# Computer Graphics 2014

# 8. Hidden Surface Elimination

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# Visual Realism

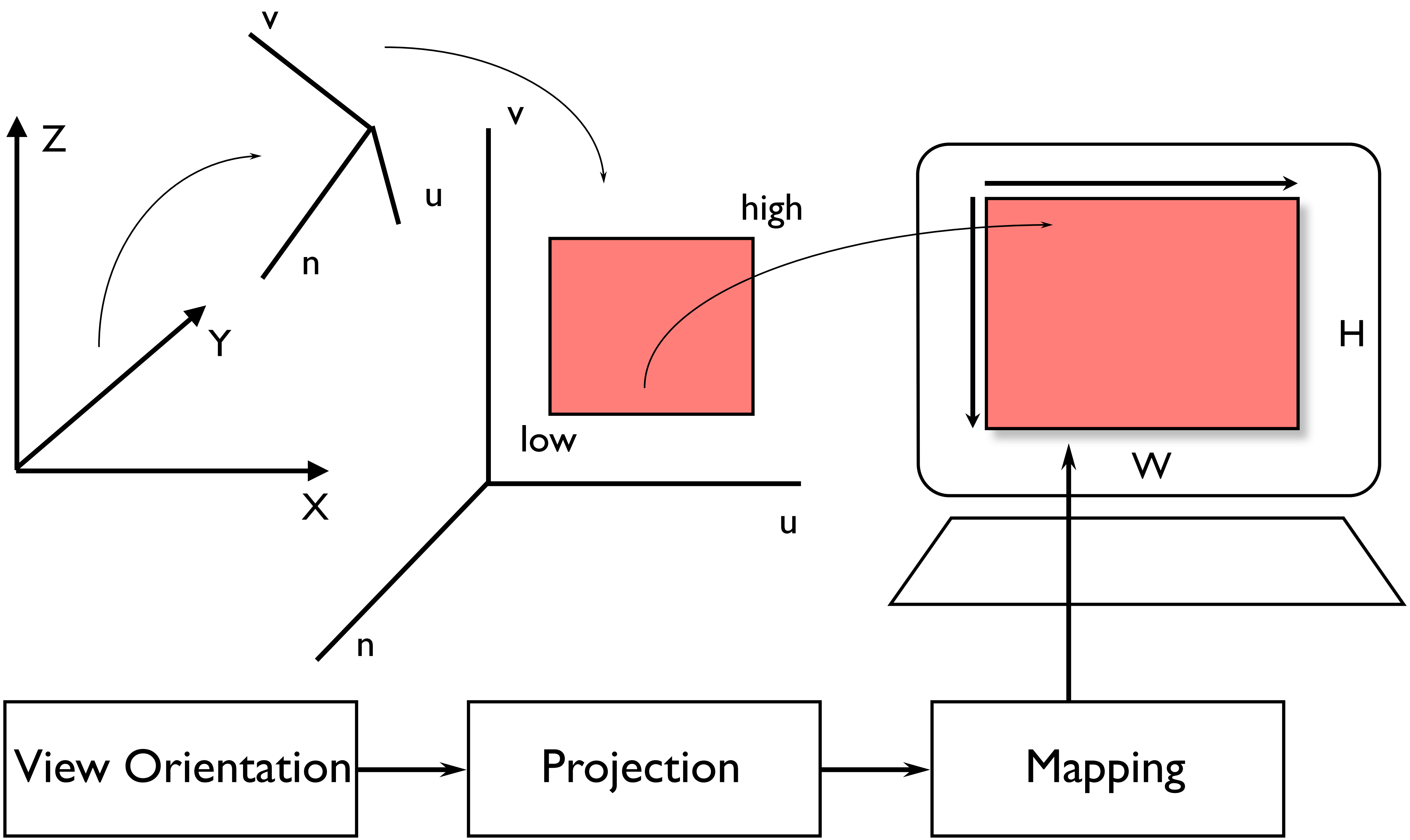
- Achieved by correct rendering of :
  - View (perspective)
  - Field of view (Clip outside the window)
  - Omit hidden parts
  - Surface details like texture
  - Light effects on surfaces like continuous shading, shadows, and caustics.
  - Volumetric effects like transparency and translucency through participating media like water, steam, smoke, …
  - Dynamic effects like movement, elasticity, …

# OpenGL functions

- glEnable / glDisable (GL_CULL_FACE);
- glCullFace(mode)

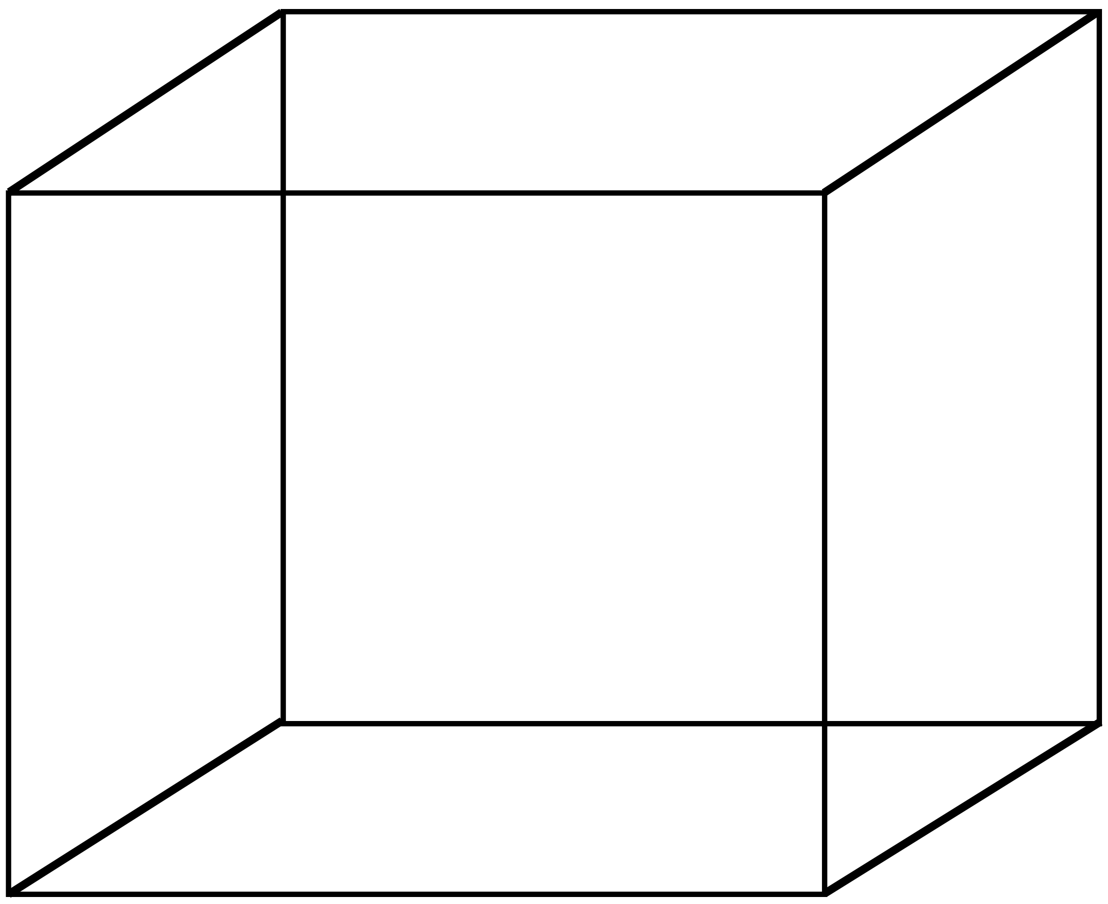
- glutInitDisplayMode( ... | GLUT_DEPTH )
- glEnable(GL_DEPTH_TEST)
- glEnable(GL_FOG) glFog*()

# Viewing Pipeline Review

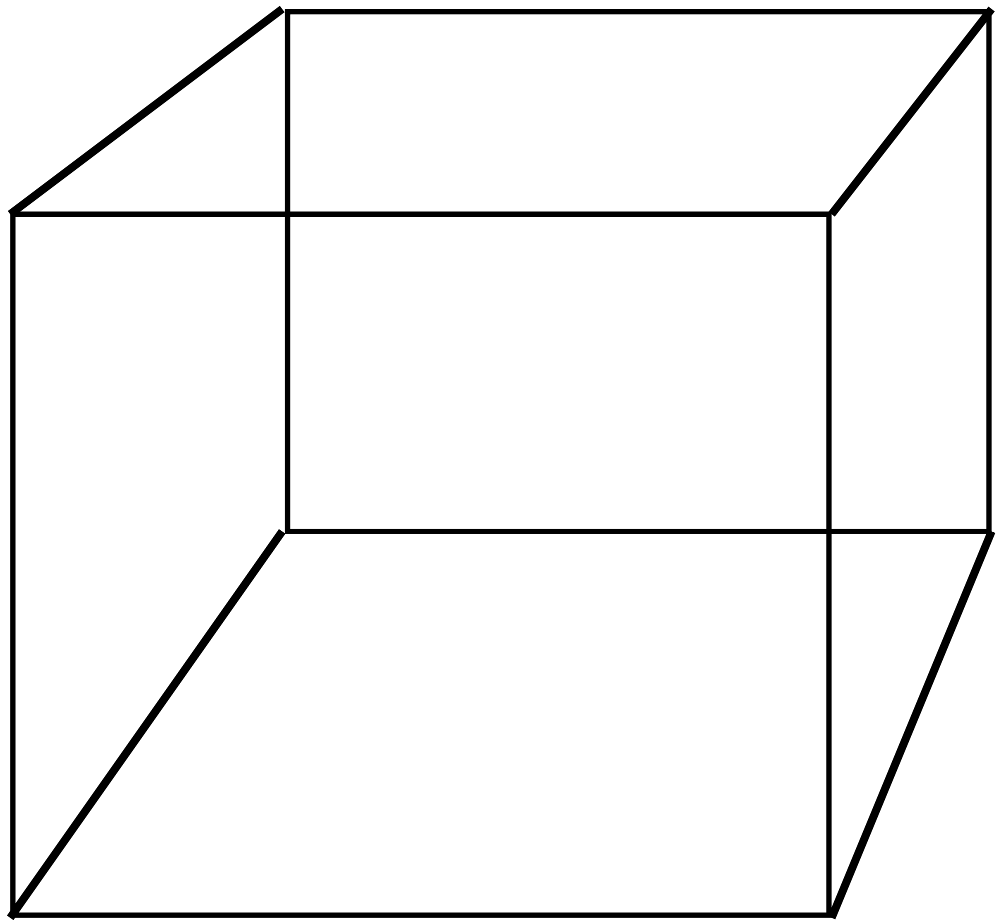


# Projection

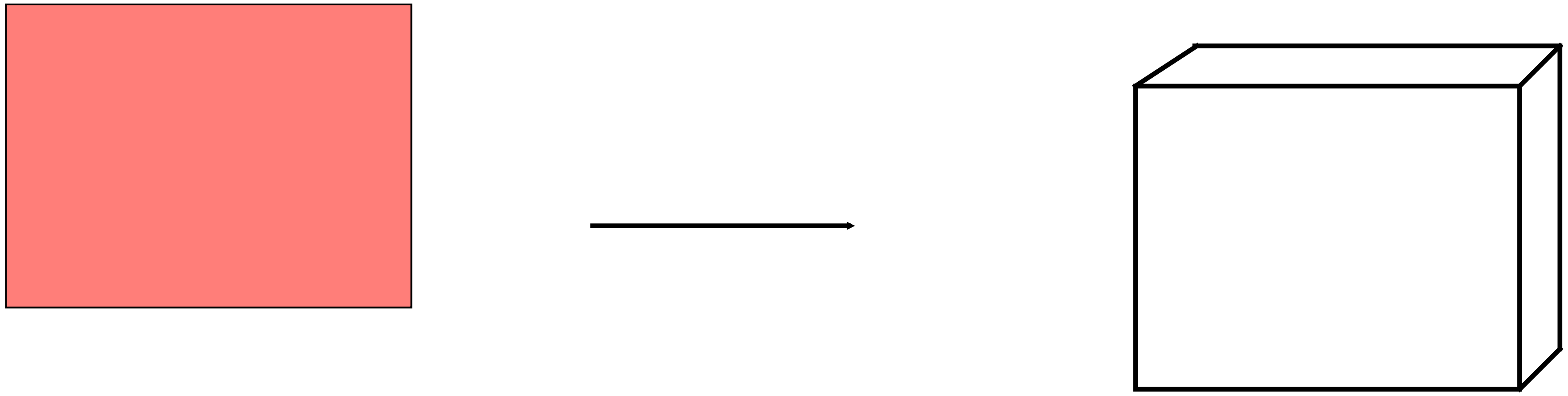
Orthographic



Perspective



# Visible Line Drawing



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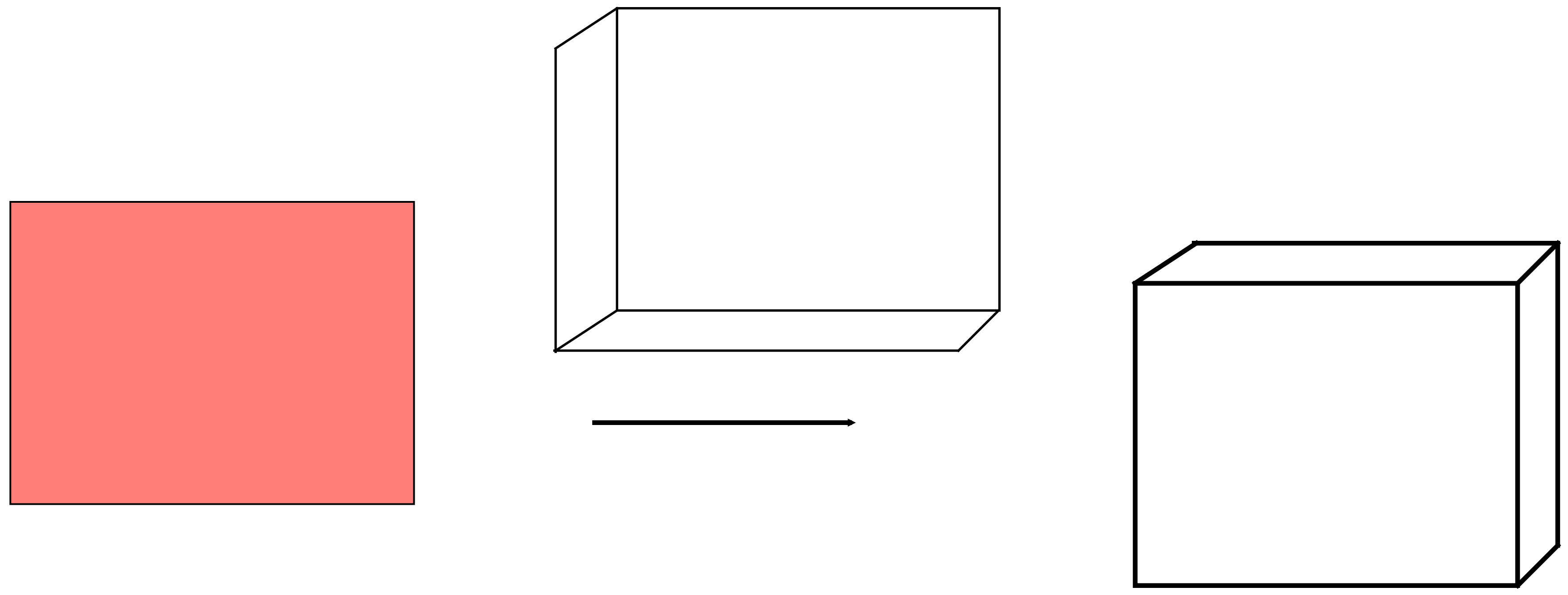

