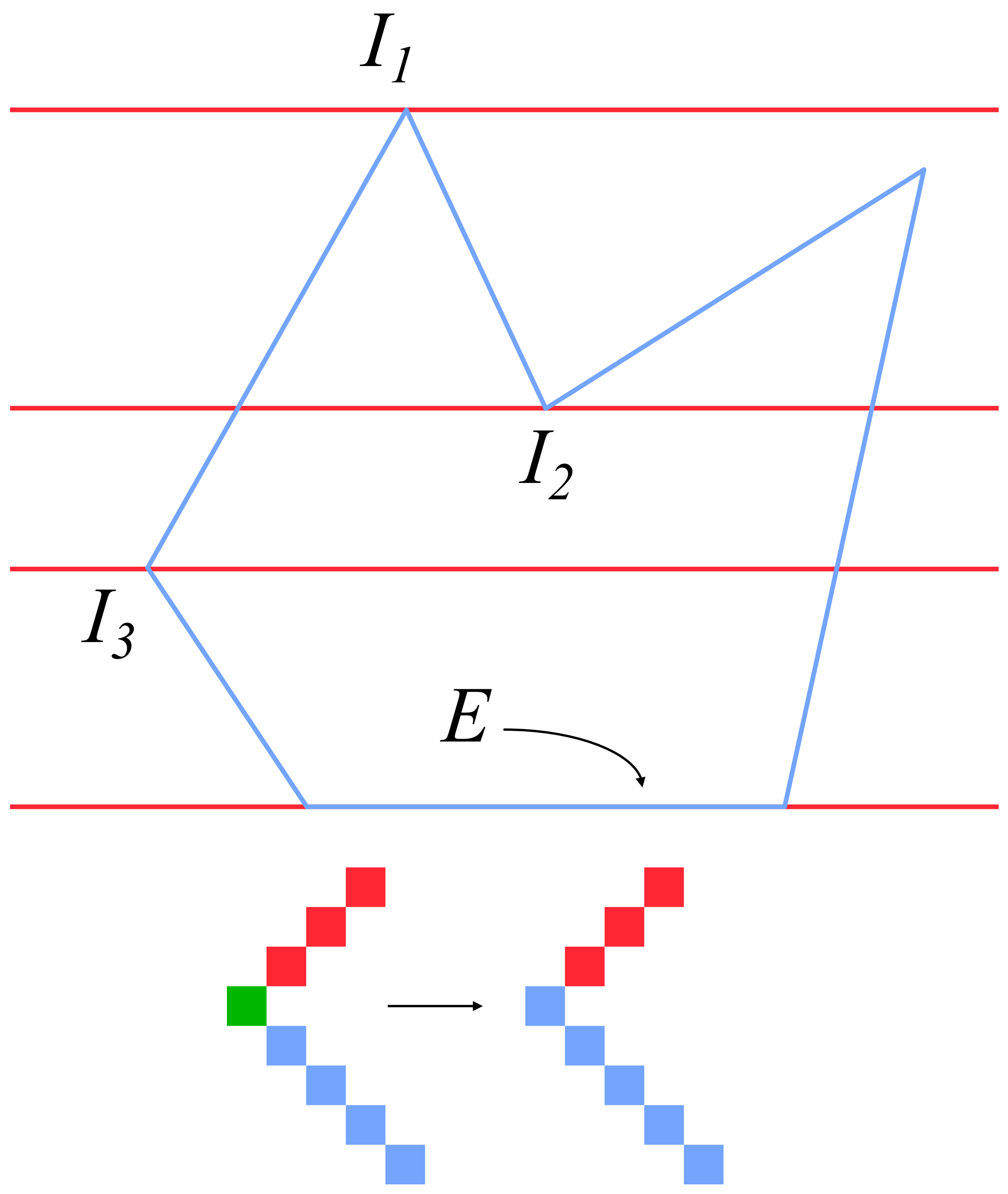
$$x_{i+1} = x_i + 1/m$$



- List of active edges could be maintained to increase efficiency
- Efficiency could be further improved if polygons are convex, much better if they are only triangles

Special cases for Scan Line Method

- Overall topology should be considered for intersection at the vertices
- Intersections like I_1 and I_2 should be considered as two intersections
- Intersections like I_3 should be considered as one intersection
- Horizontal edges like E need not be considered

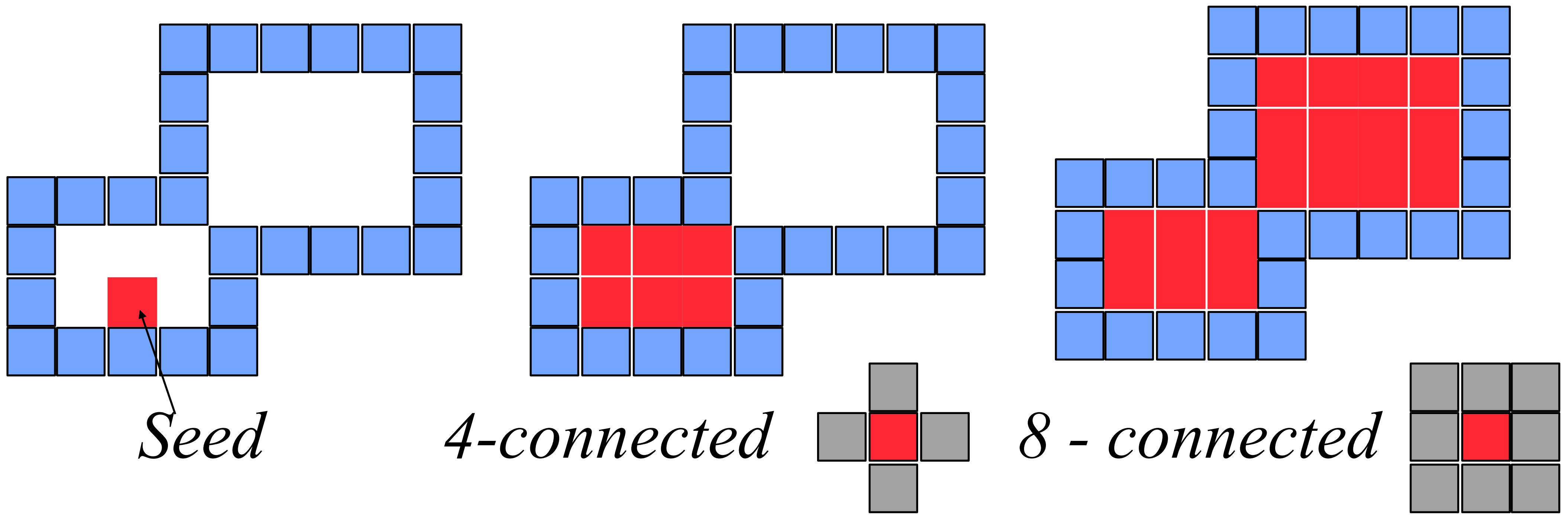


Advantages of Scan Line method

- The algorithm is efficient
- Each pixel is visited only once
- Shading algorithms could be easily integrated with this method to obtain shaded area

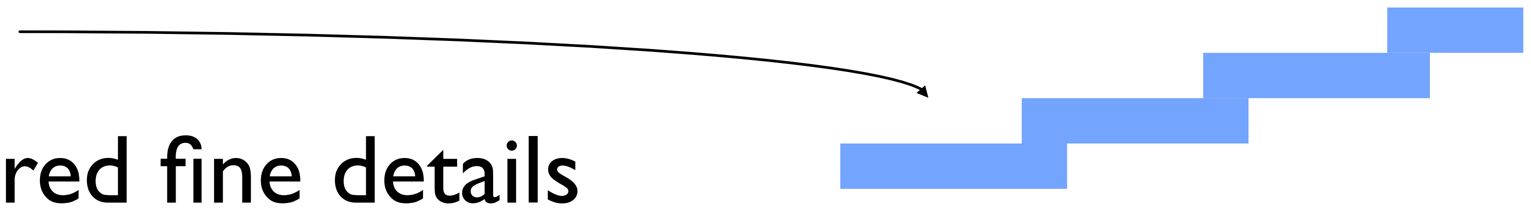
Seed Fill Algorithms

- Assumes that atleast one pixel interior to the polygon is known
- It is a recursive algorithm
- Useful in interactive paint packages