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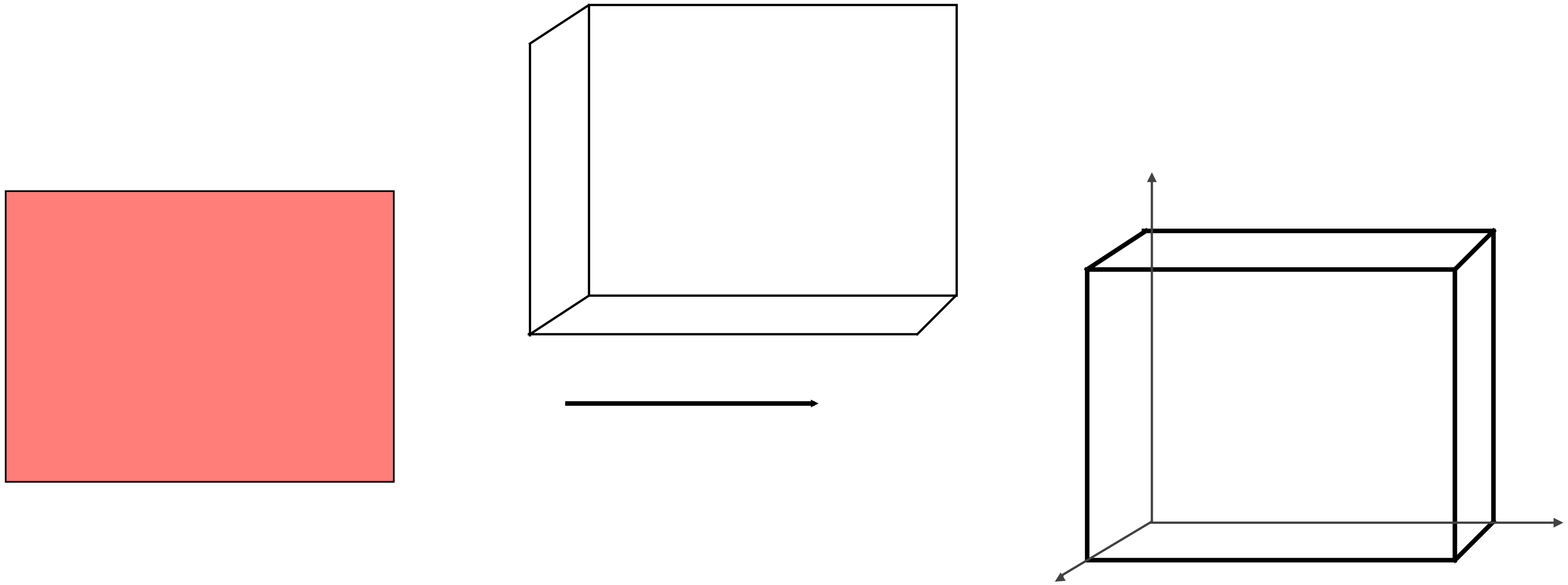

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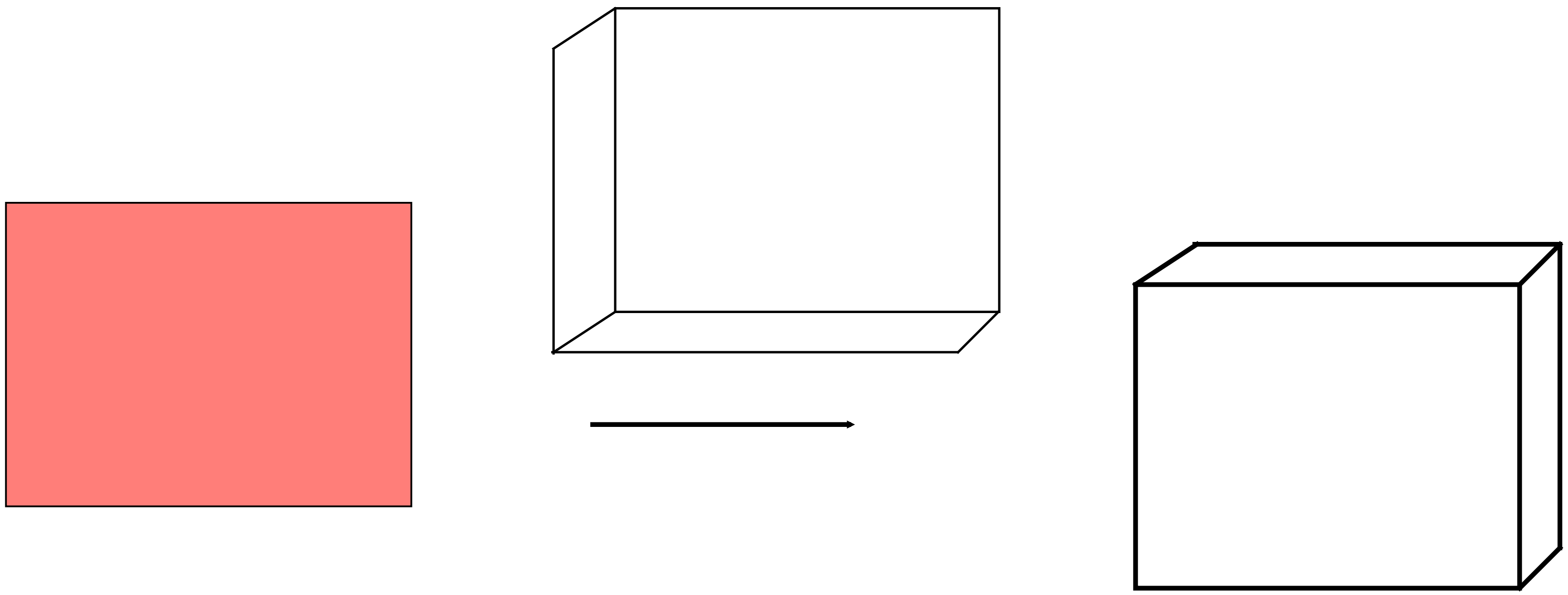

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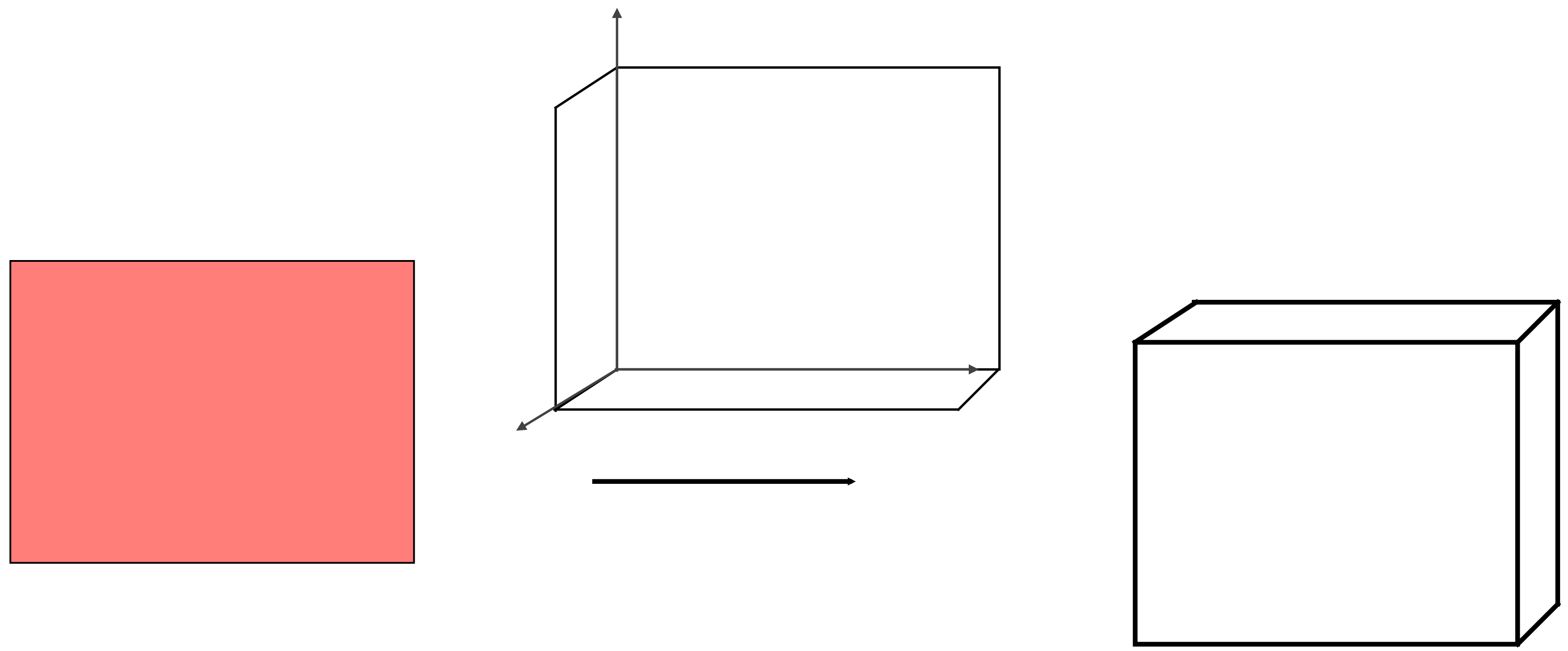


# Visible Surface Determination

- Goal
  - Given: a set of 3D objects and Viewing specification,
  - Determine: those parts of the objects that are visible when viewed along the direction of projection
- Or, equivalently, elimination of hidden parts (hidden lines and surfaces)
- Visible parts will be drawn/shown with proper colors and shades

# HLHSR Algorithms

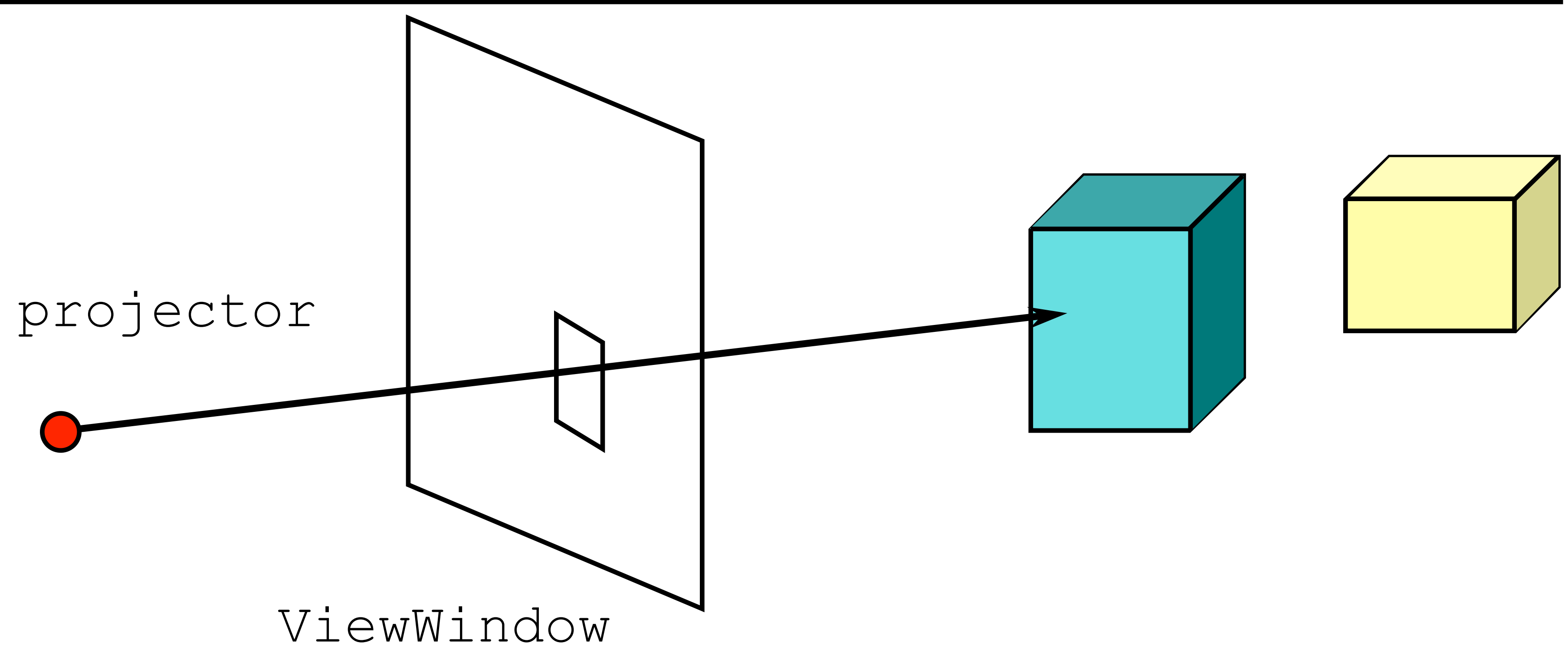
- Two Fundamental Approach
  - Object space algorithm
    - a.k.a. **Object Precision** ~
    - hidden line remove
  - Image space algorithm
    - a.k.a. **Image Precision** ~
    - z-buffer

# Object Precision Algorithm

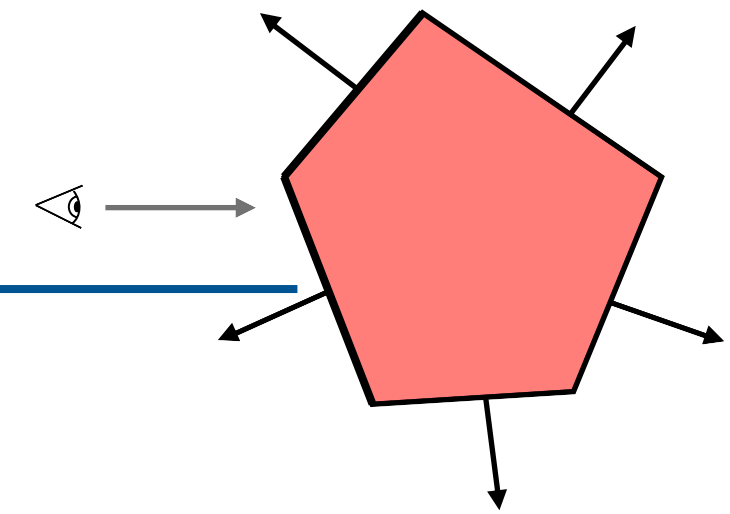
```
foreach (object in the world)  {
    determine those parts of the object whose view is
      unobstructed by other parts of it or any other object;
    draw those parts in the appropriate color;
}
```

# Image Precision Algorithms

```
foreach (pixel in the image ) {
    determine the object closest to the viewer that is pierced
      by the projector through the pixel;
    draw the pixel in the appropriate color;
}
```



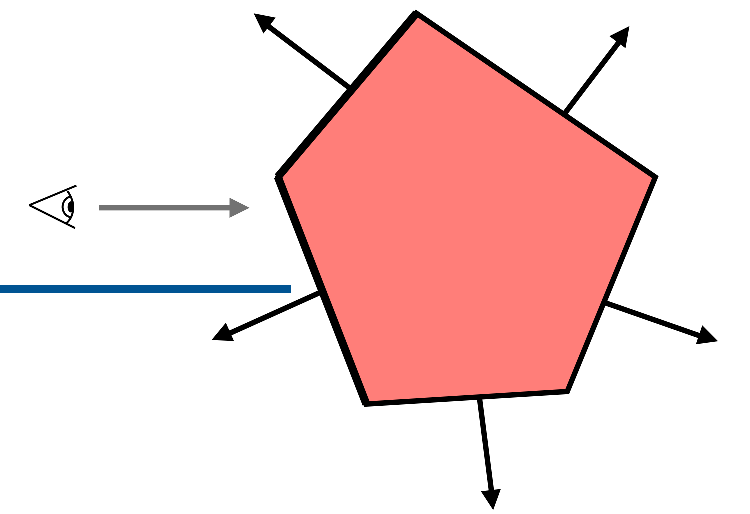
# Back-face Culling



- In a closed polygonal surface
  - i.e. the surface of a polyhedral volume or a solid polyhedron
  - The faces whose outward normals point away from the viewer are not visible
  - Such back-facing faces can be eliminated from further processing

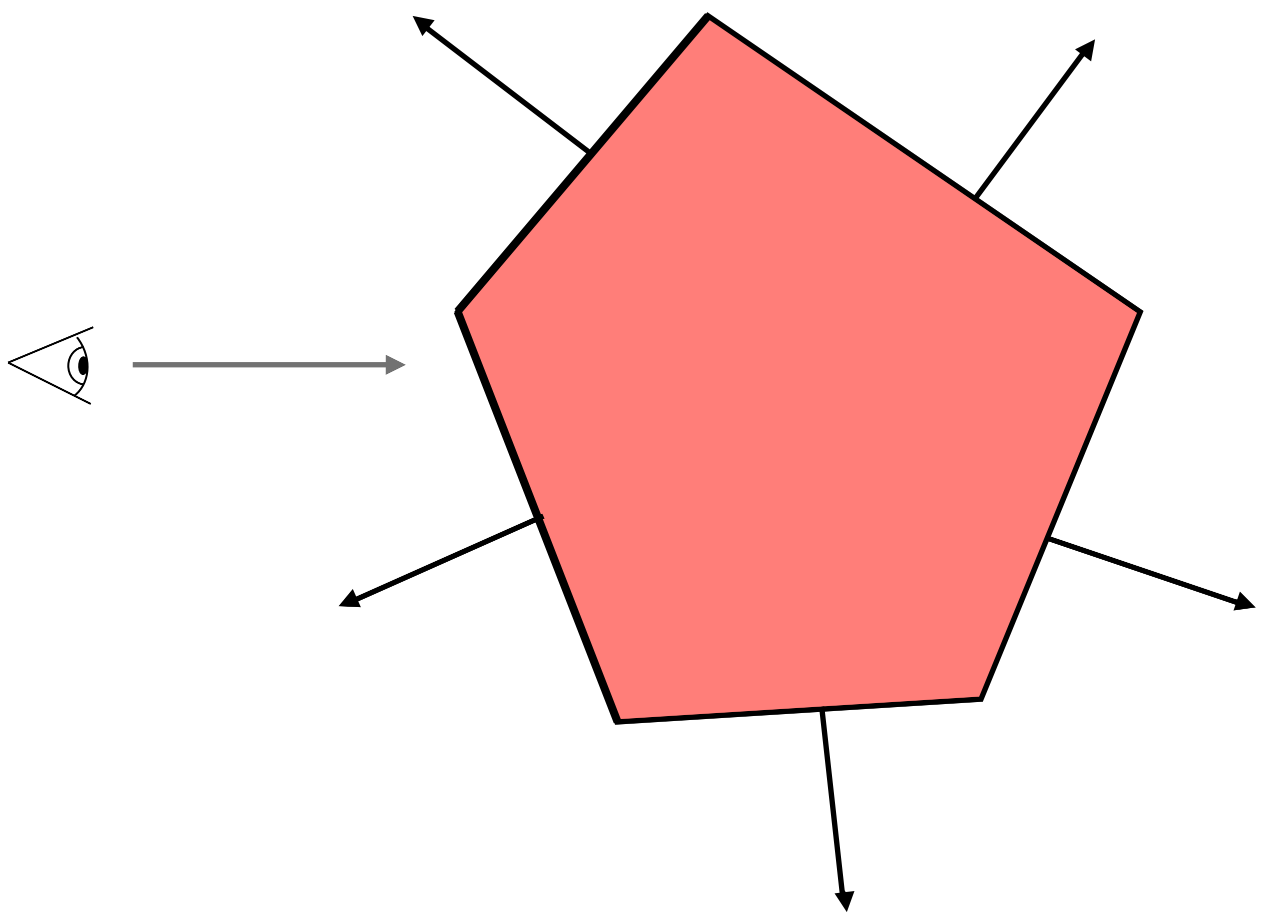
- Elimination of back-faces is called back-face culling

# Back-Face Culling



- Back Face:
    - Part of the object surface facing away from the eye.
    - i.e. surface whose normal points away from the eye position.

# Back-Face Culling



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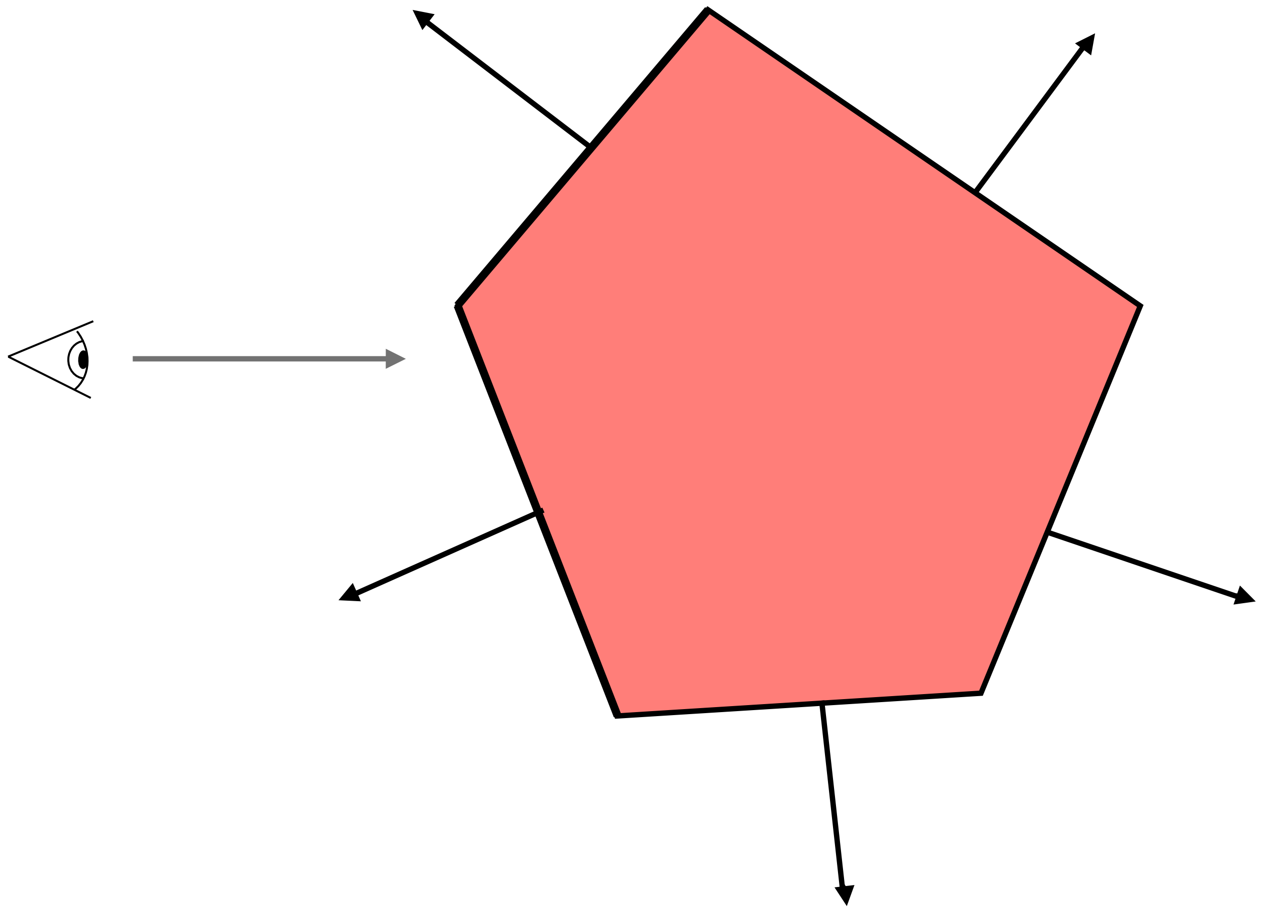


Algorithm:

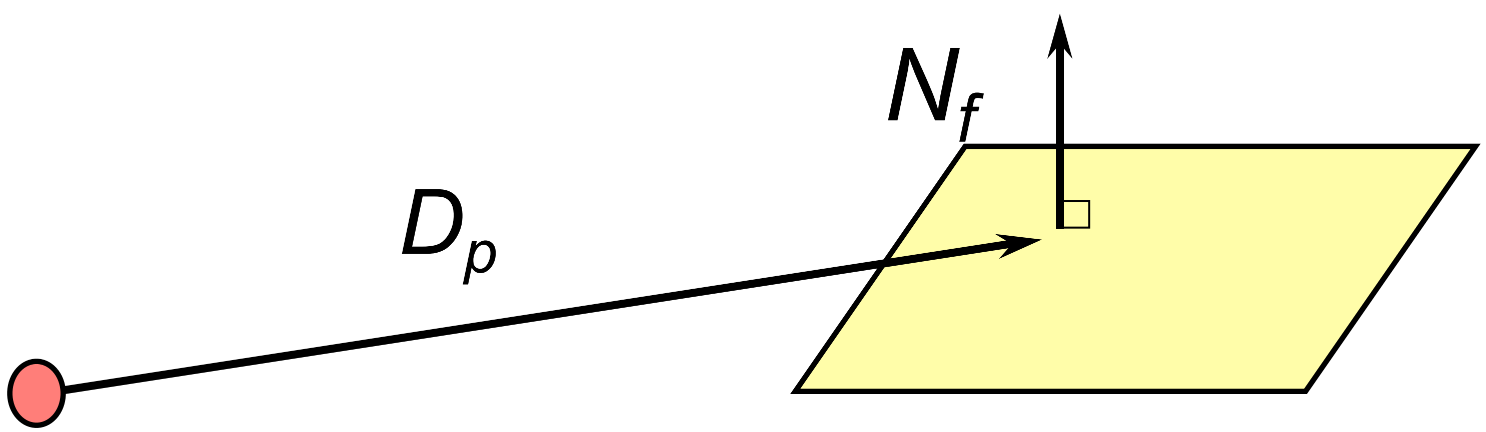
1. Find angle between the eye-vector & normal to face.
2. If between 0 to 90°, discard the face.

# Back-face Culling

- **Determination of back-faces**

A polygonal face with outward surface normal $N_f$ is a back-face if $N_f \circ D_p > 0$

where $D_p$ is the direction of projection

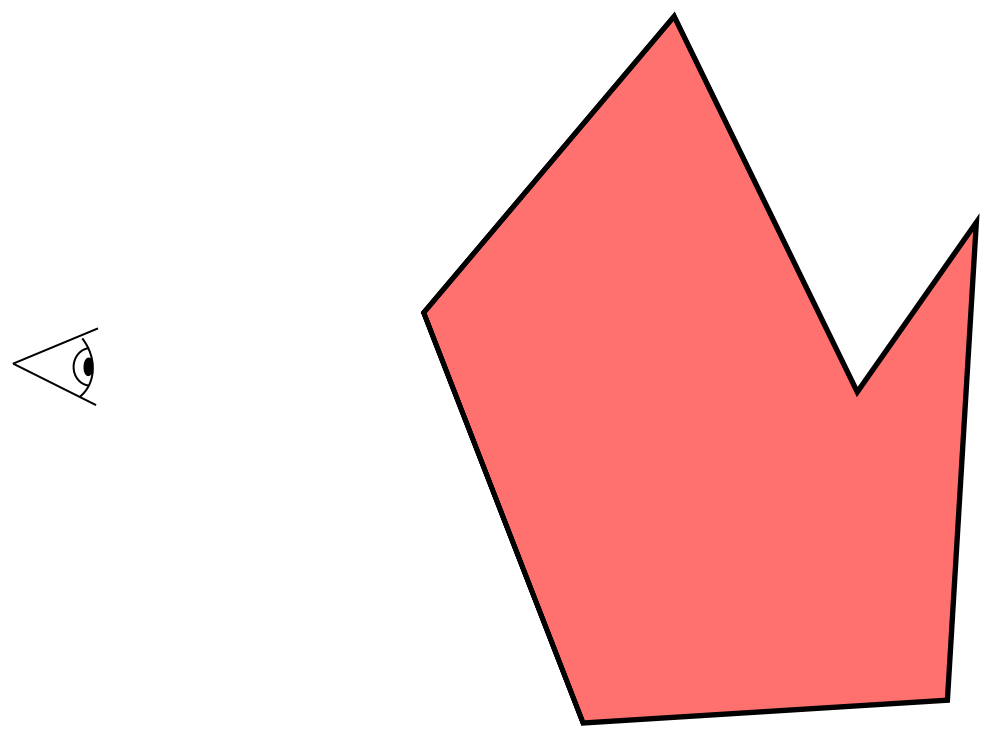



What happens when the projectors are along Z axis, i.e., (0,0,1) is the view direction.

Let $N_f = (n_x, n_y, n_z)$, the dot product now equals $n_z$. If this is +ve, then this is a back-face!

# Back-Face Culling

Back-face culling does not solve all visibility problems



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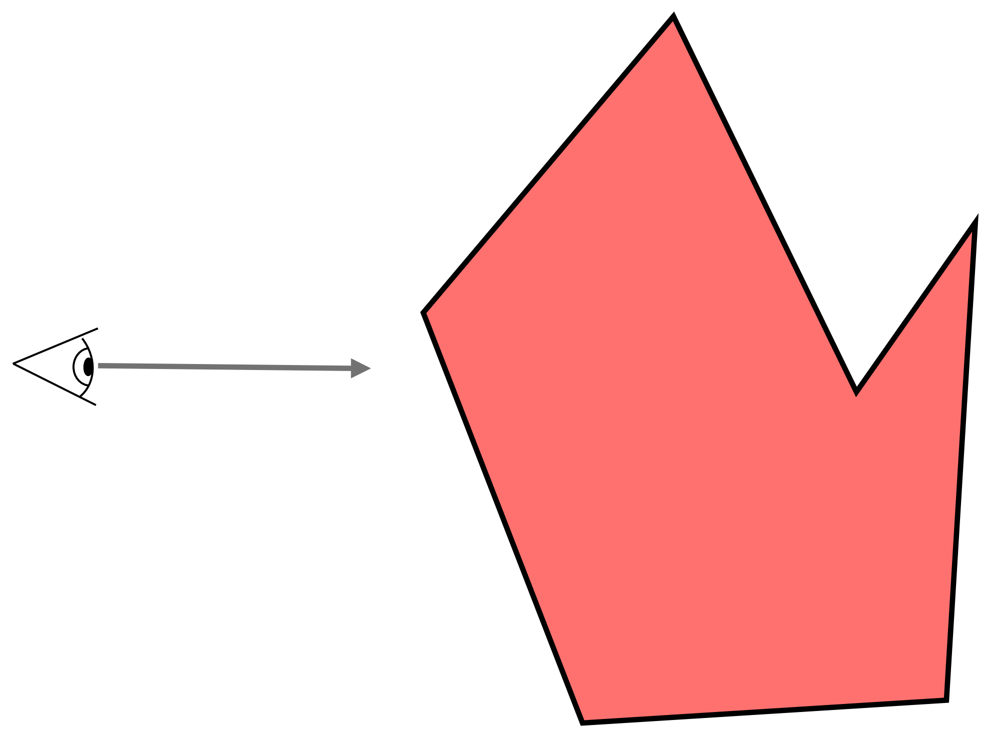

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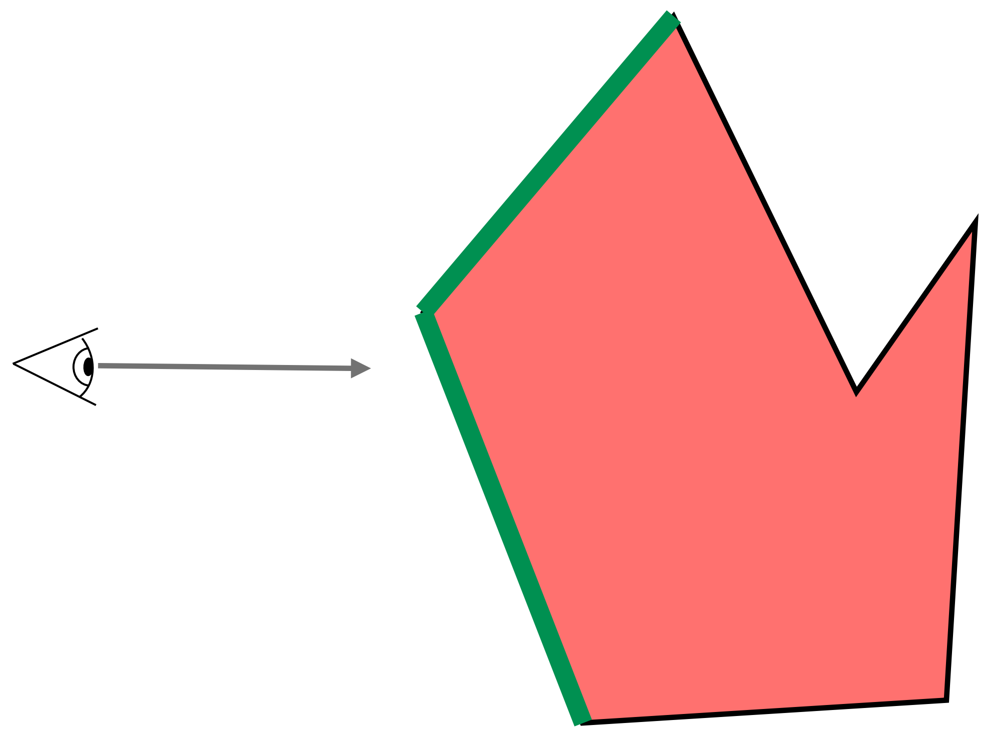