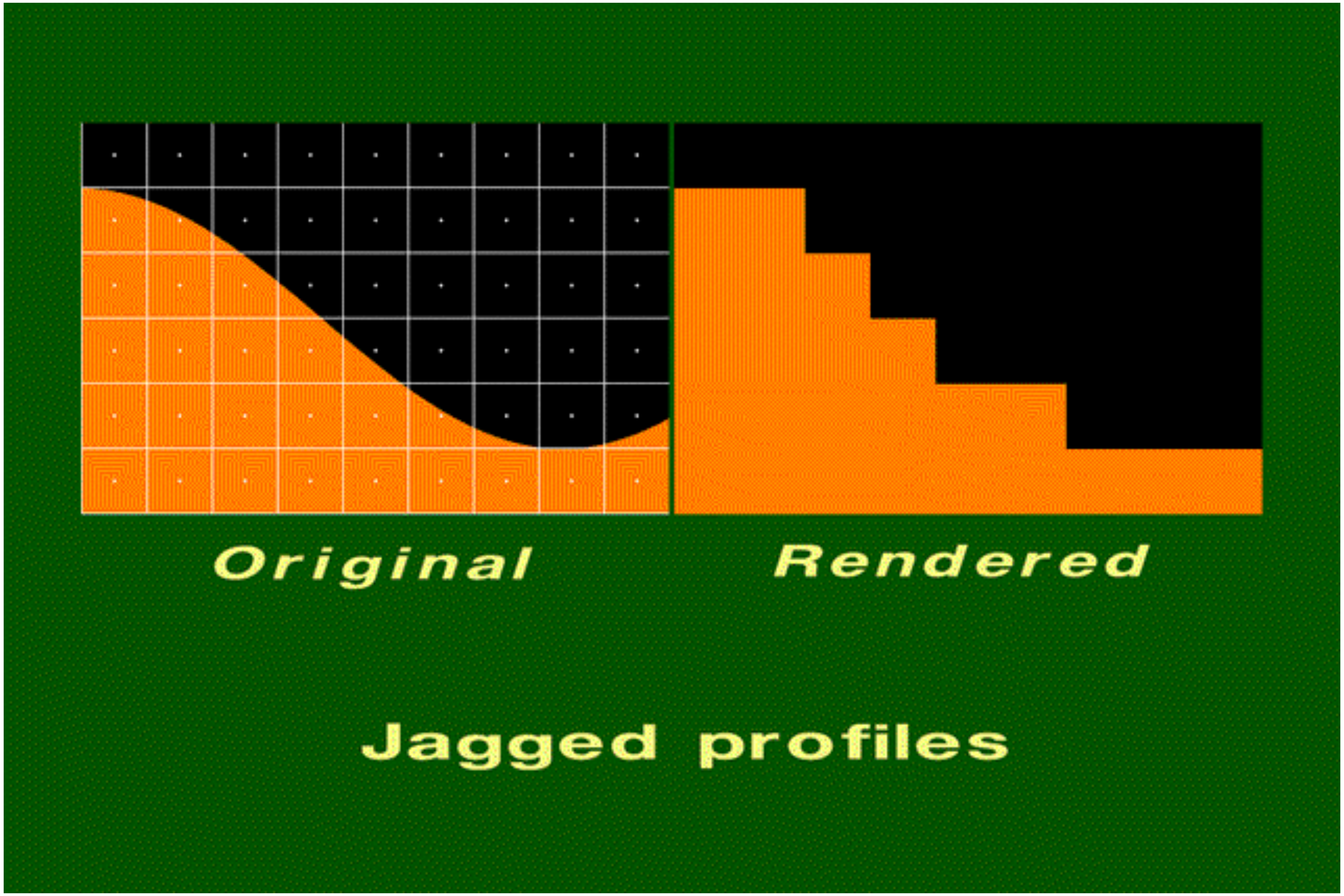


Aliasing

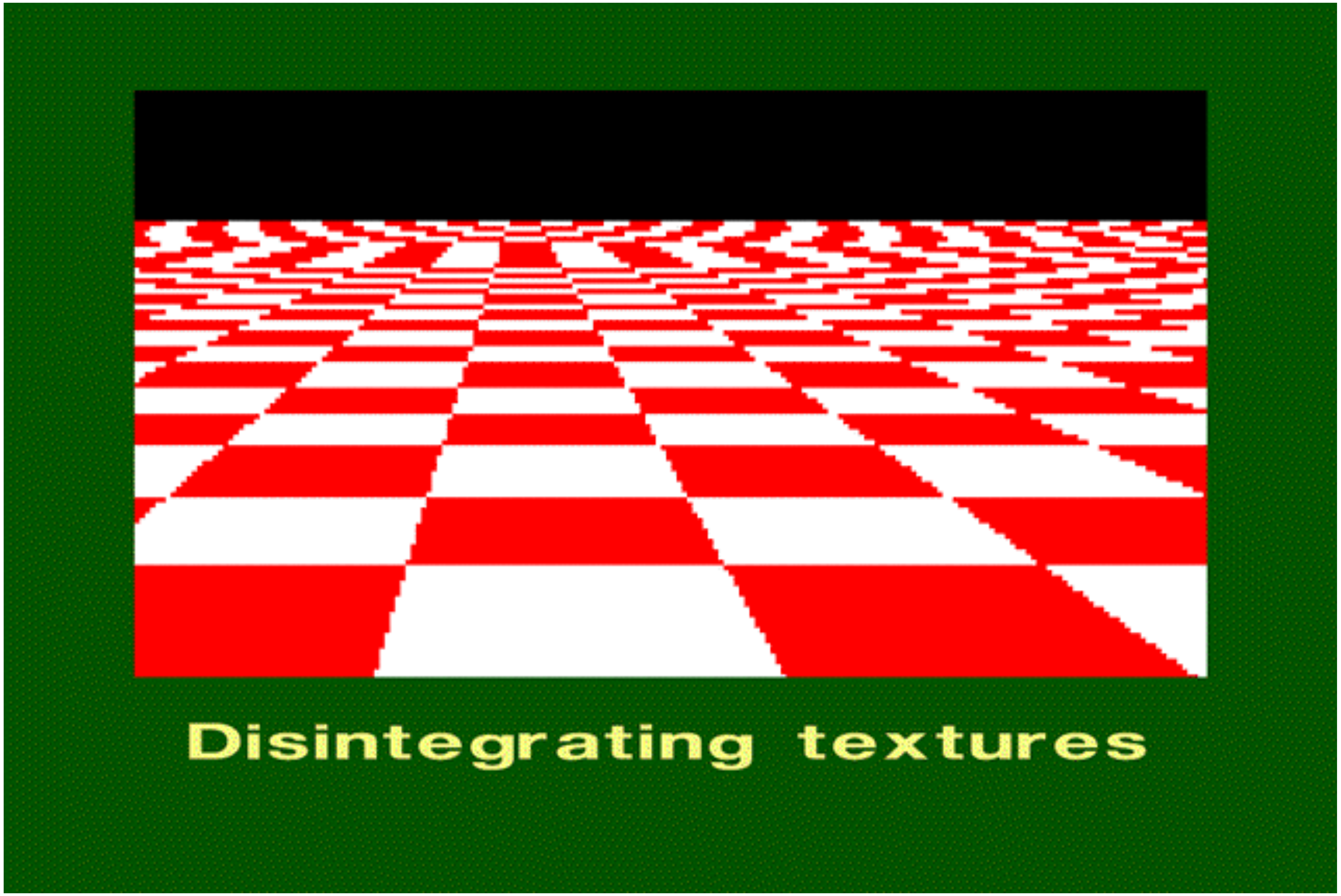
- Aliasing is caused due to the discrete nature of the display device
- Rasterizing primitives is like sampling a continuous signal by a finite set of values (point sampling)
- Information is lost if the rate of sampling is not sufficient. This sampling error is called ***aliasing***.
- Effects of aliasing are
  - Jagged edges
  - Incorrectly rendered fine details
  - Small objects might miss



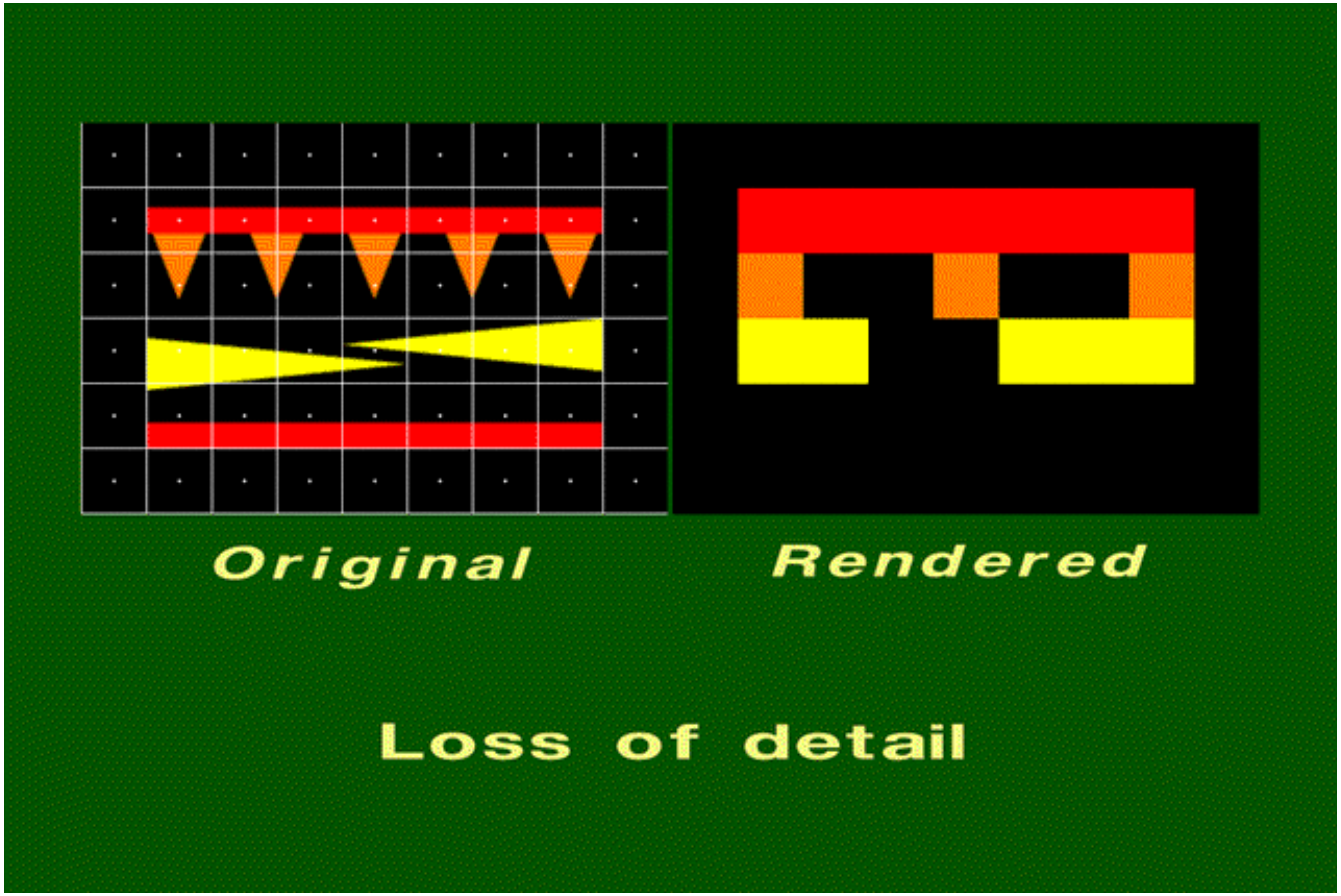
Aliasing(examples)



Aliasing(examples)



Aliasing(examples)

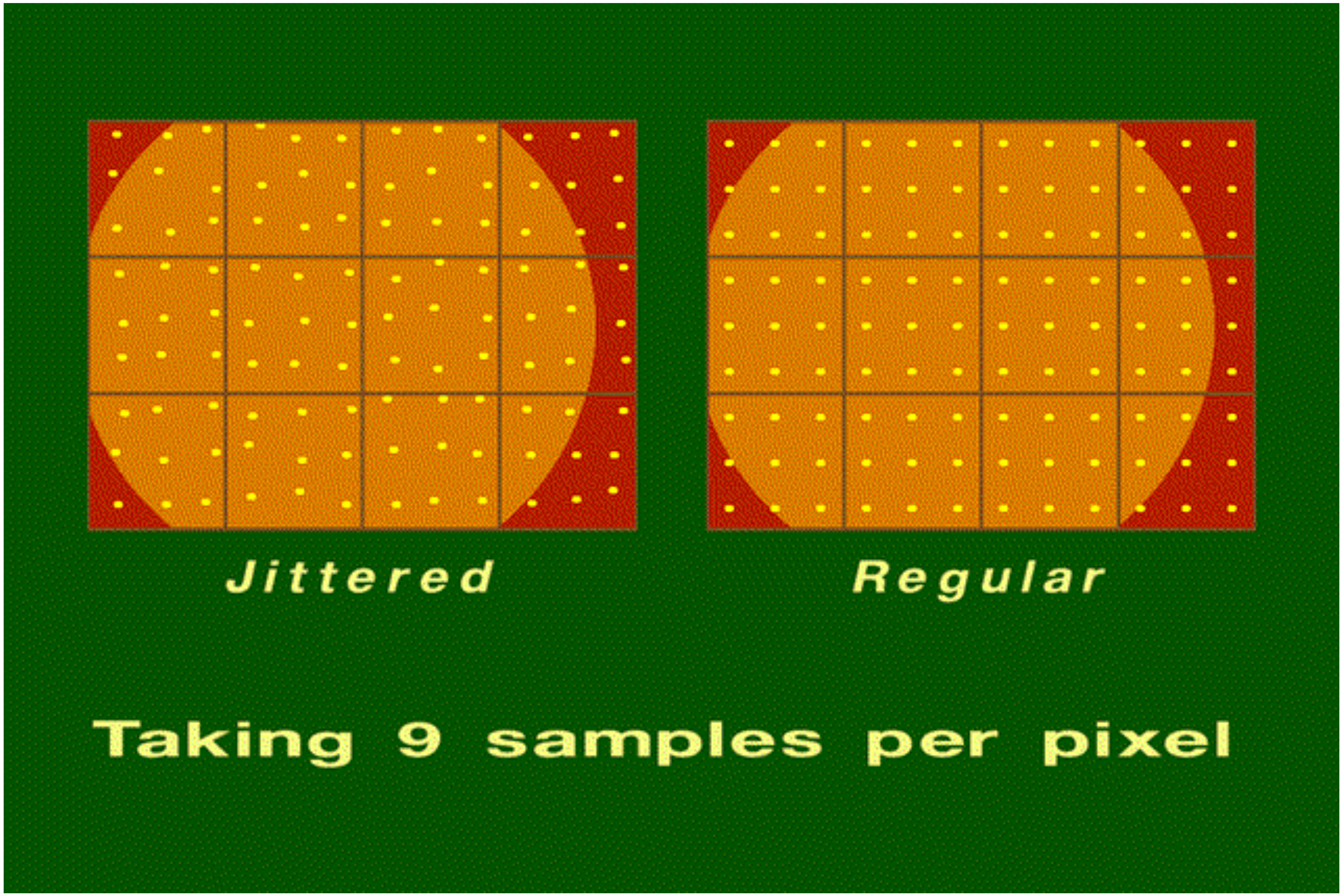


Antialiasing

- Application of techniques to reduce/eliminate aliasing artifacts
- Some of the methods are
  - increasing sampling rate by increasing the resolution. Display memory requirements increases four times if the resolution is doubled
  - averaging methods (post processing). Intensity of a pixel is set as the weighted average of its own intensity and the intensity of the surrounding pixels
  - Area sampling, more popular

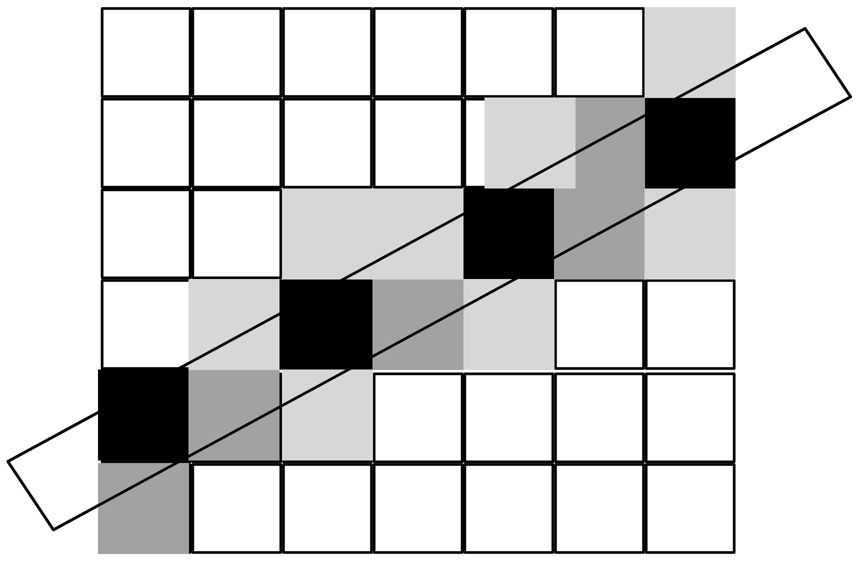
Antialiasing(postfiltering)

How should one supersample?