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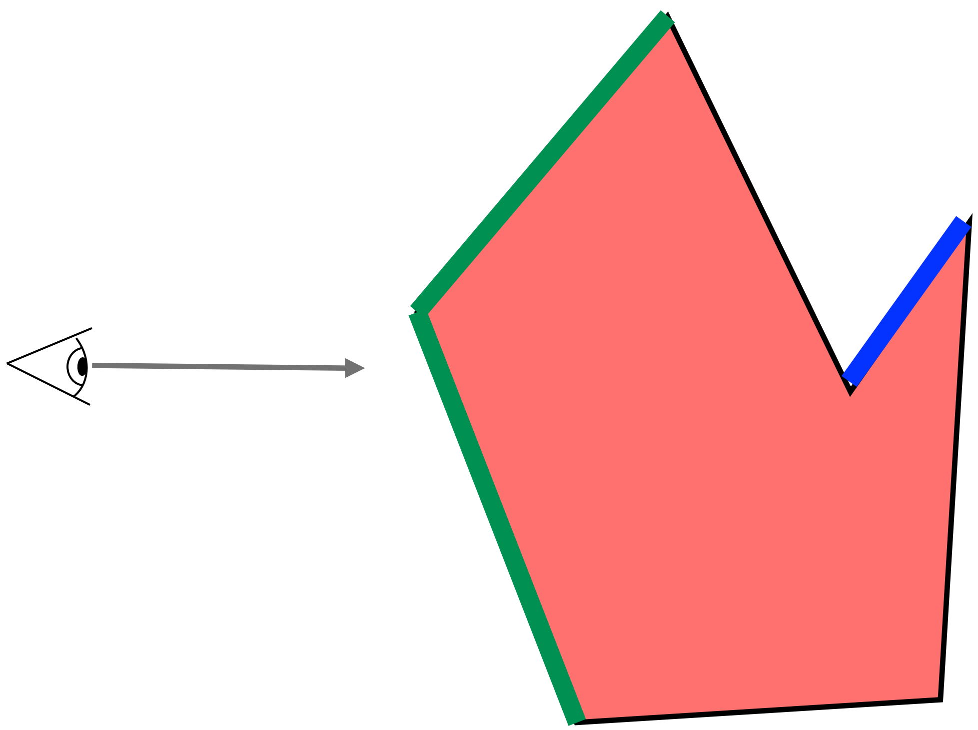

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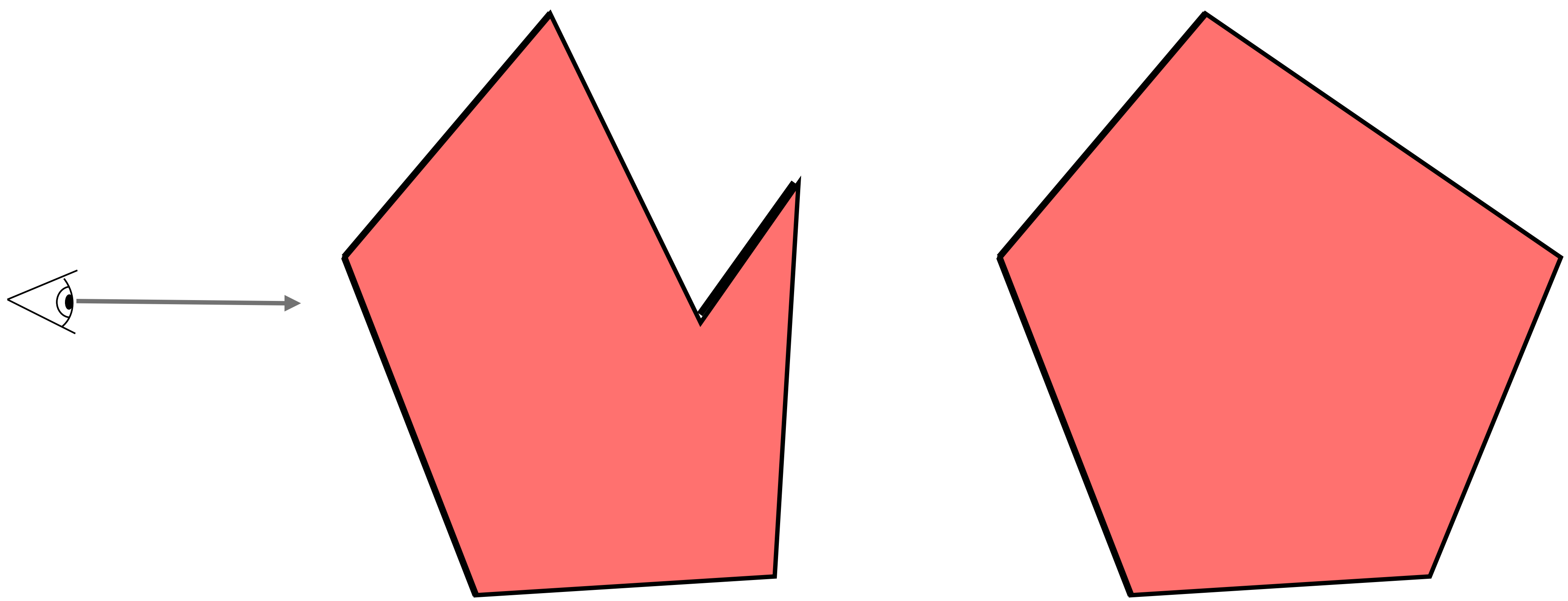
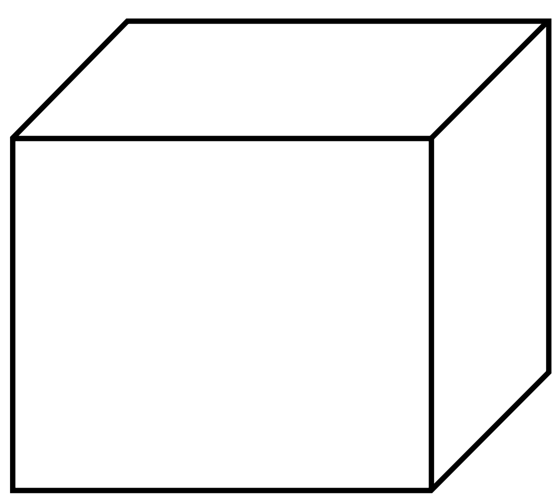
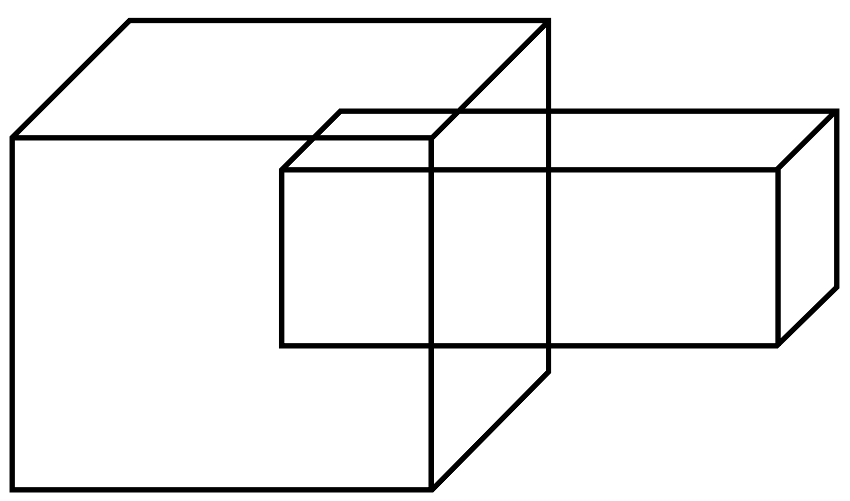


# Back-face Culling

If the scene consists of a **single convex closed** polygonal surface then back-face culling is equivalent to HLHSR





# Hidden Surface Removal

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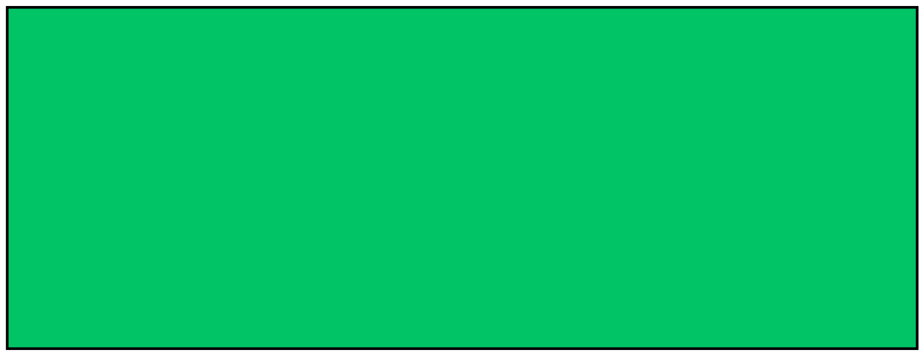

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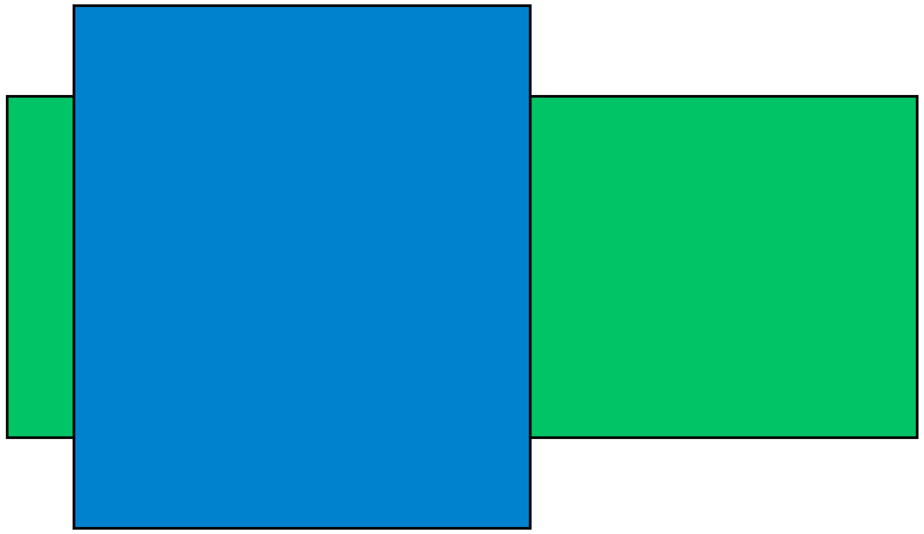

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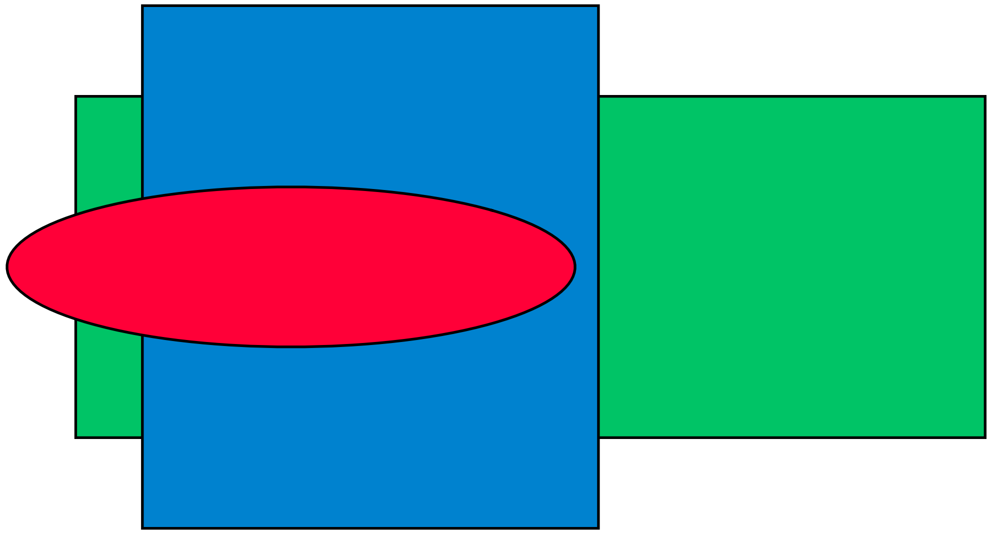

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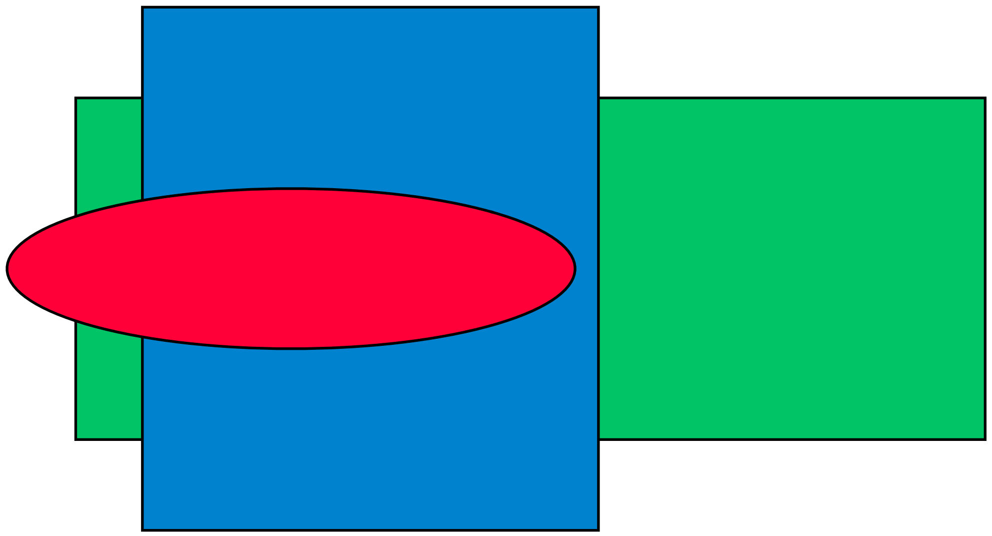
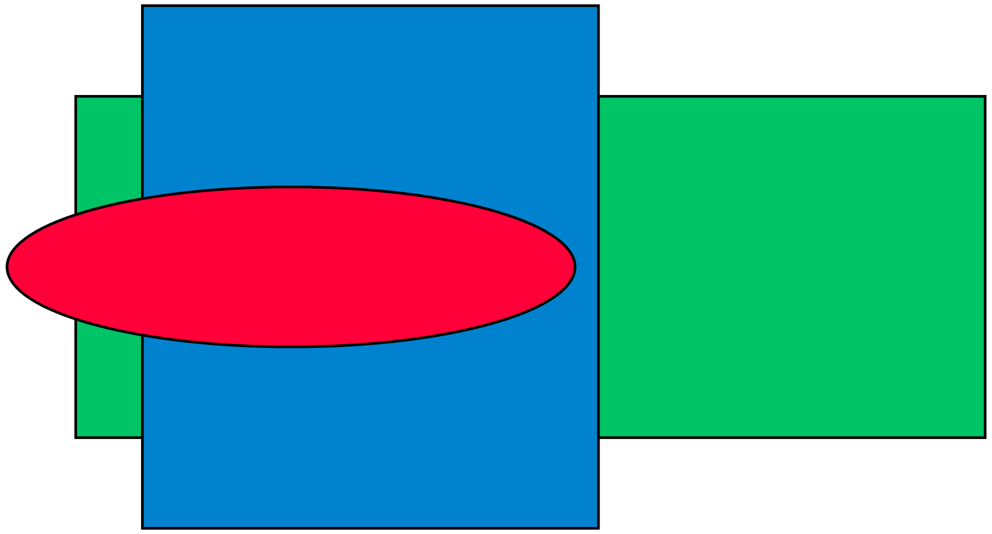


Painter's Algorithm

From back to Front

# Hidden Surface Removal



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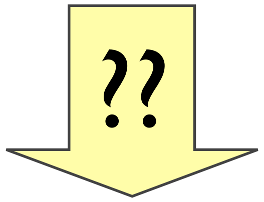

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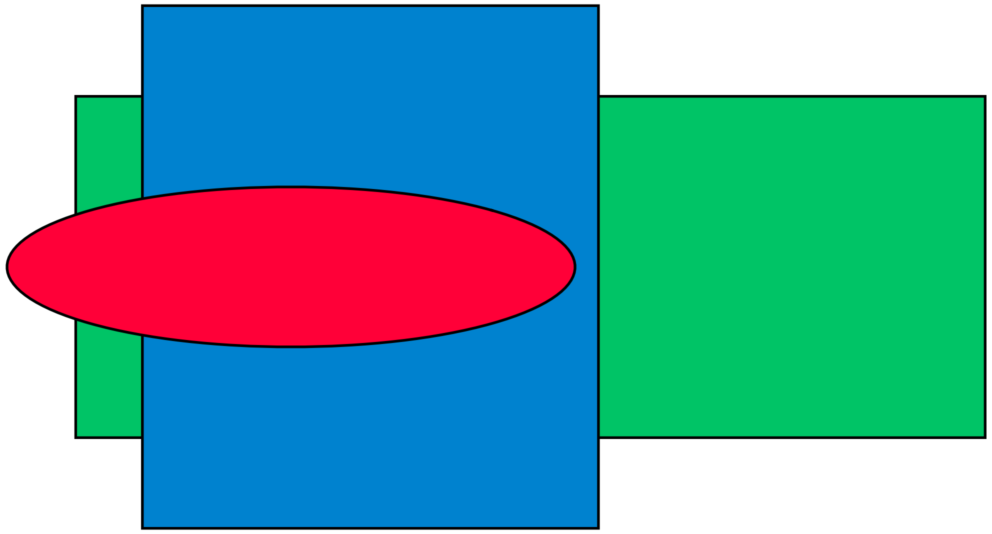

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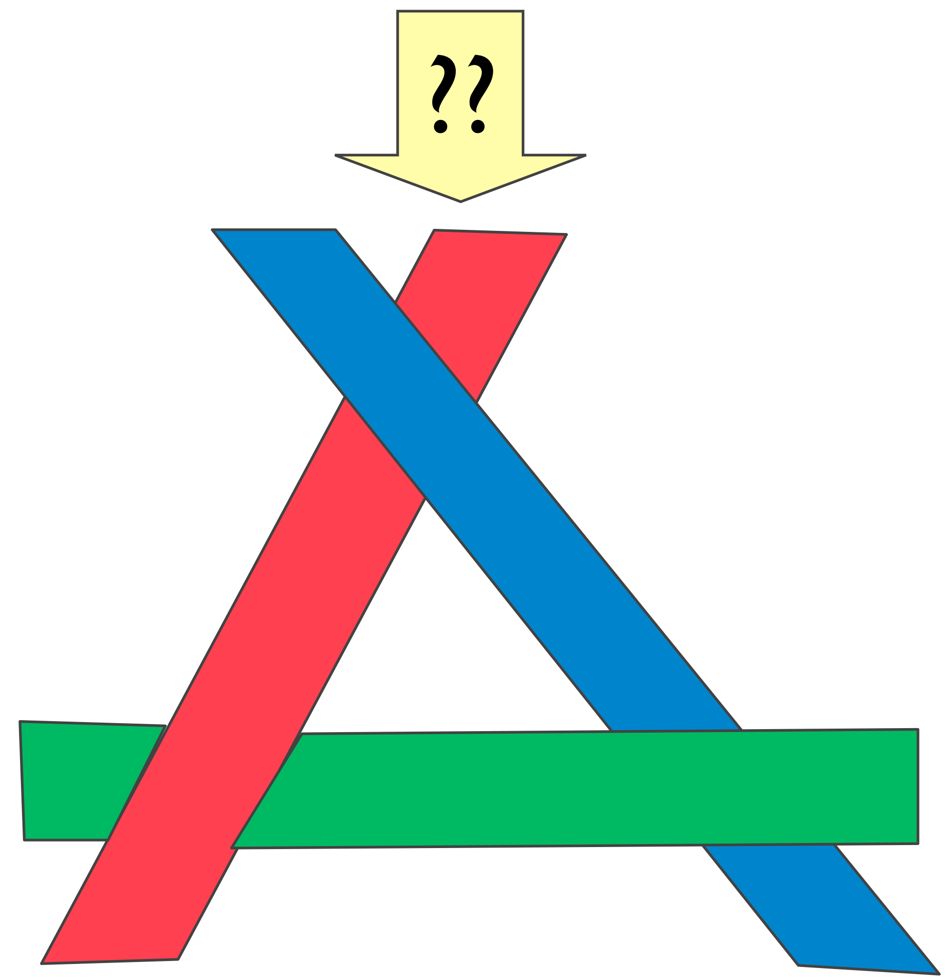

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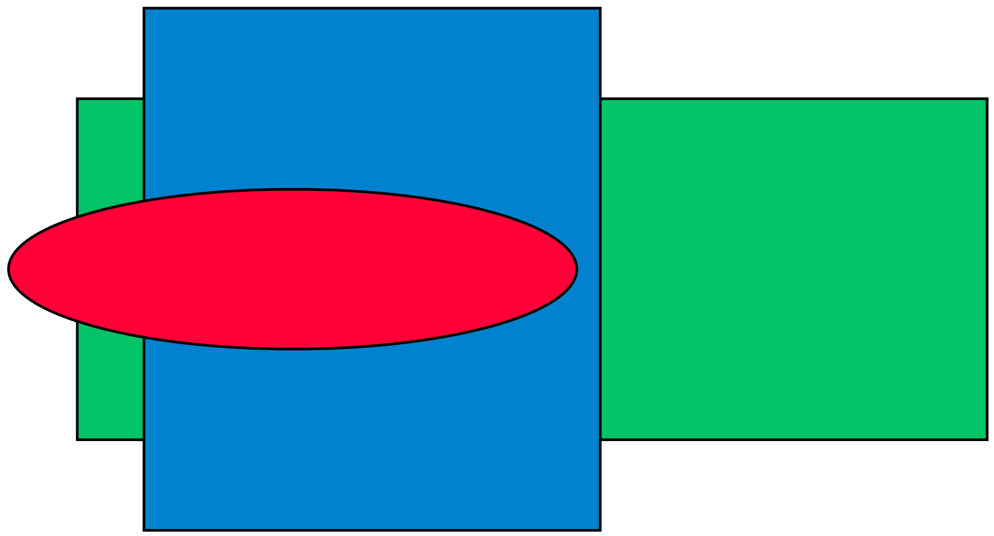

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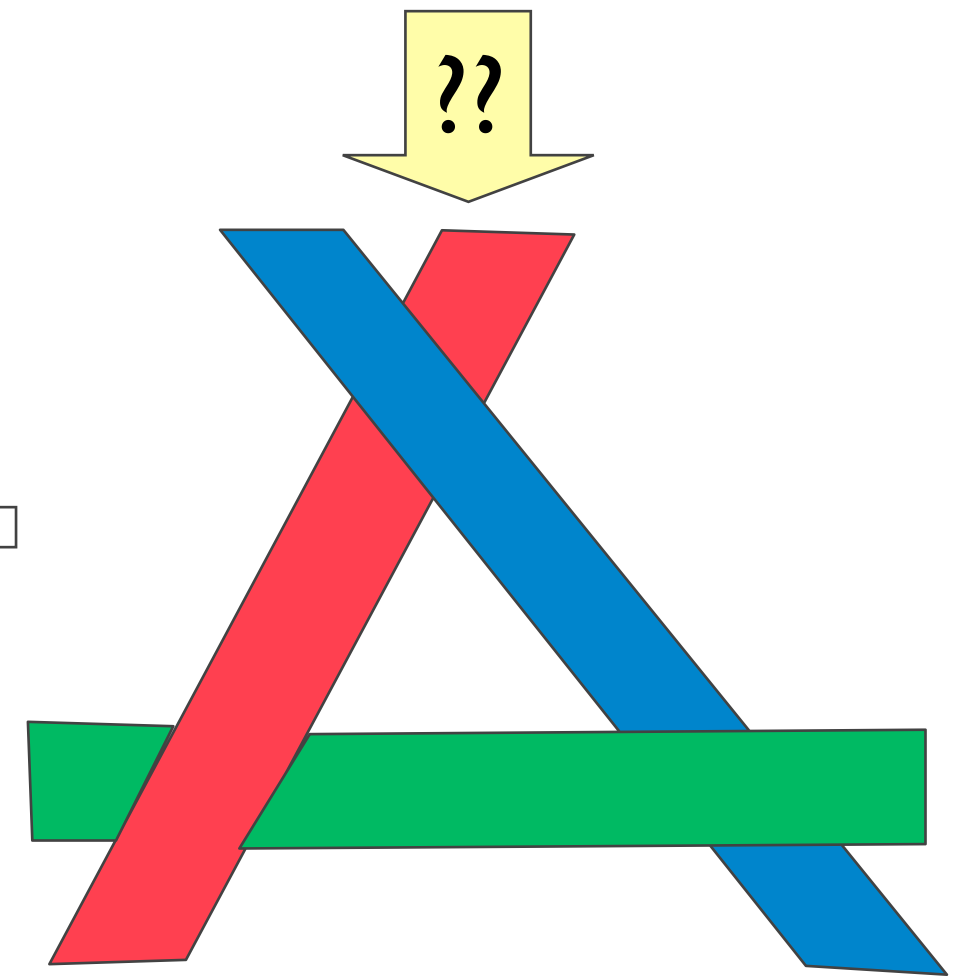
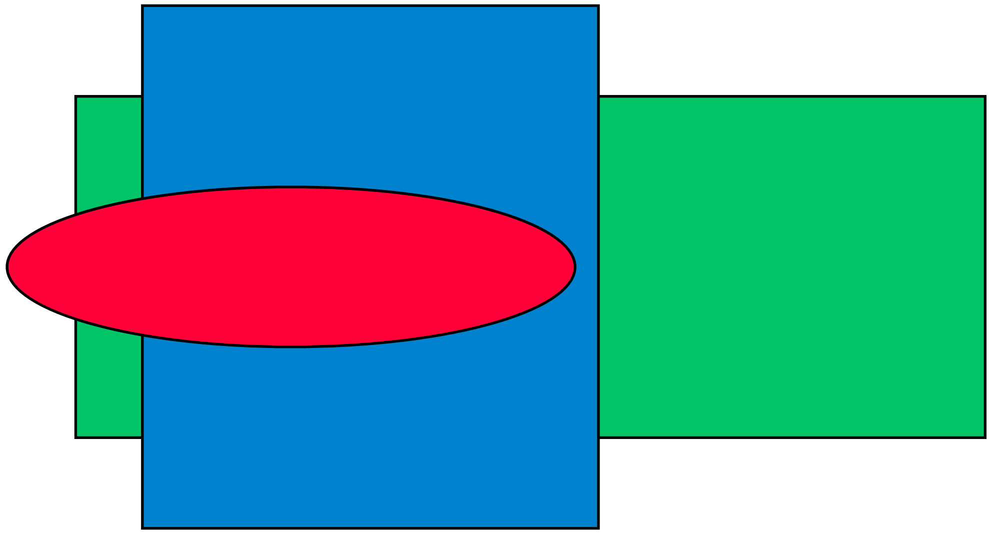



**Clipping**

# Hidden Surface Removal



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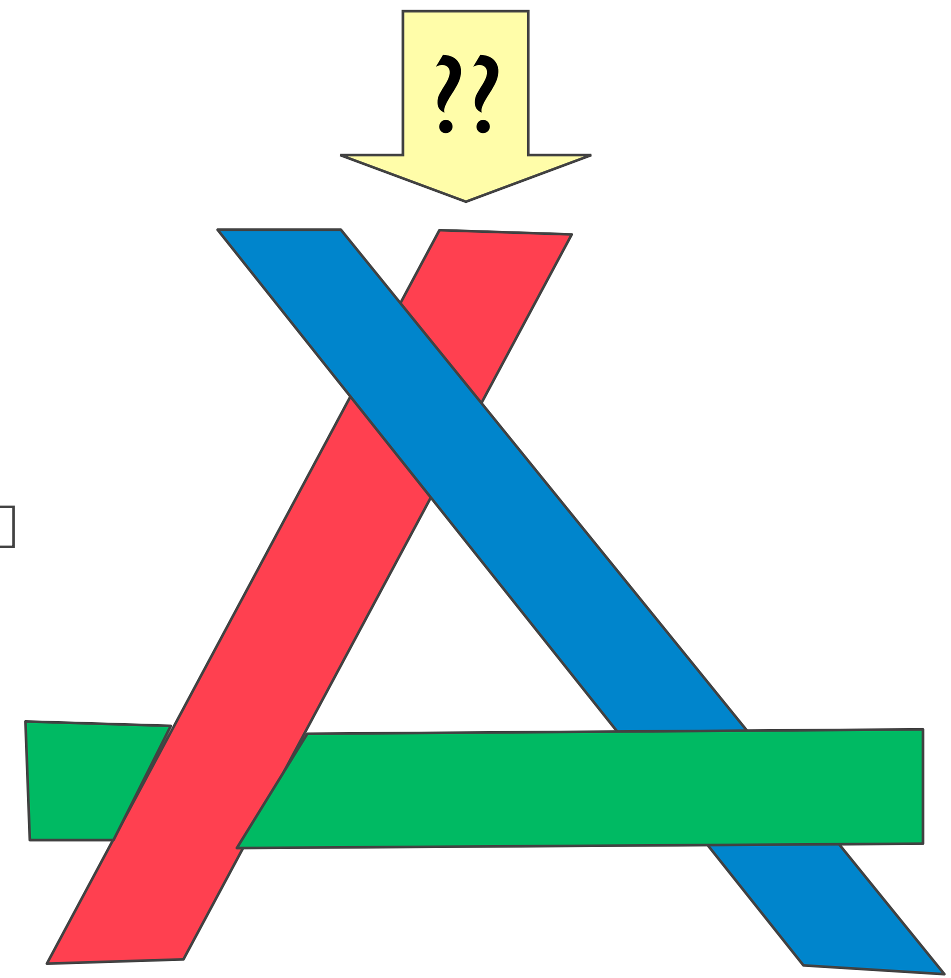