Area Sampling

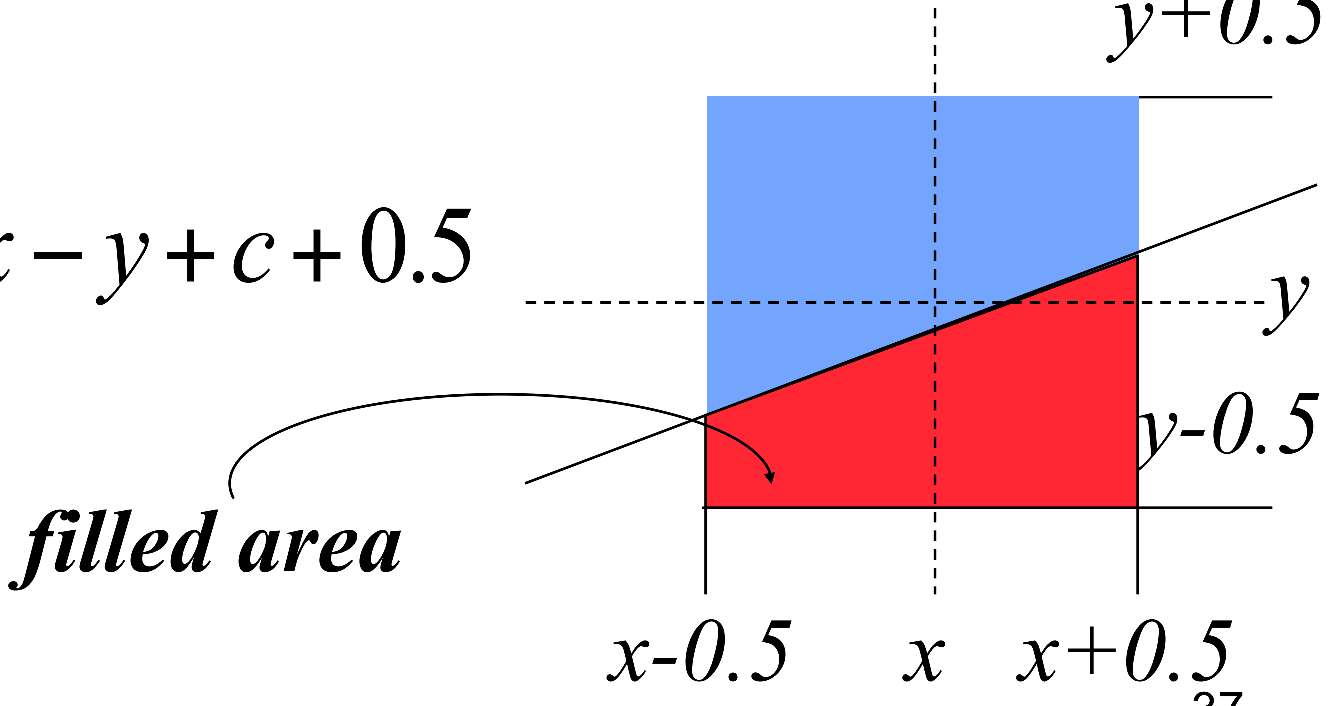
- A scan converted primitive occupies finite area on the screen
- Intensity of the boundary pixels is adjusted depending on the percent of the pixel area covered by the primitive. This is called weighted area sampling



Area Sampling

- Methods to estimate percent of pixel covered by the primitive
  - subdivide pixel into sub-pixels and determine how many sub-pixels are inside the boundary
  - Incremental line algorithm can be extended, with area calculated as

$$Area = m \times x - y + c + 0.5$$



Clipping

- Clipping of primitives is done usually before scan converting the primitives
- Reasons being
  - scan conversion needs to deal only with the clipped version of the primitive, which might be much smaller than its unclipped version
  - Primitives are usually defined in the real world, and their mapping from the real to the integer domain of the display might result in the overflowing of the integer values resulting in unnecessary artifacts

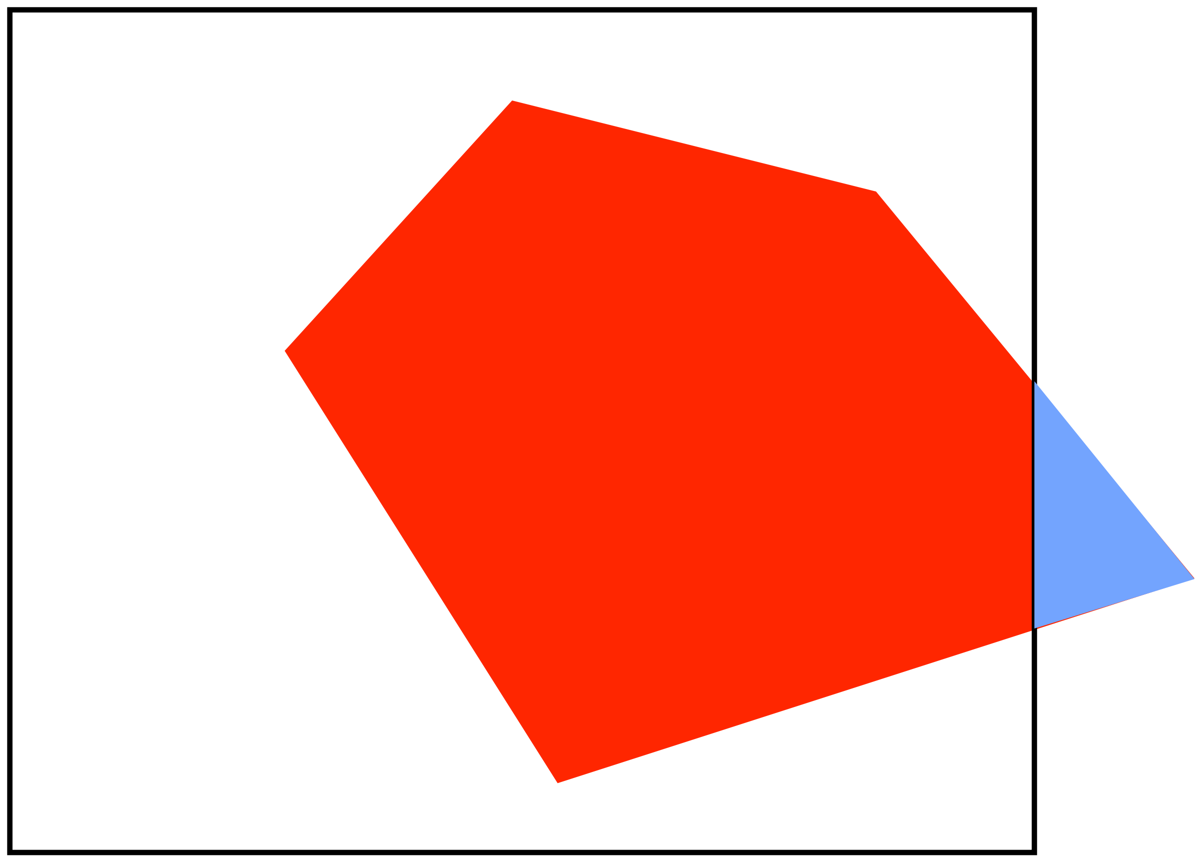
Clipping

- Why Clipping?
- How Clipping?
  - Lines
  - Polygons

- Note: Content from chapter 4.
  - Lots of stuff about rendering systems and mathematics in that chapter.

Definition

- Clipping – Removal of content that is not going to be displayed
  - Behind camera
  - Too close
  - Too far
  - Off sides of the screen

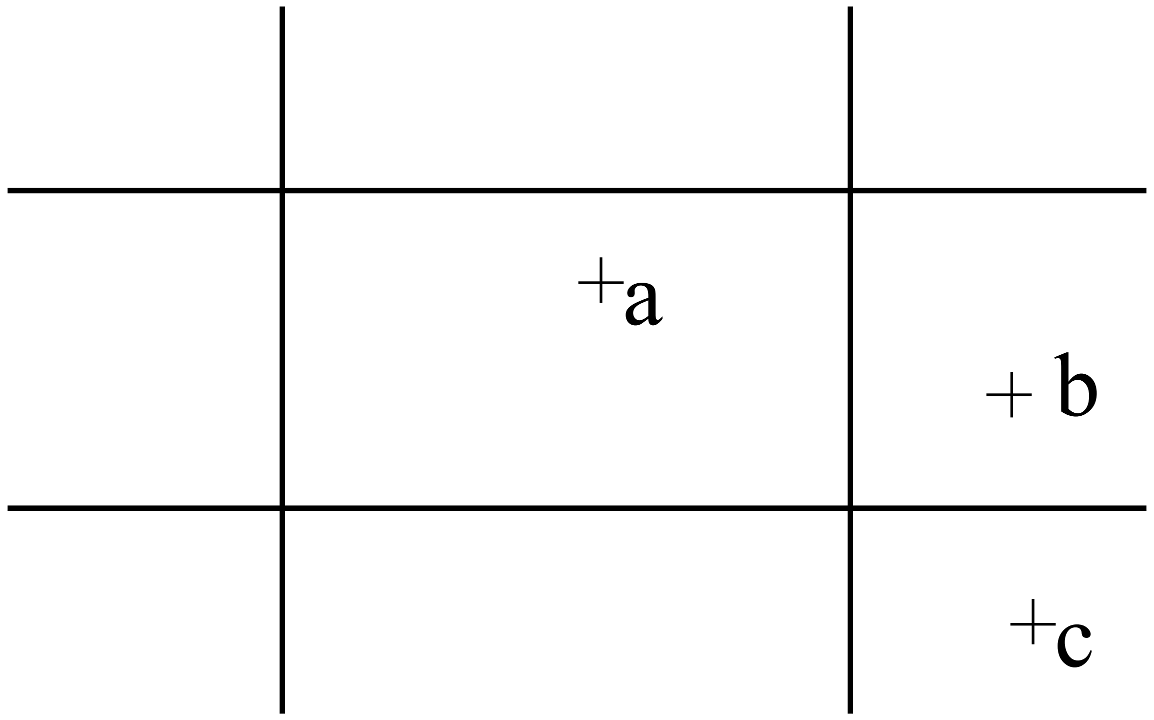


How would we clip?

- Points?
- Lines?
- Polygons?
- Other objects?

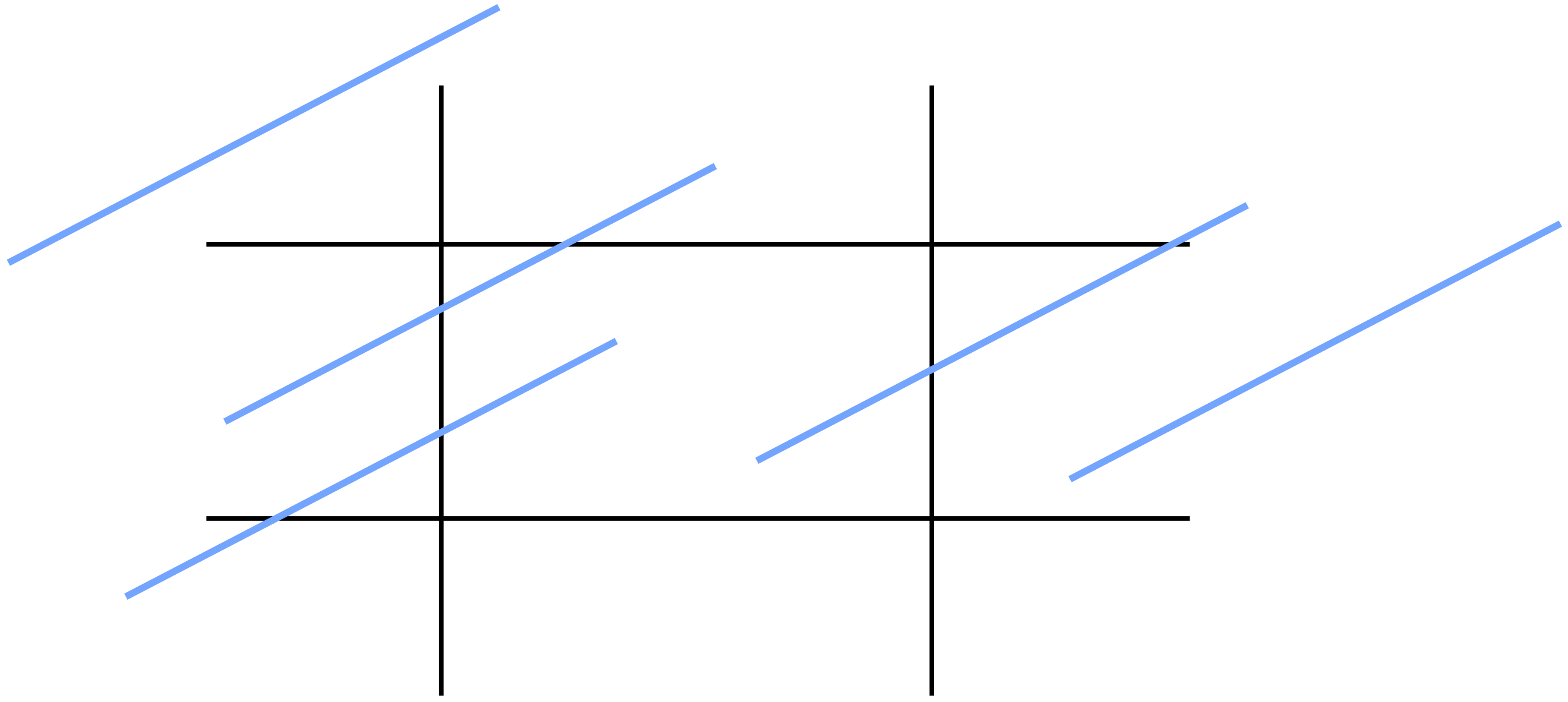
We’ll start in 2D

- Assume a 2D upright rectangle we are clipping against
  - Common in windowing systems
  - Points are trivial
    - >= minx and <= maxx and >= miny and <= maxy