**Area Sorting**

**Clipping**

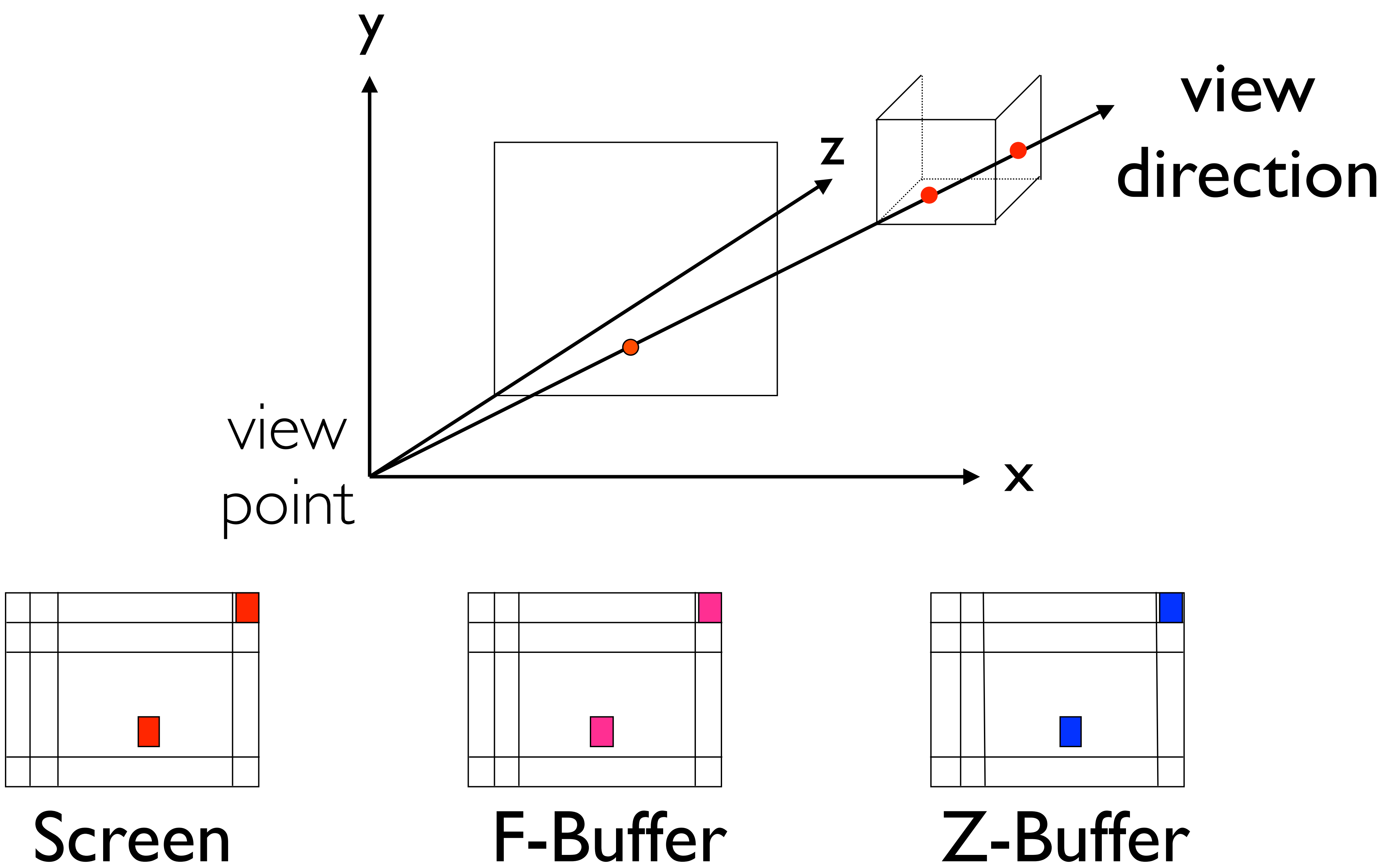
# Z-Buffer Algorithm

- Image precision algorithm
  - Apart from a frame buffer F in which color values are stored,
  - it also needs a z-buffer, of the same size as the frame buffer, to store depth (z) values

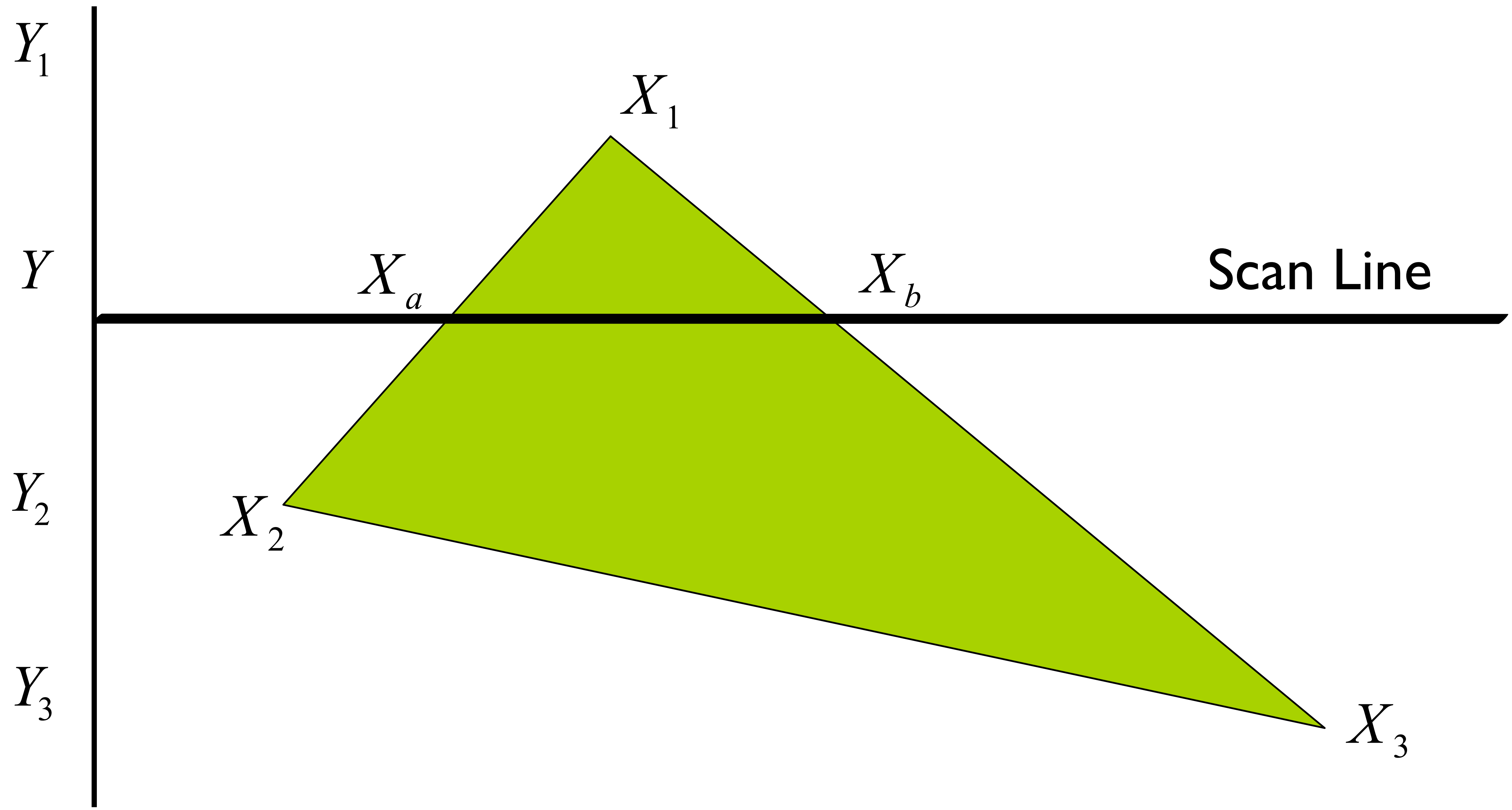
F-Buffer Z-Buffer

A.K.A. depth-buffer method

# Z-Buffer



# Polygon Scan Conversion



# Z-Buffer Pseudo-code

- for ( j=0; j<SCREEN_HEIGHT; j++ )
  - for ( i=0; i<SCREEN_WIDTH; i++ ) {
    - WriteToFrameBuffer(i, j, BackgroundColor);
    - WriteToZBuffer(i, j, MAX);
  - }

- for ( each polygon )
  - for ( each pixel in polygon's projection ) {
    - z = polygon's z value at (i, j) ;
    - if ( z < ReadFromZBuffer(i, j) ) {
      - WriteToFrameBuffer(i, j, polygon's color at (i, j));
      - WriteToZBuffer(i, j, z);
    - }
  - }

# Z-buffer：

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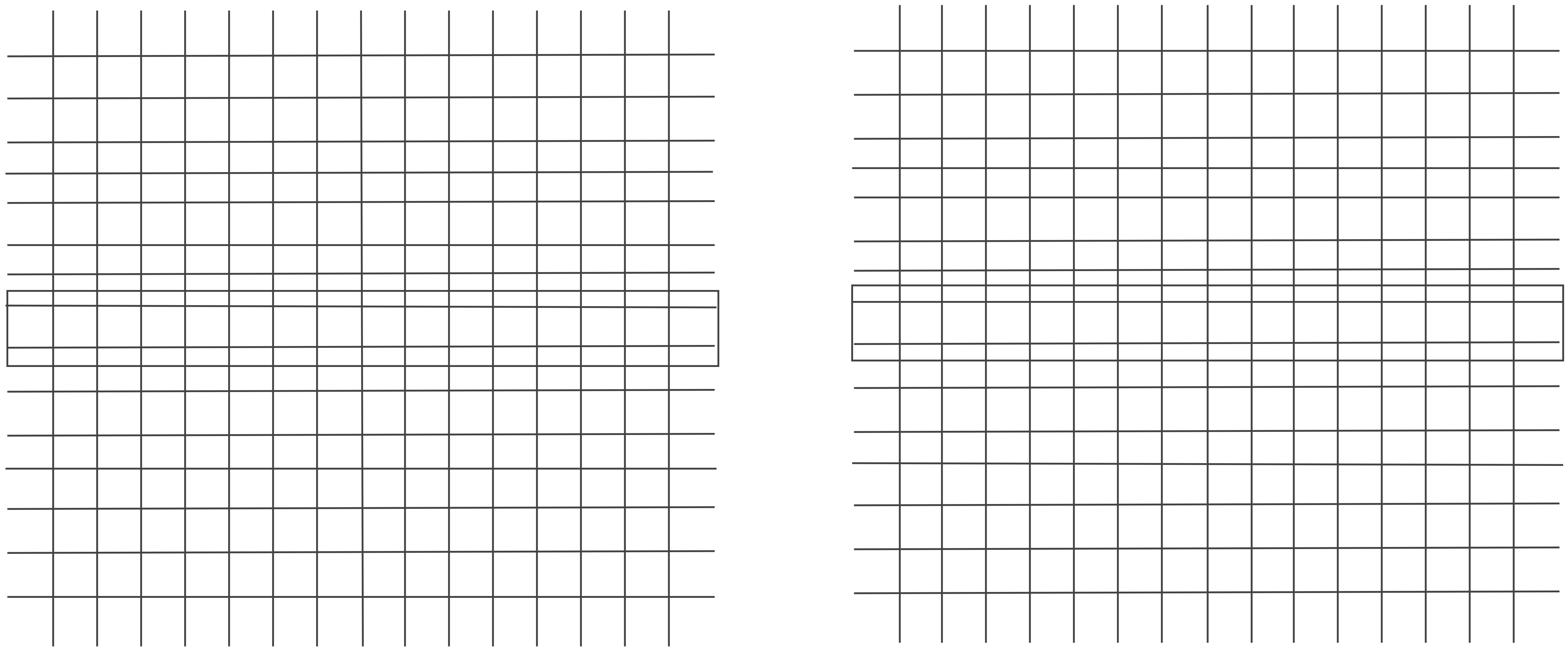

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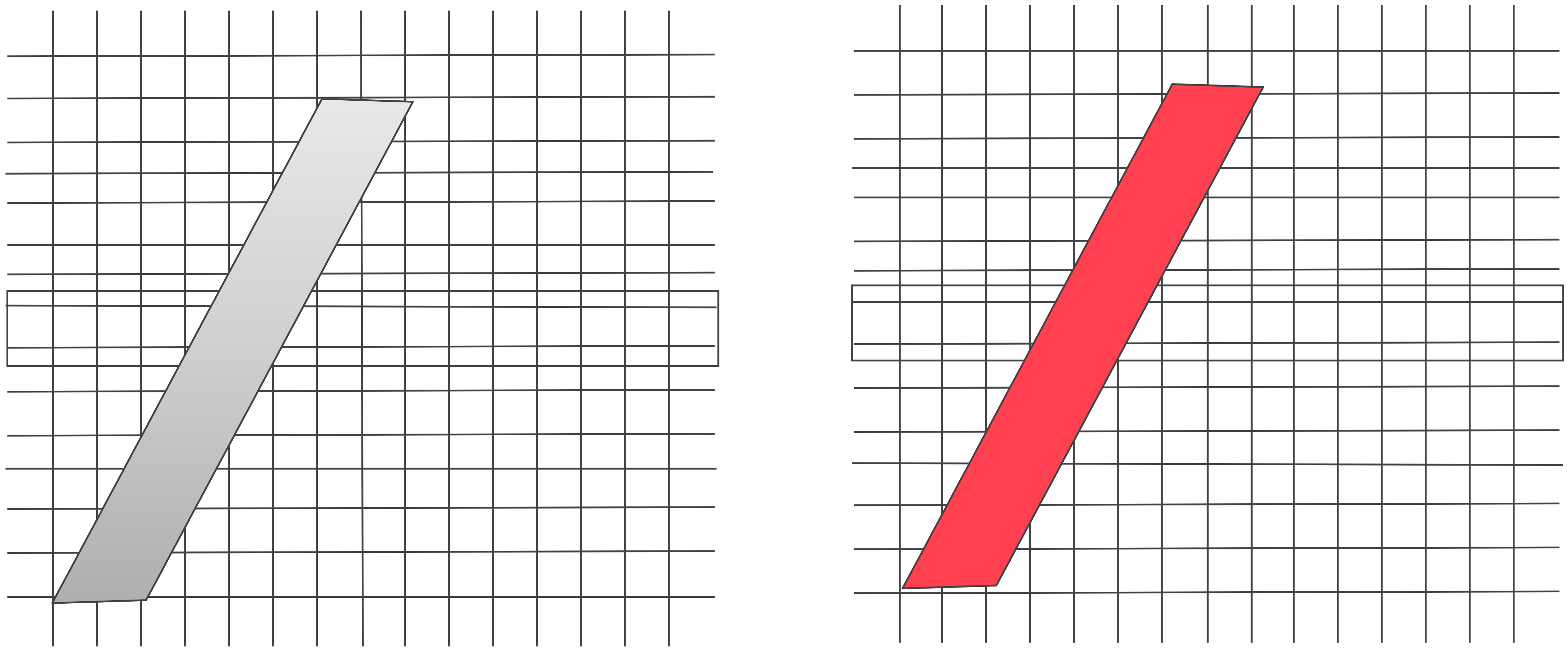

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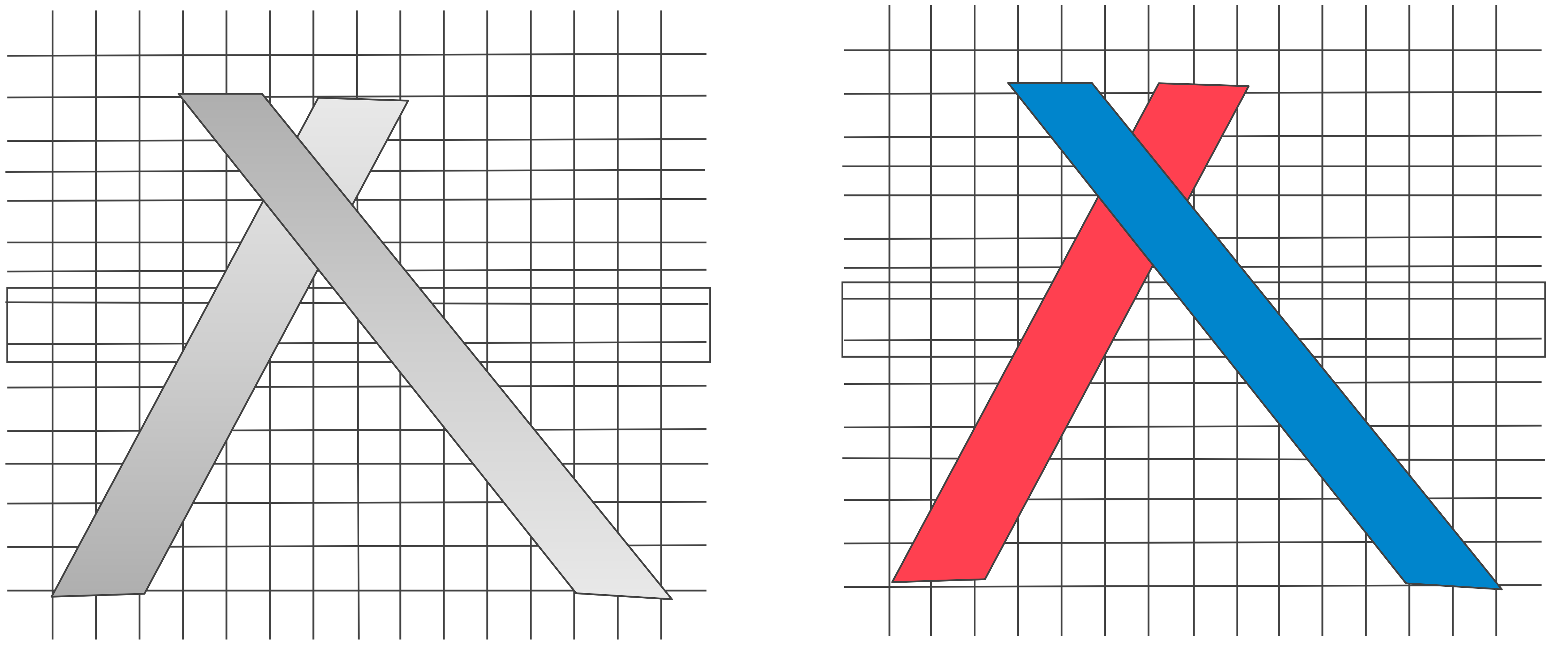