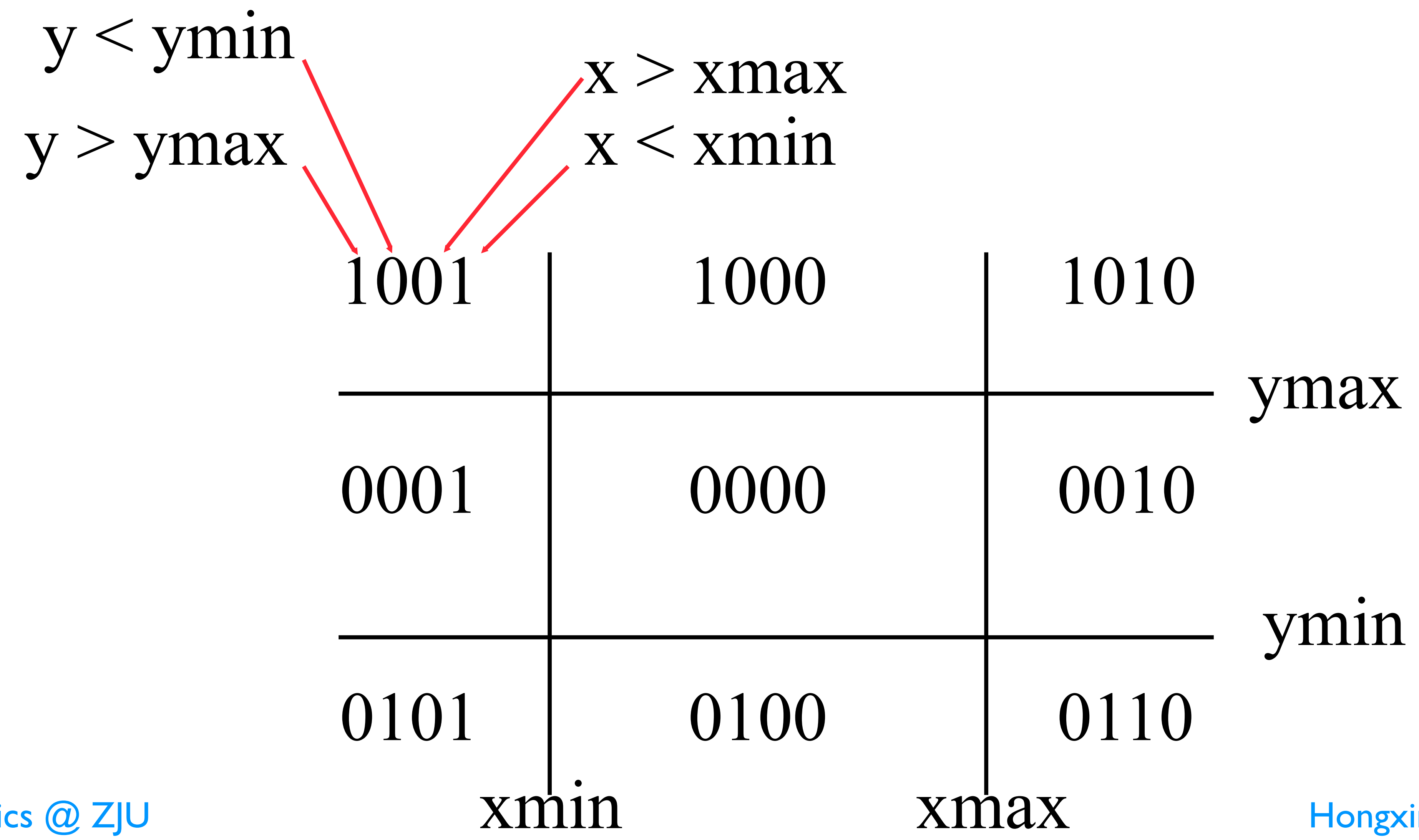
Line Segments

- What can happen when a line segment is clipped?



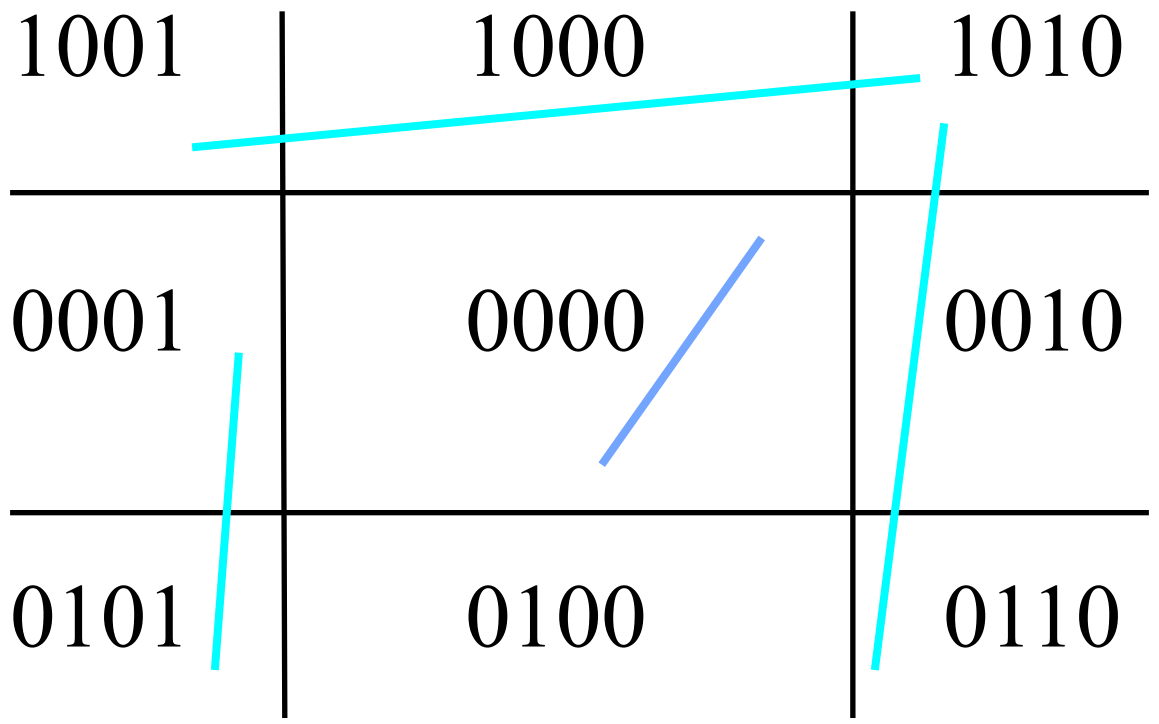
Cohen-Sutherland Line Clipping

- We'll assign the ends of a line "outcodes", 4 bit values that indicate if they are inside or outside the clip area.



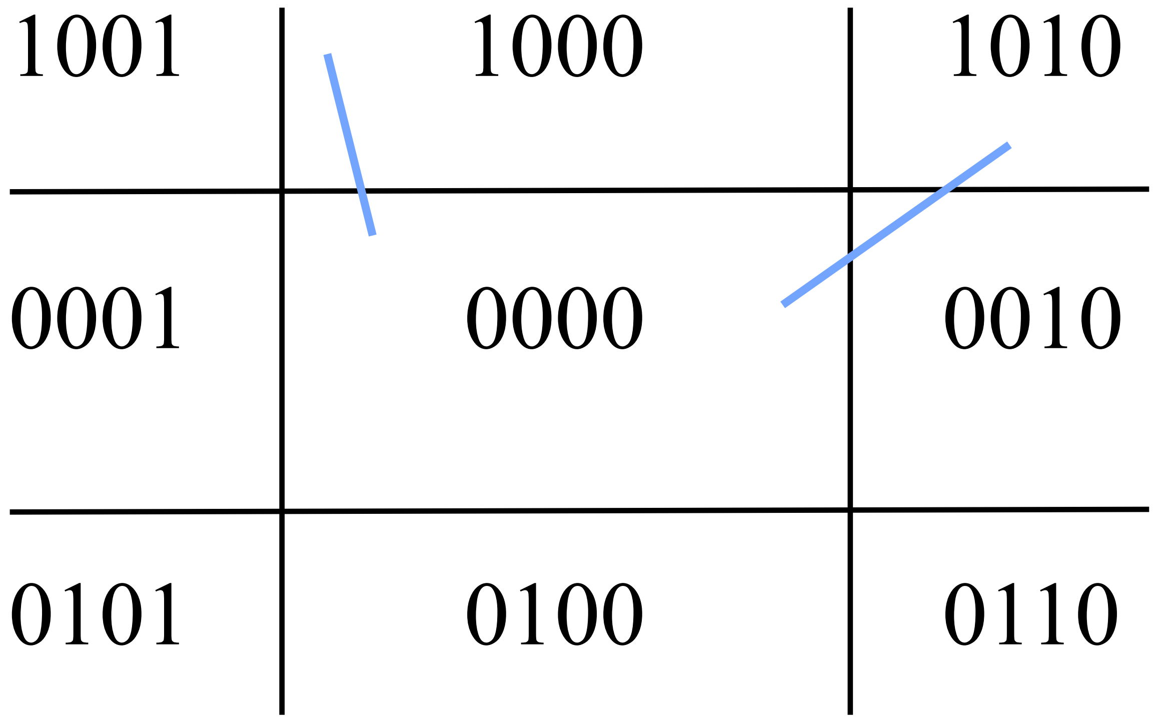
Outcode cases

- We'll call the two endpoint outcodes o_1 and o_2.
  - If $o_1 = o_2 = 0$, both endpoints are <u>inside</u>.
  - else if $(o_1 \& o_2) != 0$, both ends points are on the <u>same side</u>, the edge is discarded.



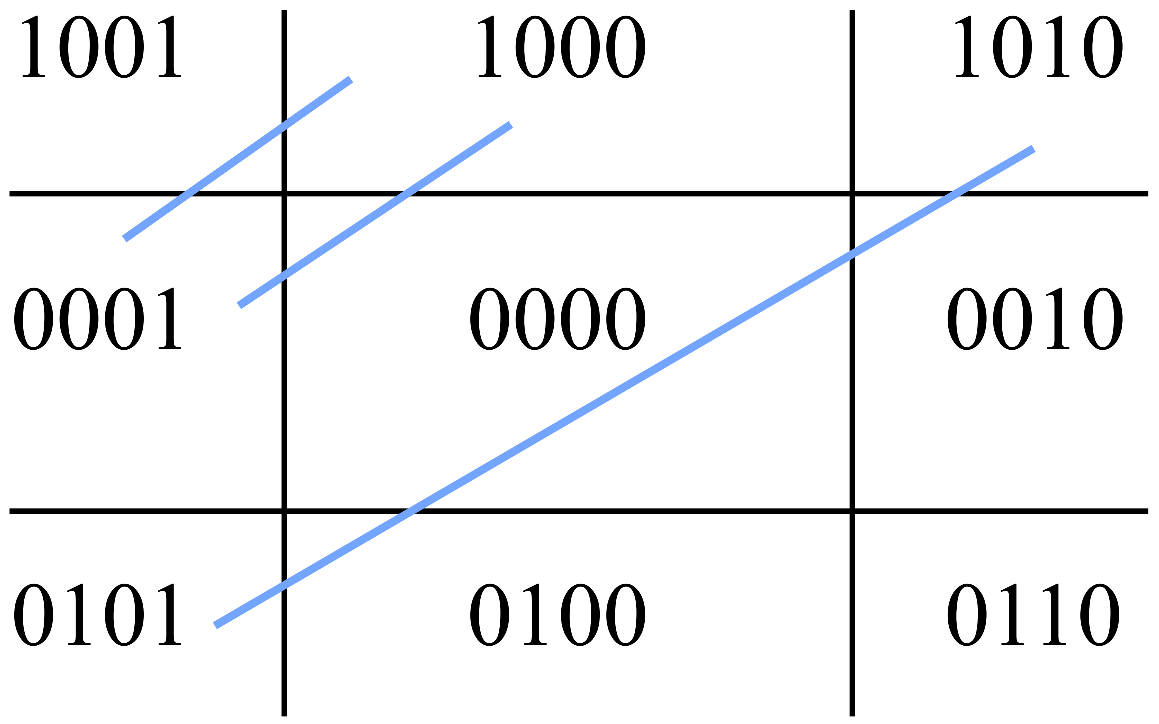
More cases

- else if (o_1 != 0) and (o_2 = 0), (or vice versa), one end is inside, other is outside.
  - Clip and recompute *one that's outside* until inside.
  - Clip edges with bits set…
  - May require two clip computations



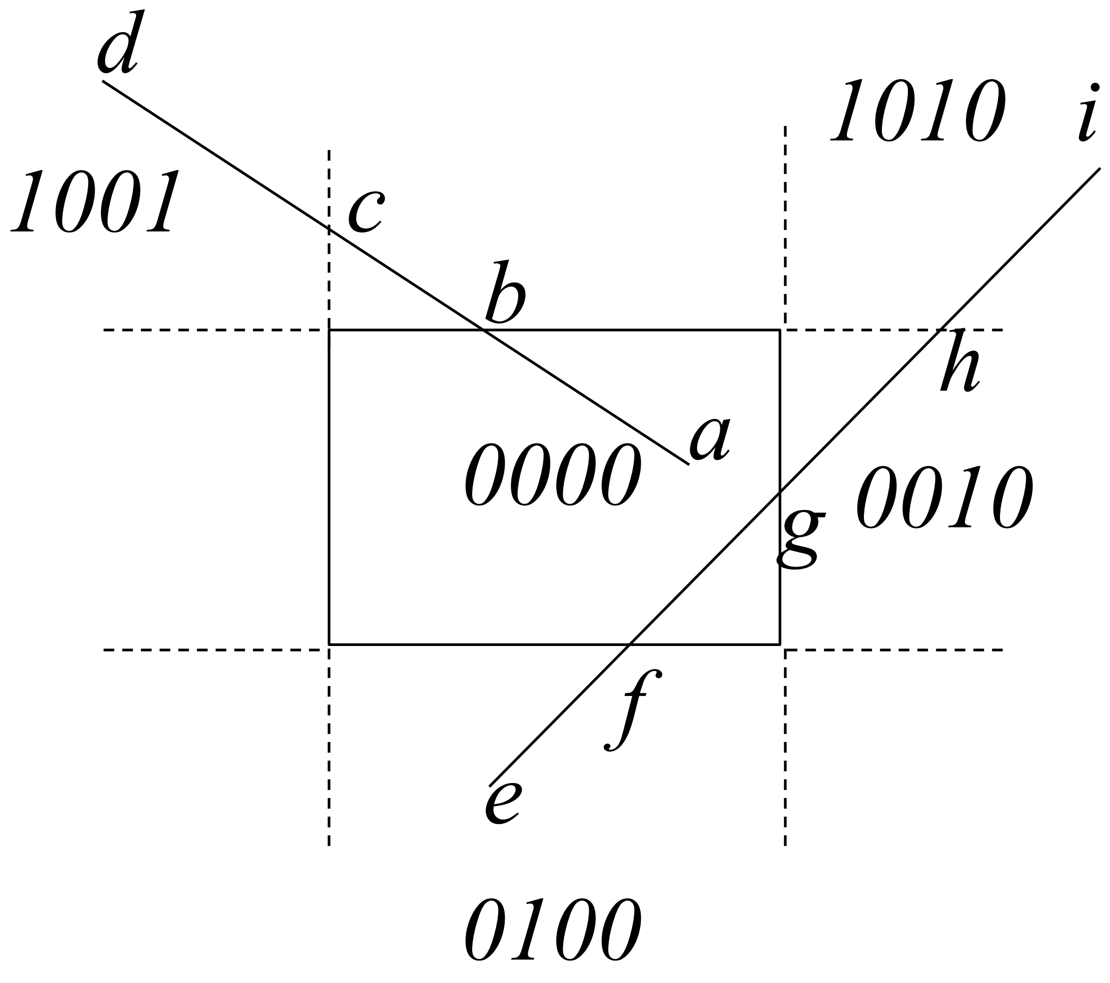
Last case…

- else if (o1 & o2) = 0, end points are on different sides.
  - Clip and recompute.
  - May have some inside part or may not…
  - May require up to 4 clips!

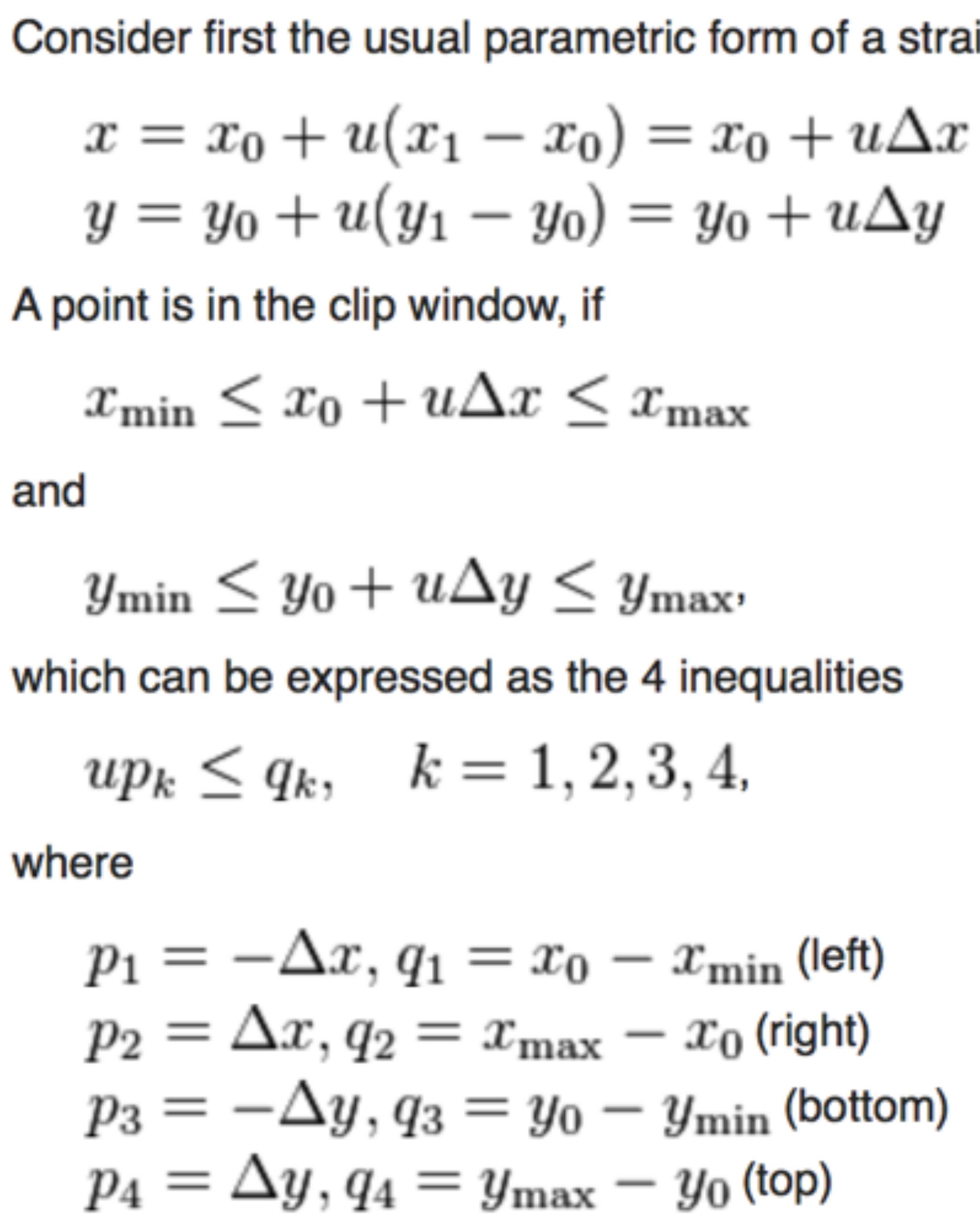


Cohen-Sutherland Line-Clipping Algorithm

- To do the clipping find the end point that lies outside
- Test the outcode to find the edge that is crossed and determine the corresponding intersection point
- Replace the outside end-point by intersection-point
- Repeat the above steps for the new line



Liang–Barsky algorithm



Consider first the usual parametric form of a straight line:

$$x = x_0 + u(x_1 - x_0) = x_0 + u\Delta x$$
$$y = y_0 + u(y_1 - y_0) = y_0 + u\Delta y$$

A point is in the clip window, if

$$x_{\min} \le x_0 + u\Delta x \le x_{\max}$$

and

$$y_{\min} \le y_0 + u\Delta y \le y_{\max},$$

which can be expressed as the 4 inequalities

$$up_k \le q_k, \quad k = 1, 2, 3, 4,$$

where

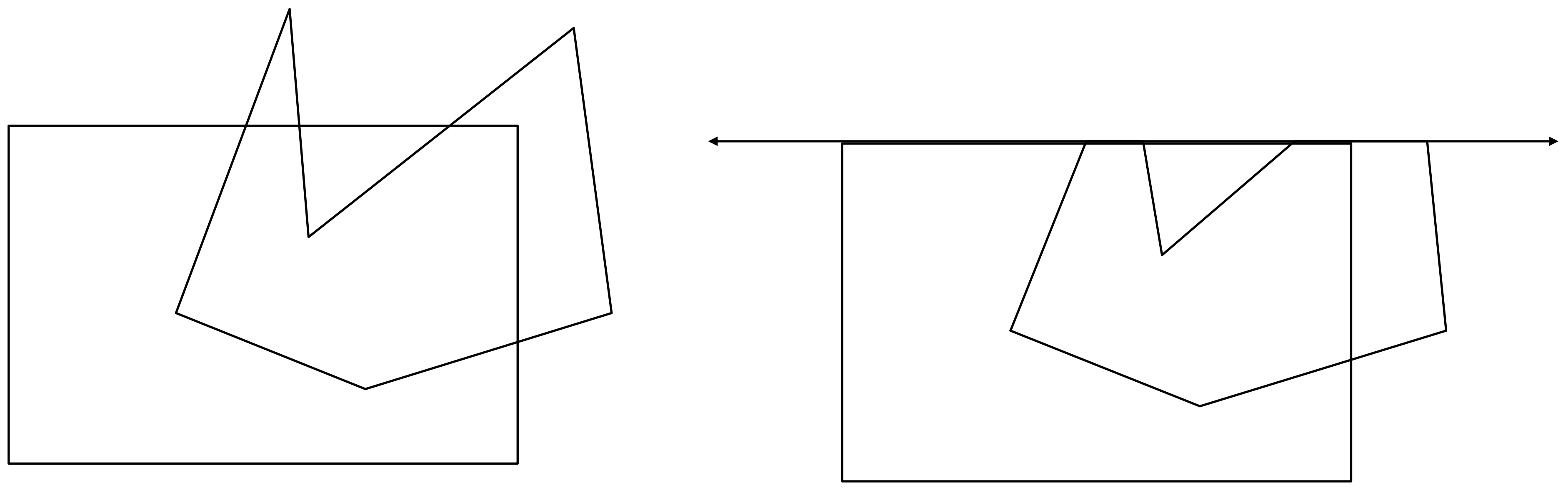
$p_1 = -\Delta x, q_1 = x_0 - x_{\min}$ (left)
$p_2 = \Delta x, q_2 = x_{\max} - x_0$ (right)
$p_3 = -\Delta y, q_3 = y_0 - y_{\min}$ (bottom)
$p_4 = \Delta y, q_4 = y_{\max} - y_0$ (top)

Liang–Barsky algorithm

To compute the final line segment:

1. A line parallel to a clipping window edge has $p_k = 0$ for that boundary.
2. If for that k, $q_k < 0$, the line is completely outside and can be eliminated.
3. When $p_k < 0$ the line proceeds outside to inside the clip window and when $p_k > 0$, the line proceeds inside to outside.
4. For nonzero p_k, $u = \dfrac{q_k}{p_k}$ gives the intersection point.
5. For each line, calculate u_1 and u_2. For u_1, look at boundaries for which $p_k < 0$ (i.e. outside to inside). Take u_1 to be the largest among $\left\{0, \dfrac{q_k}{p_k}\right\}$. For u_2, look at boundaries for which $p_k > 0$ (i.e. inside to outside). Take u_2 to be the minimum of $\left\{1, \dfrac{q_k}{p_k}\right\}$. If $u_1 > u_2$, the line is outside and therefore rejected.