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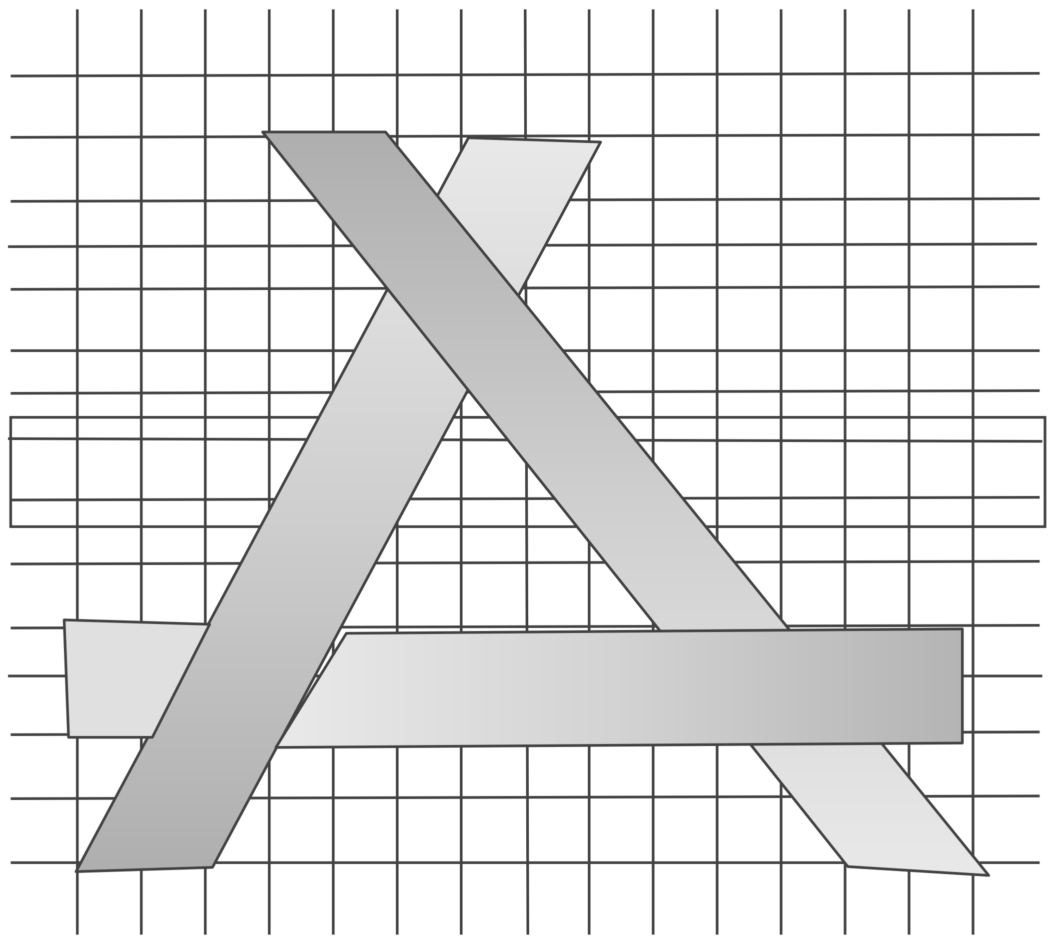

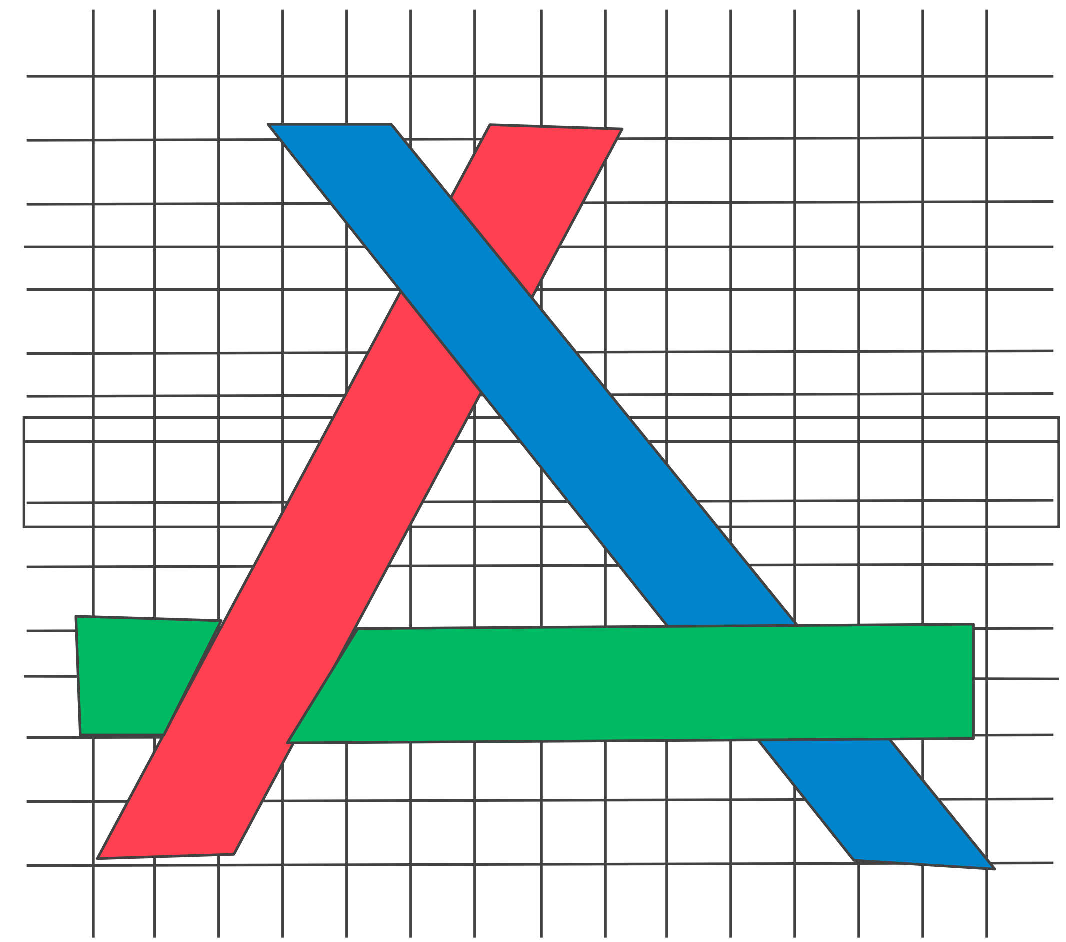

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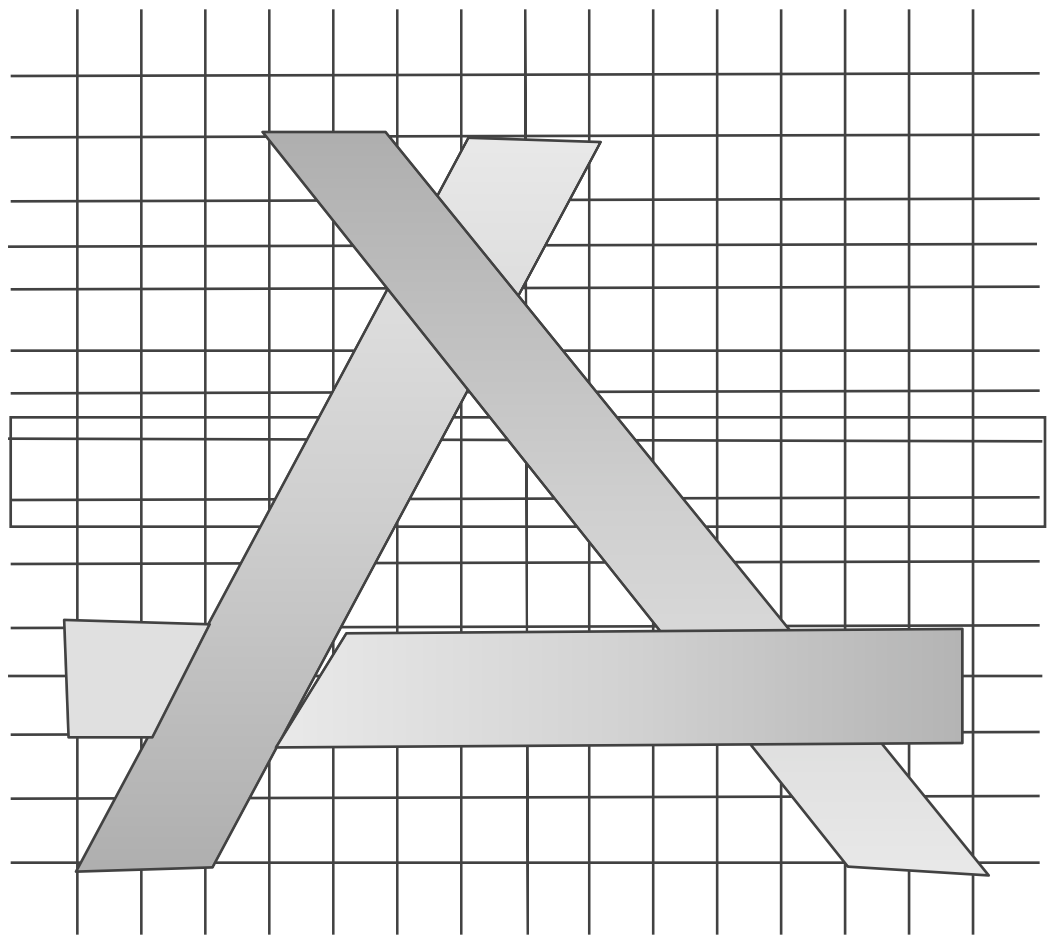
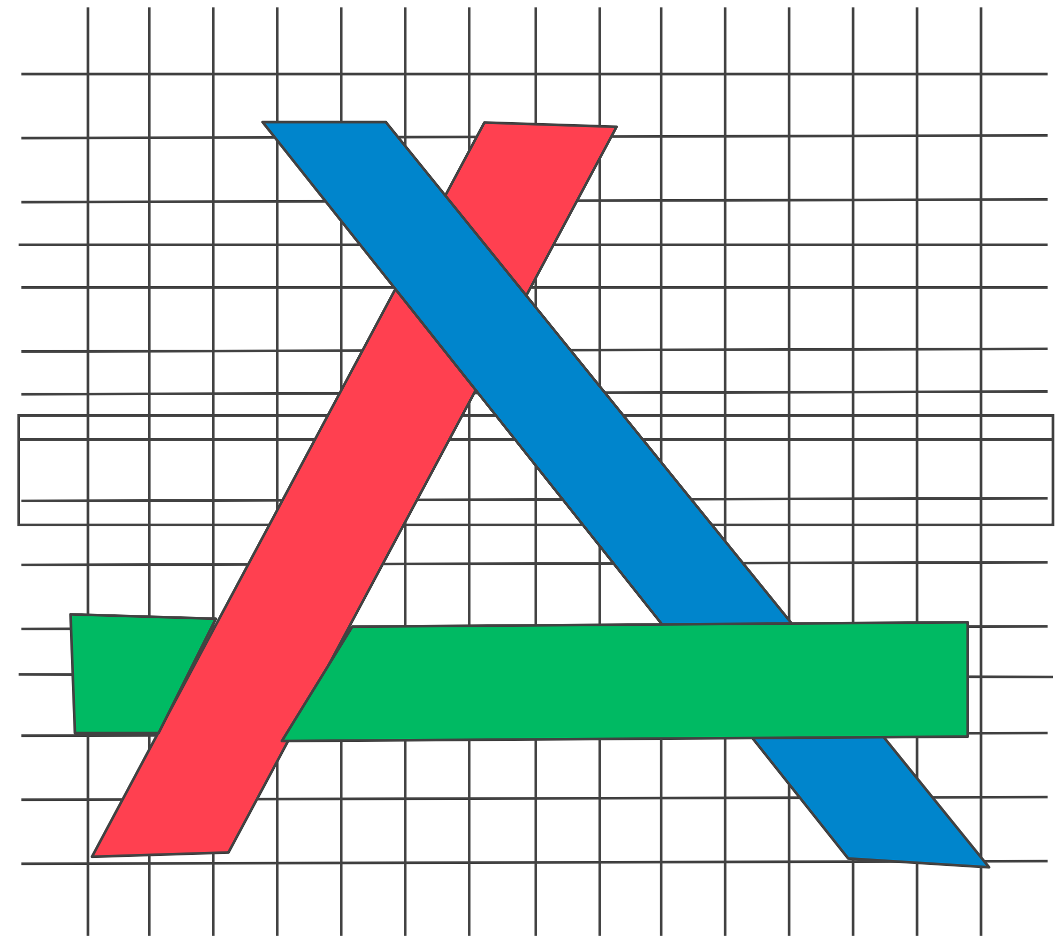
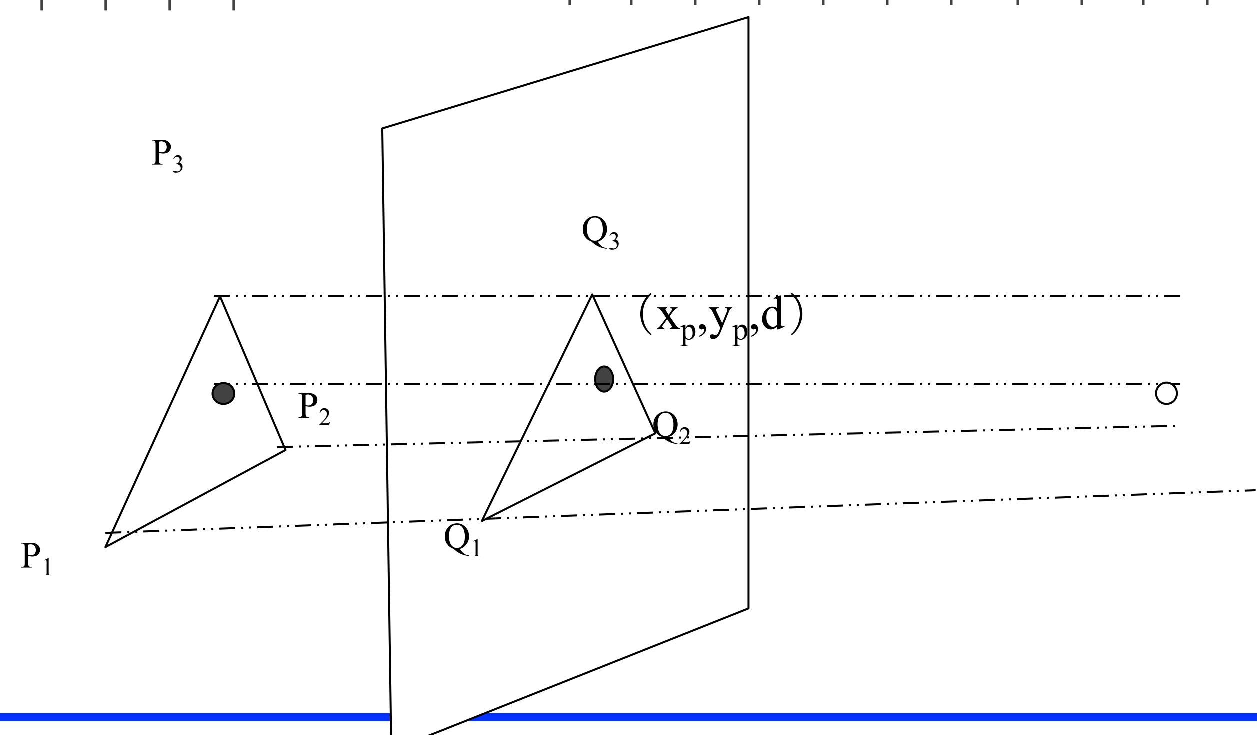
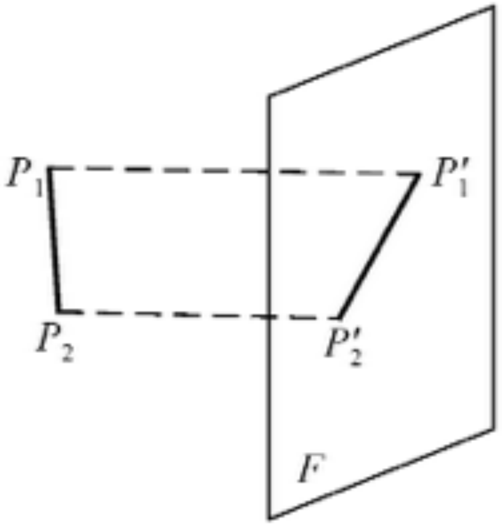