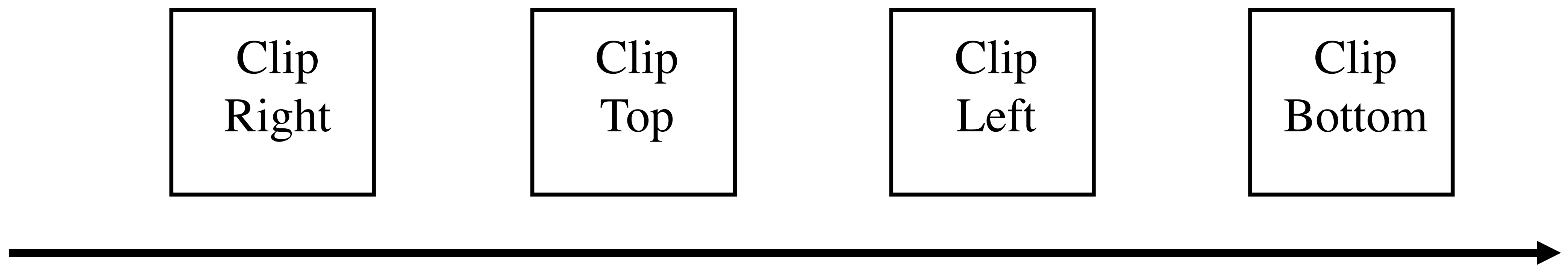
Sutherland-Hodgeman Polygon-Clipping Algorithm



- Polygons can be clipped against each edge of the window one edge at a time. Window/edge intersections, if any, are easy to find since the X or Y coordinates are already known.
- Vertices which are kept after clipping against one window edge are saved for clipping against the remaining edges.

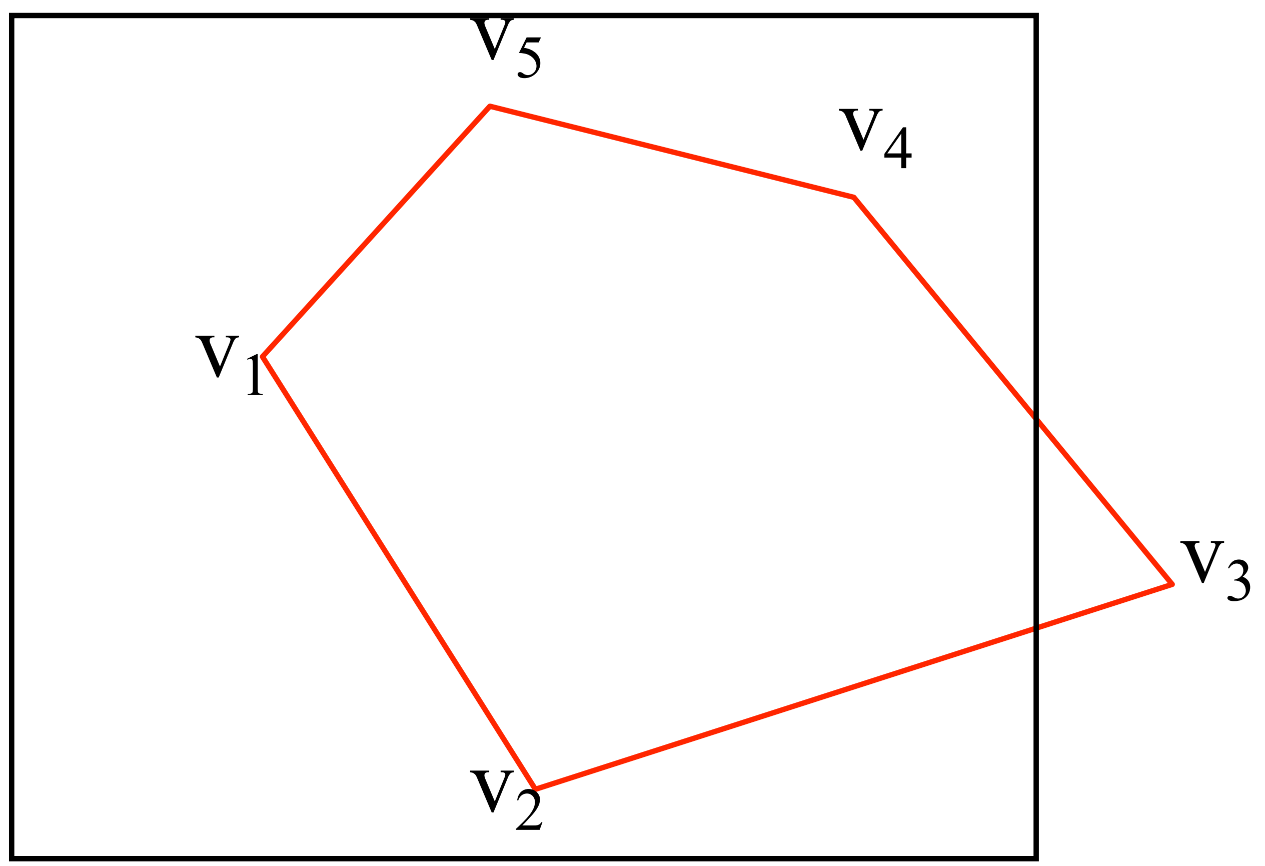
Pipelined Polygon Clipping

- Because polygon clipping does not depend on any other polygons, it is possible to arrange the clipping stages in a **pipeline**. the input polygon is clipped against one edge and any points that are kept are passed on as input to the next *stage* of the pipeline.
- This way four polygons can be at different *stages* of the clipping process simultaneously. This is often implemented in hardware.



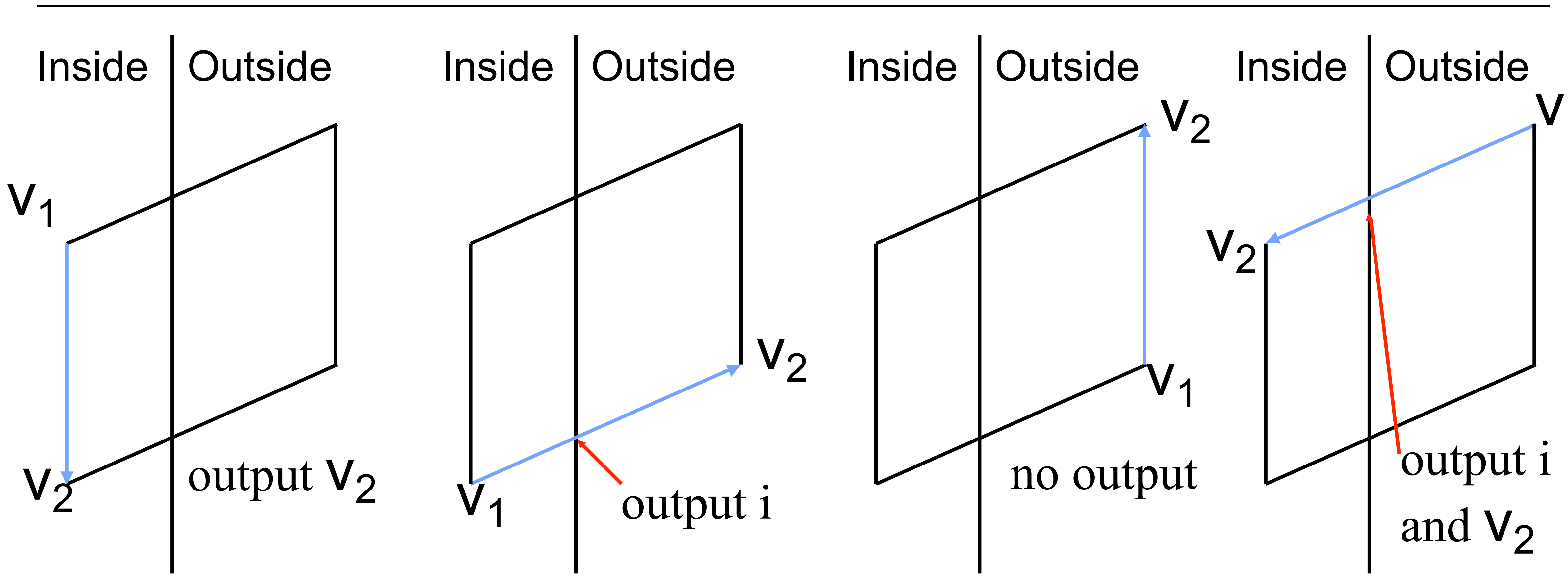
Sutherland-Hodgeman Polygon Clipping Algorithm

- Polygon clipping is similar to line clipping except we have to keep track of inside/outside relationships
  - Consider a polygon as a list of vertices
  - Note that clipping can increase the number of vertices!
  - Typically clip one edge at a time…

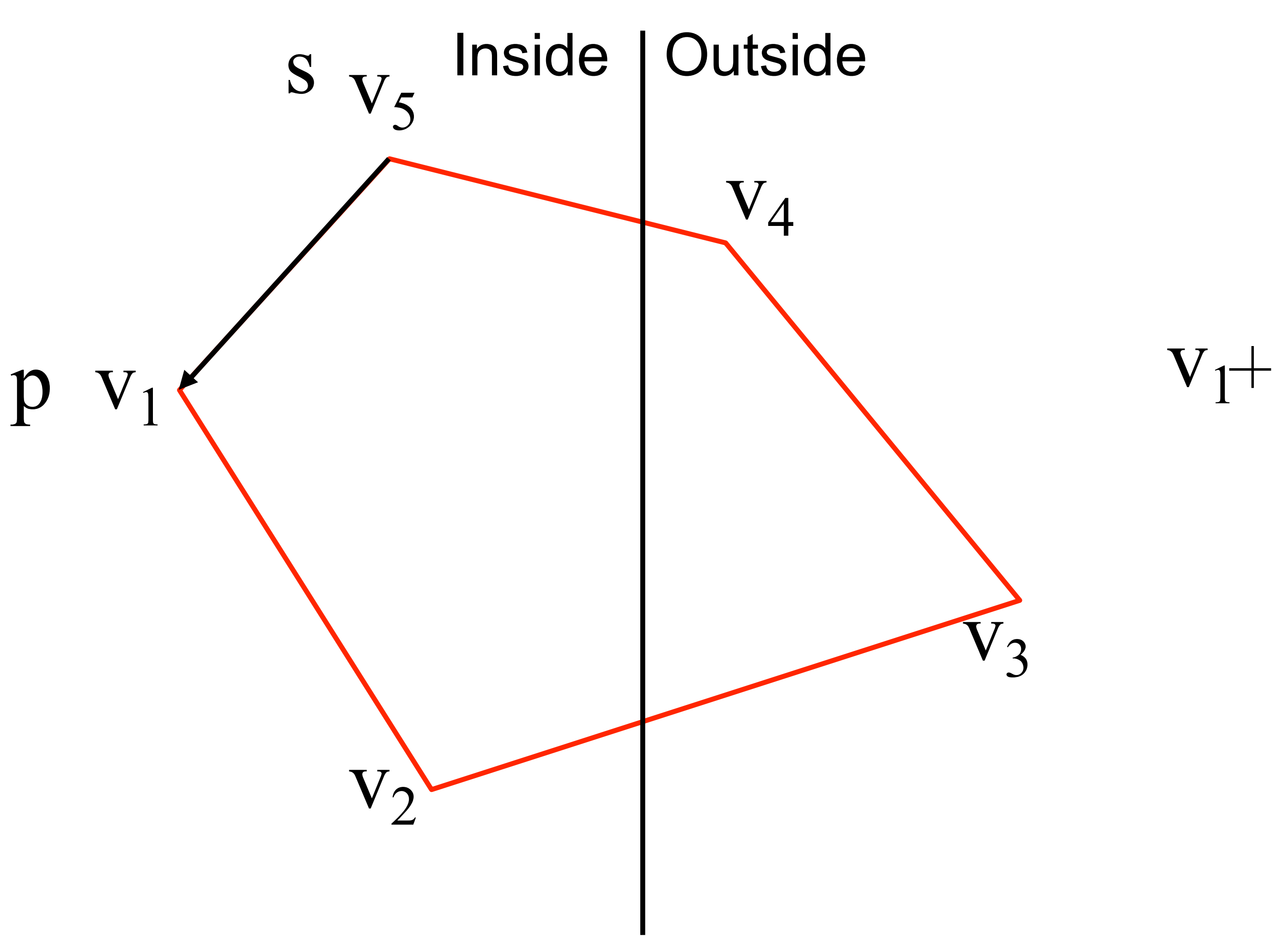


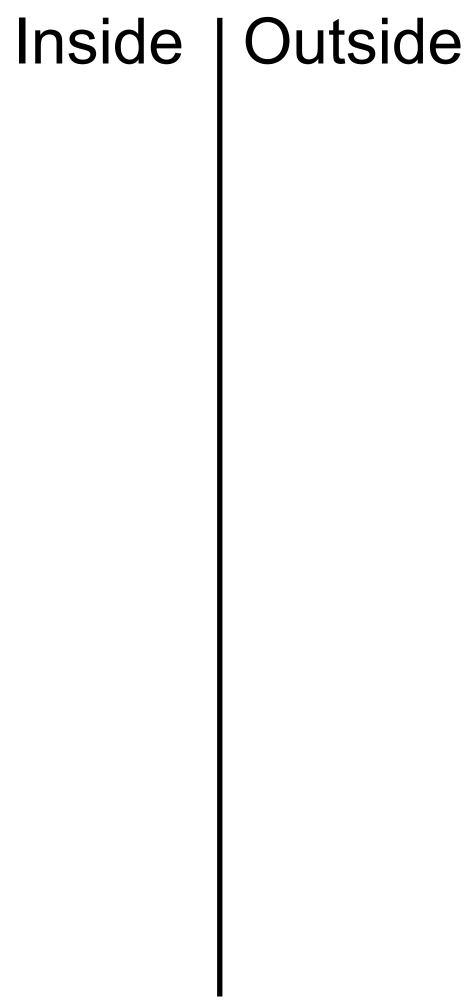
Sutherland-Hodgeman algorithm

- Present the vertices in pairs
  - $(v_n,v_1), (v_1, v_2), (v_2,v_3), \ldots, (v_{n-1},v_n)$
  - For each pair, what are the possibilities?
  - Consider v_1, v_2



Example: v_5, v_1

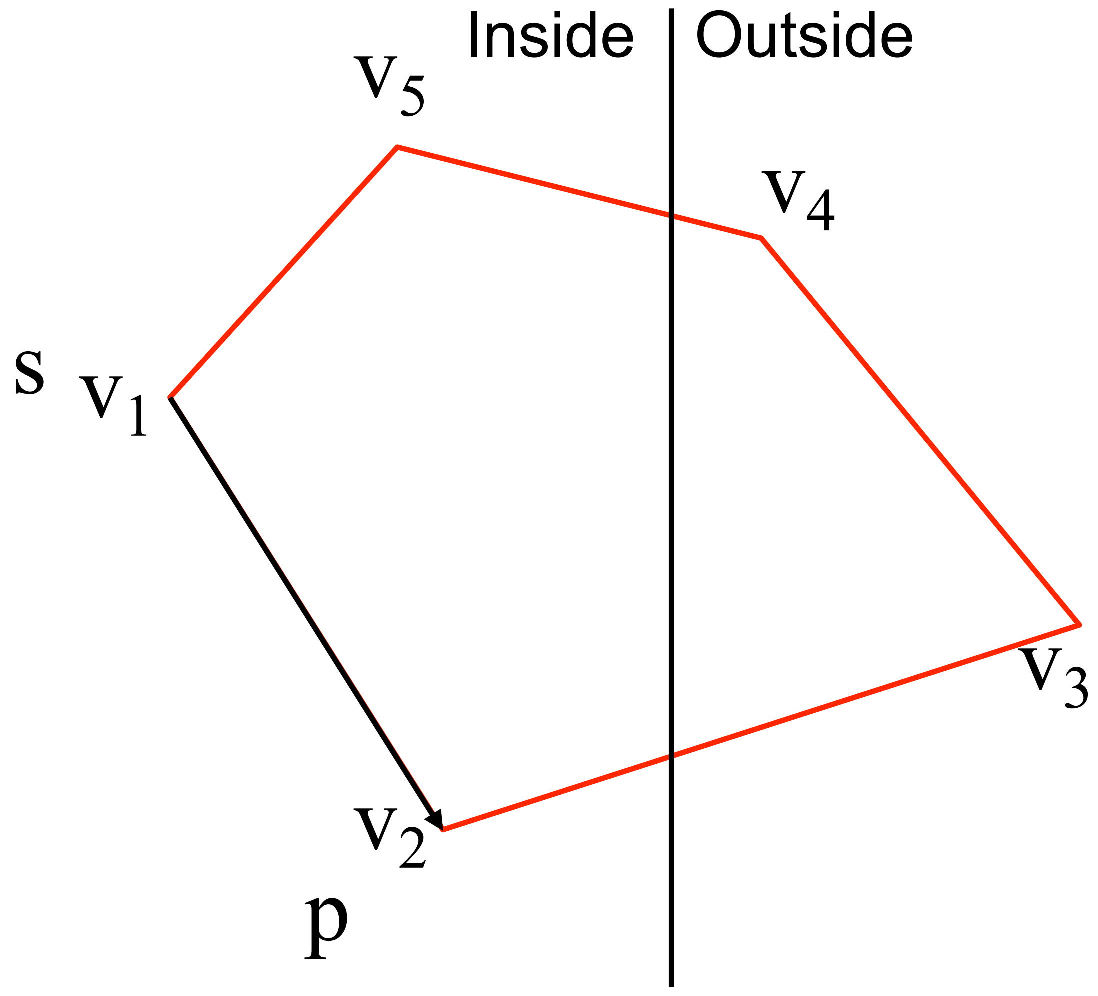




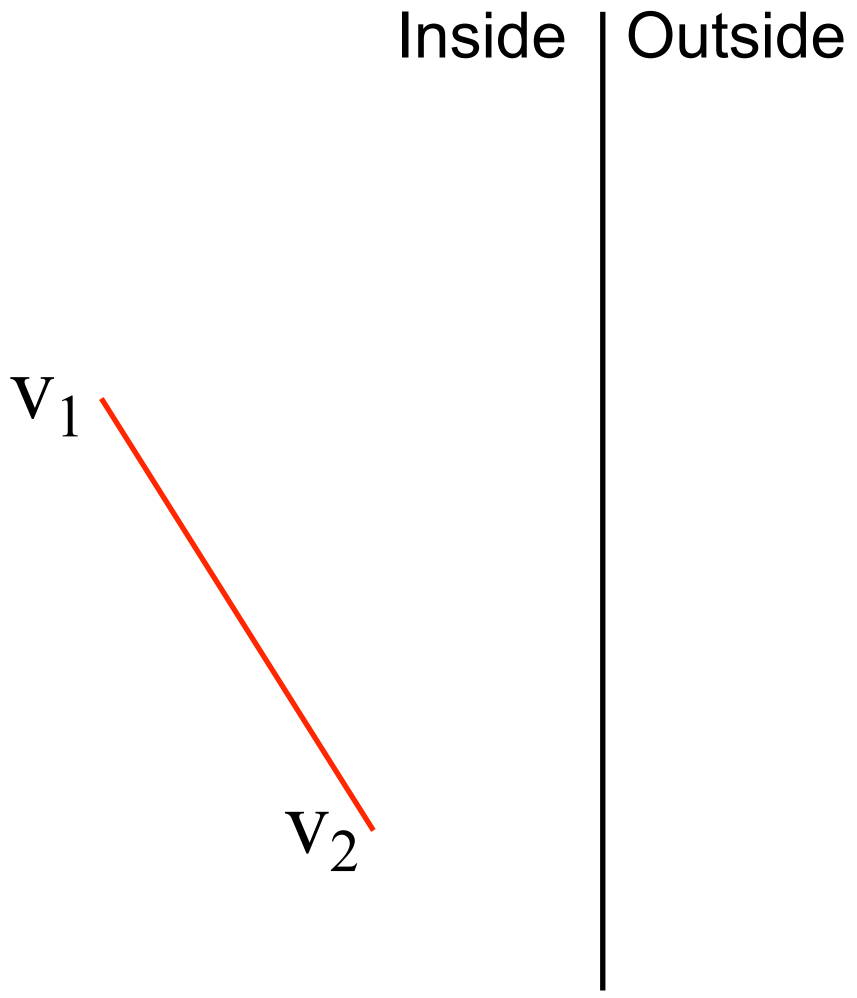
Inside, Inside
Output v_1

Current
Output

v_1, v_2

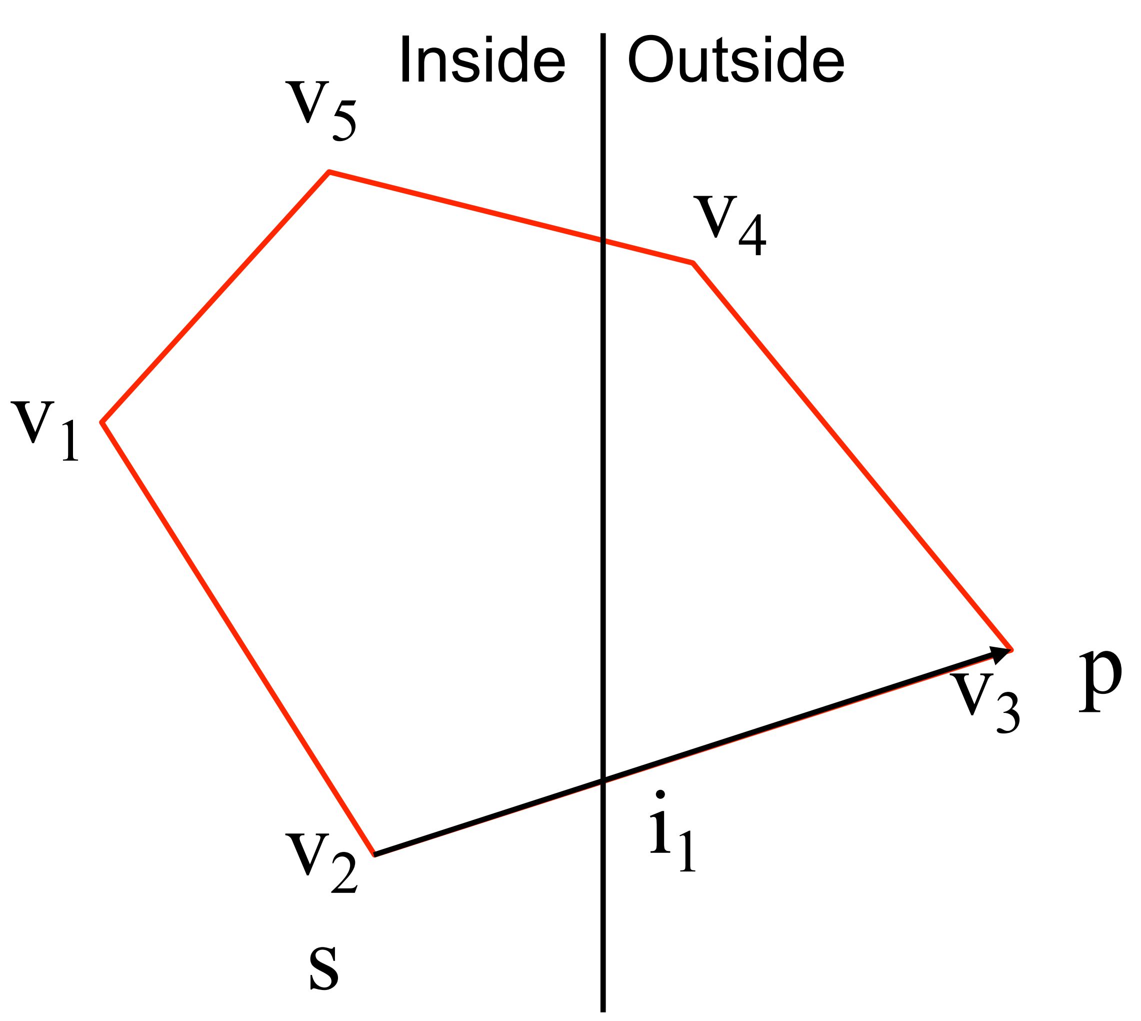


Inside, Inside
Output v_2

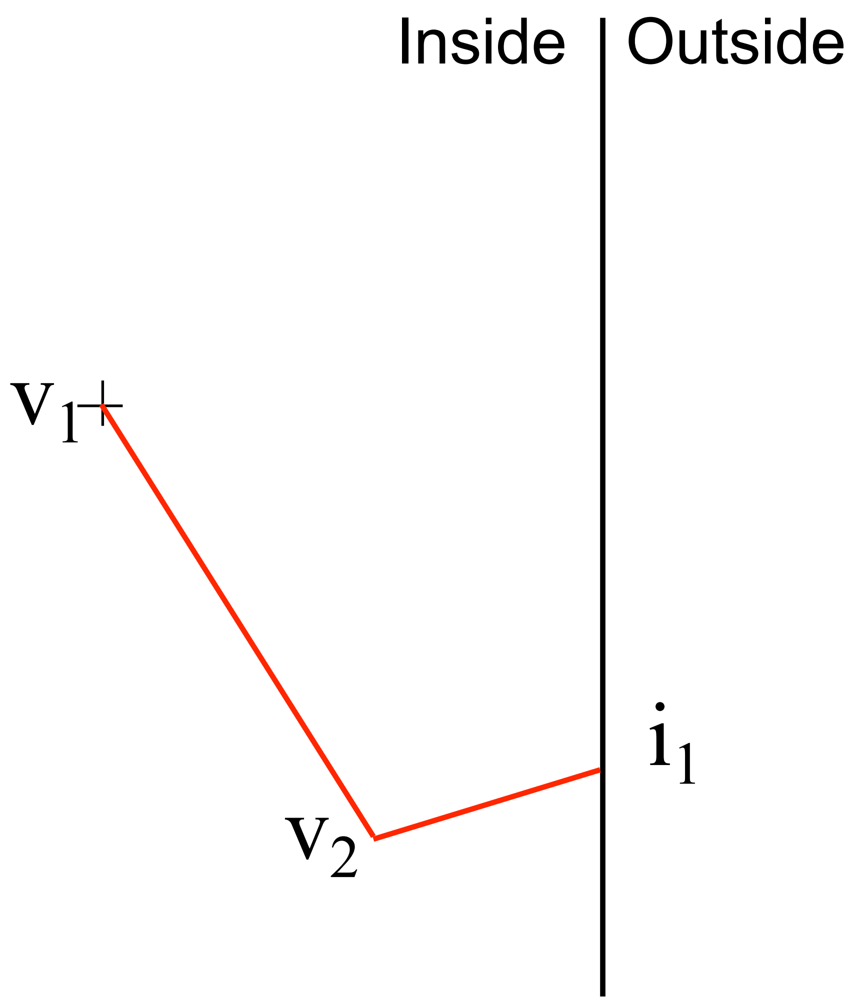


Current
Output

v_2, v_3

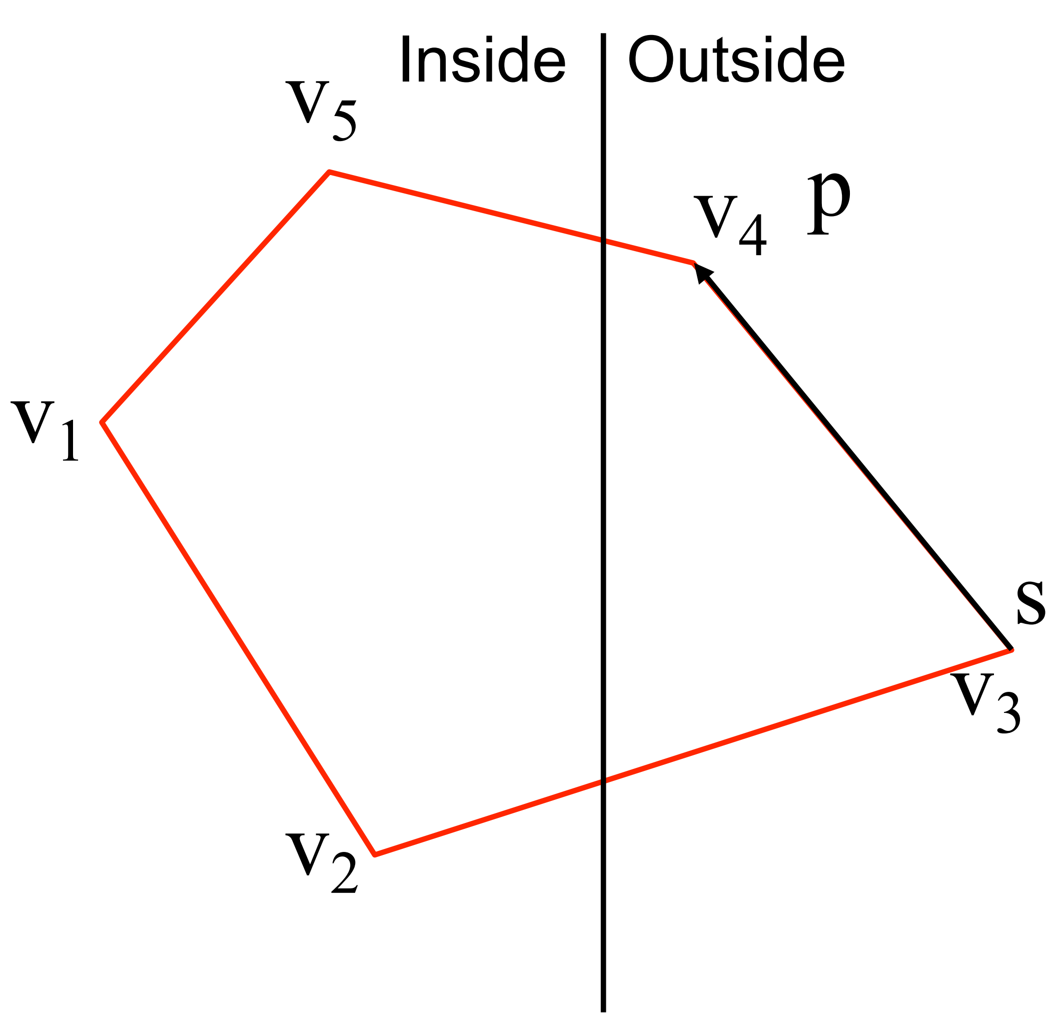


Inside, Outside
Output i_1

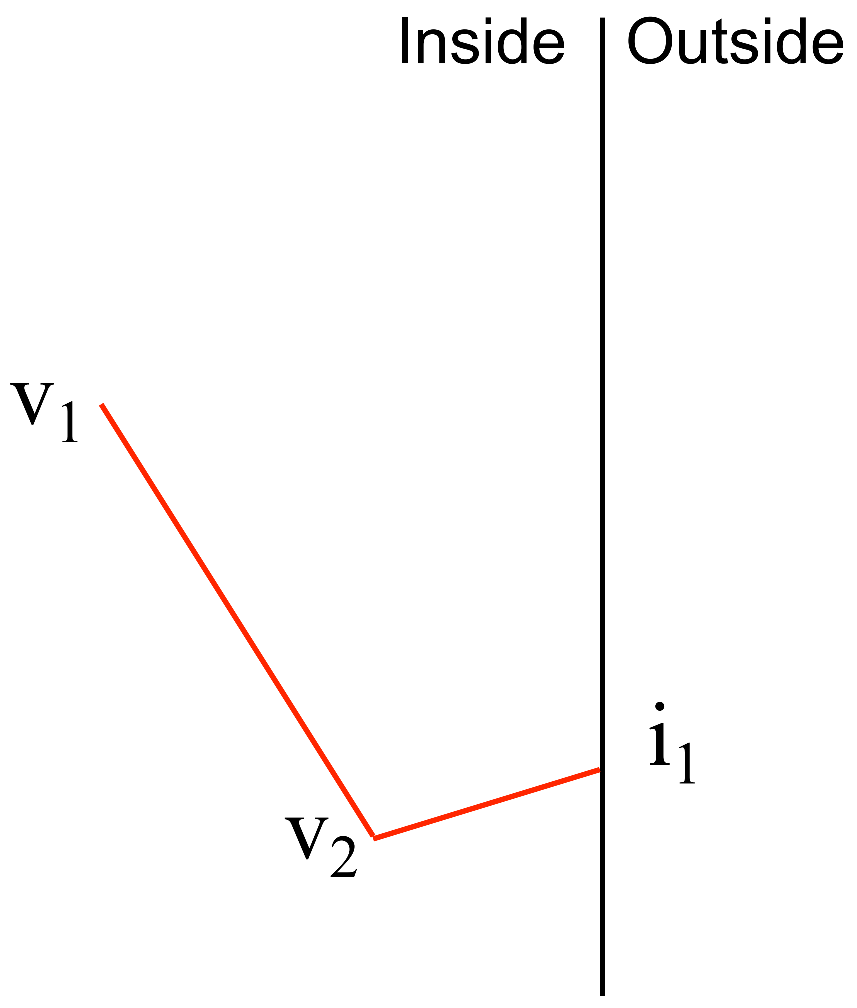


Current
Output

v_3, v_4

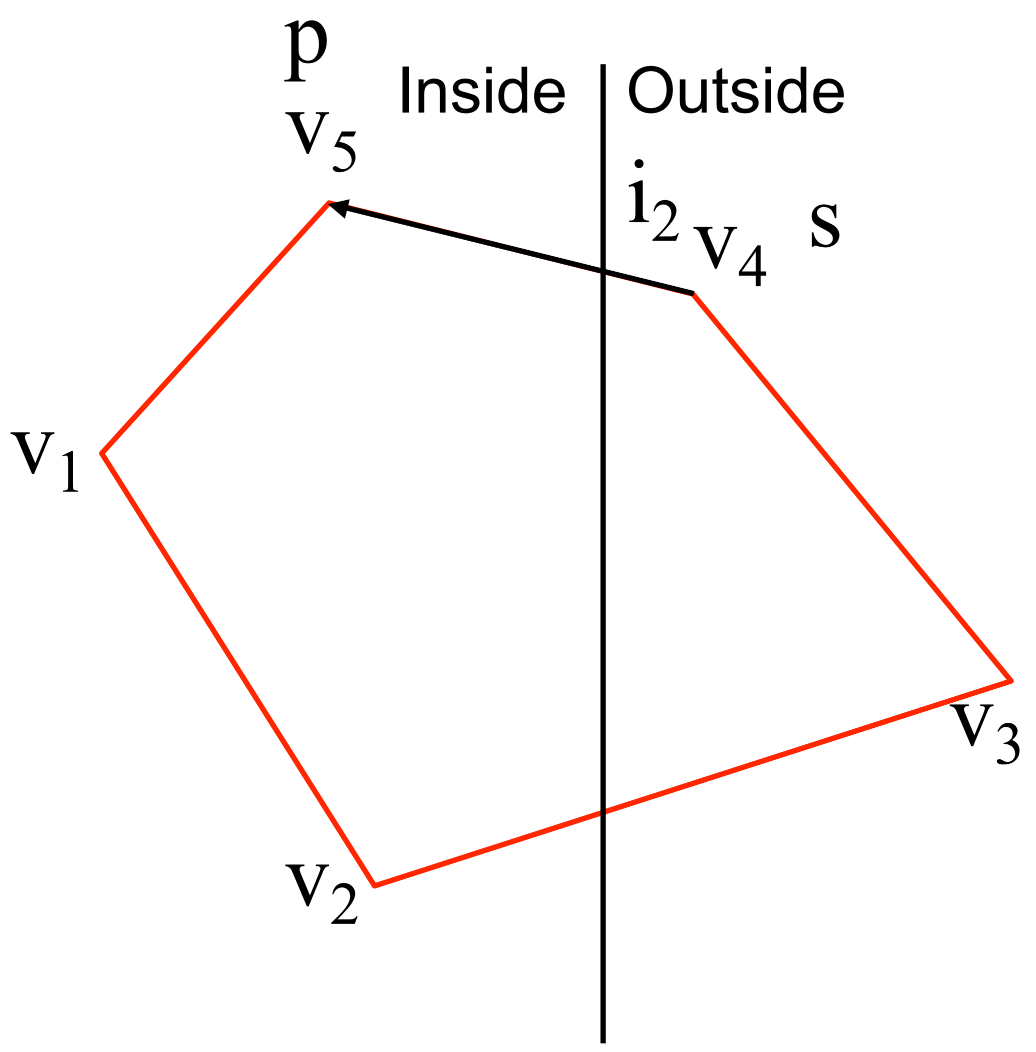


Outside, Outside
No output

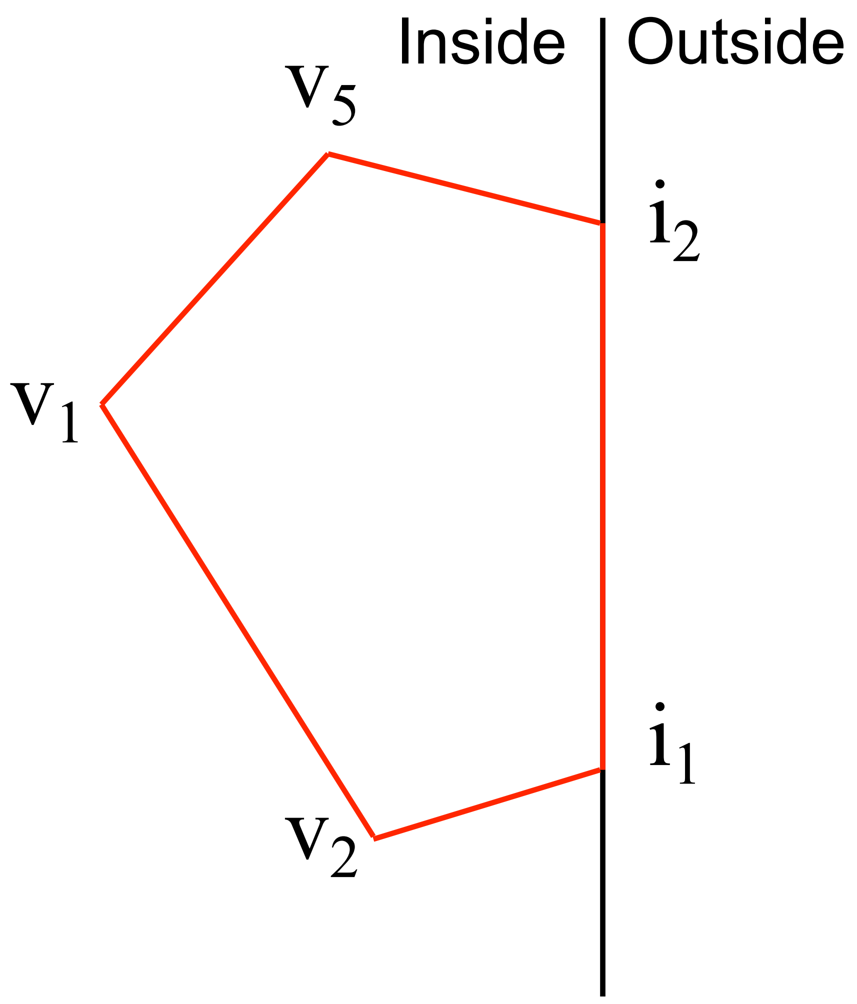


Current
Output

v_4, v_5 – last edge…



Outside, Inside
Output i_2, v_5



Current
Output