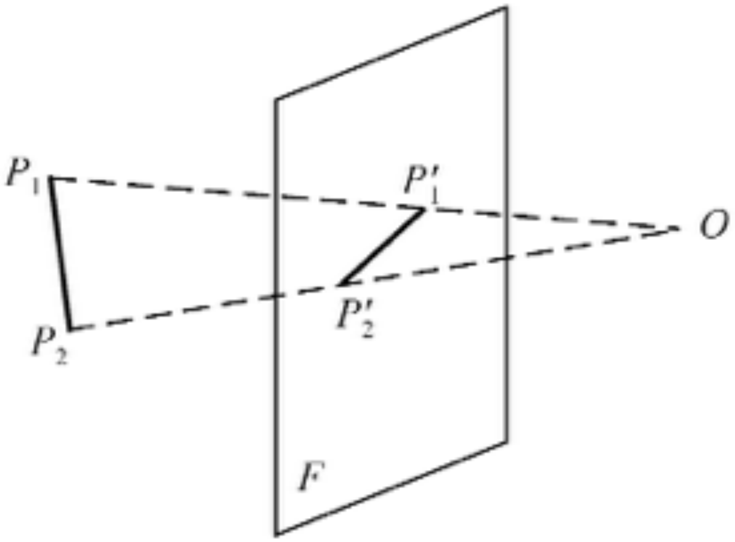


# Z-buffer

**Project**:




**Orthographic**




**Perspective**

**Calculate the z of the point**

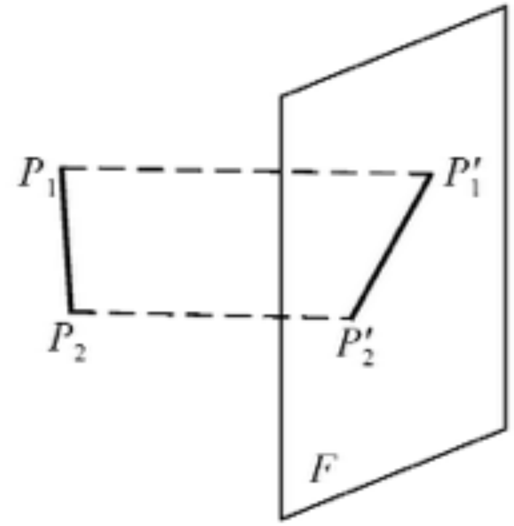
$$Ax + By + Cz + D = 0$$

$$z = \frac{-Ax - By - D}{C}$$

Question: how?

# Z-buffer

**Project**:

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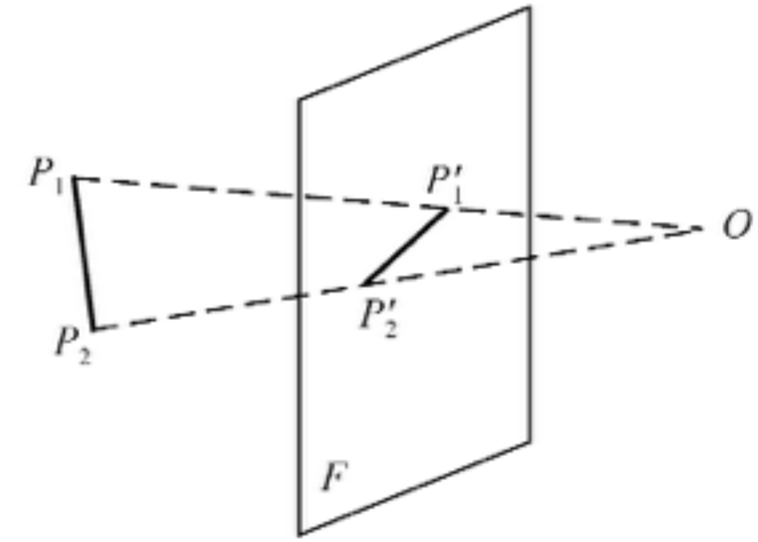


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**DDA**

$x$++, $y$++

$z$+??

Question: how?

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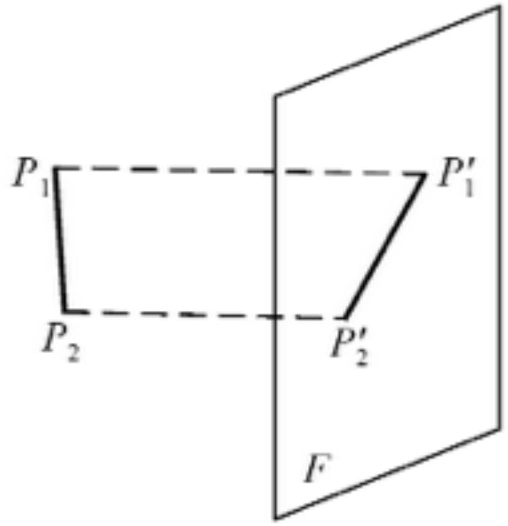


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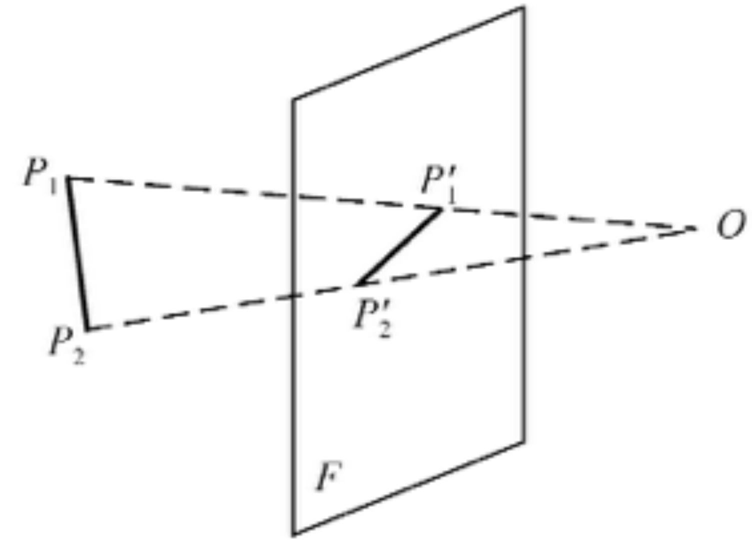


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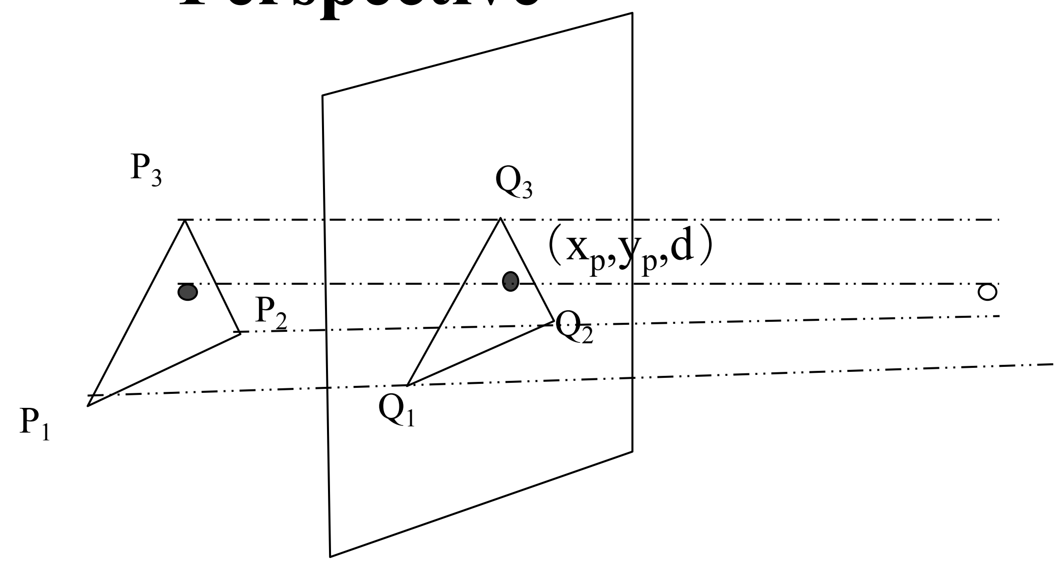


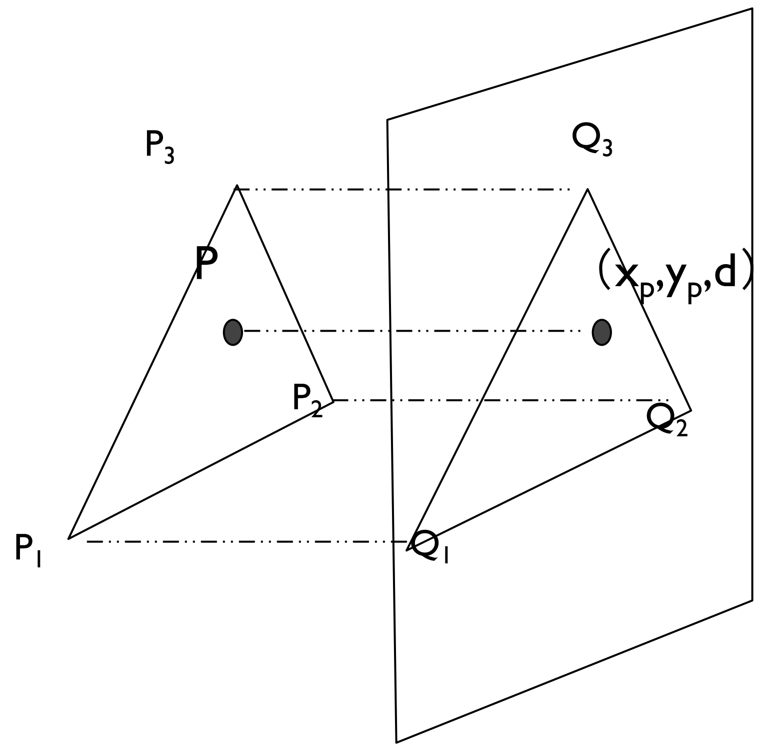

$$z = \frac{-Ax - By - D}{C}$$

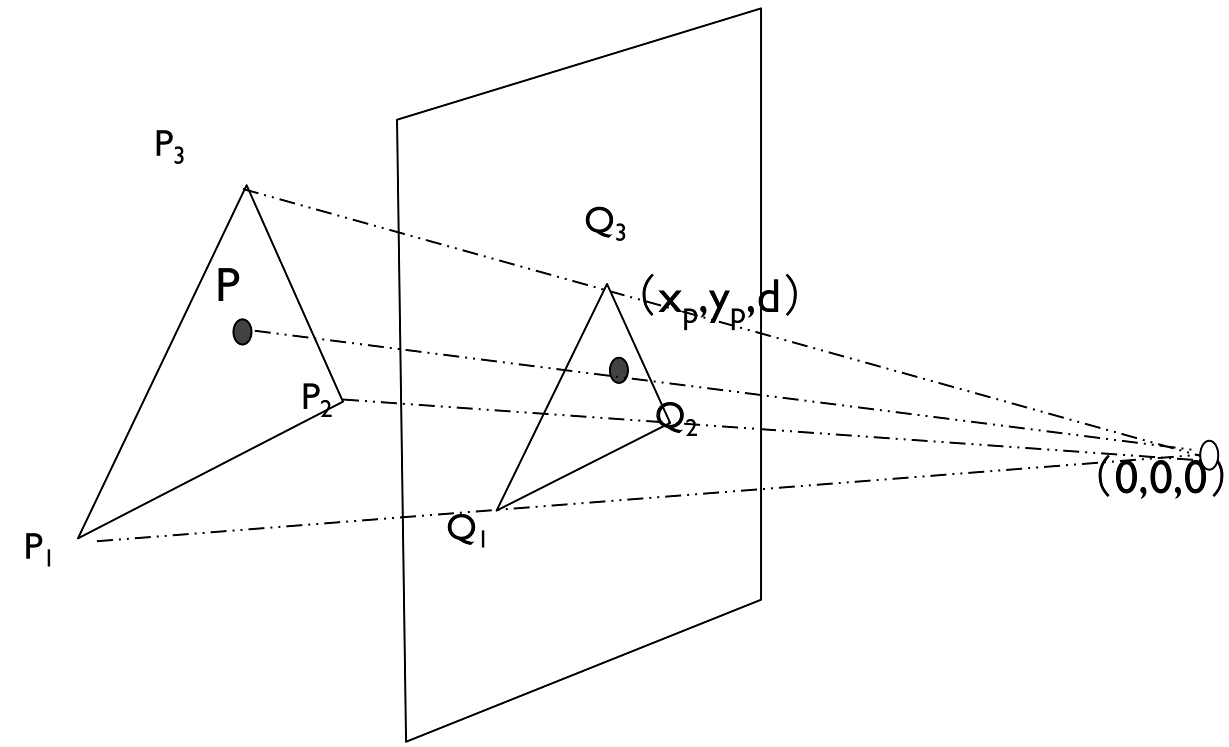
**DDA**

***x*++, *y*++**

***z*+??**

Question: how?



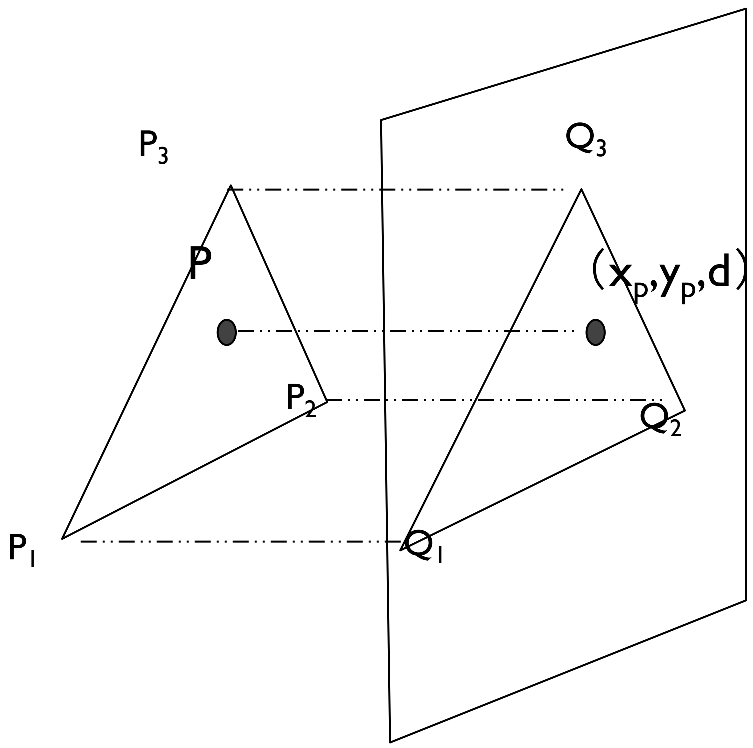



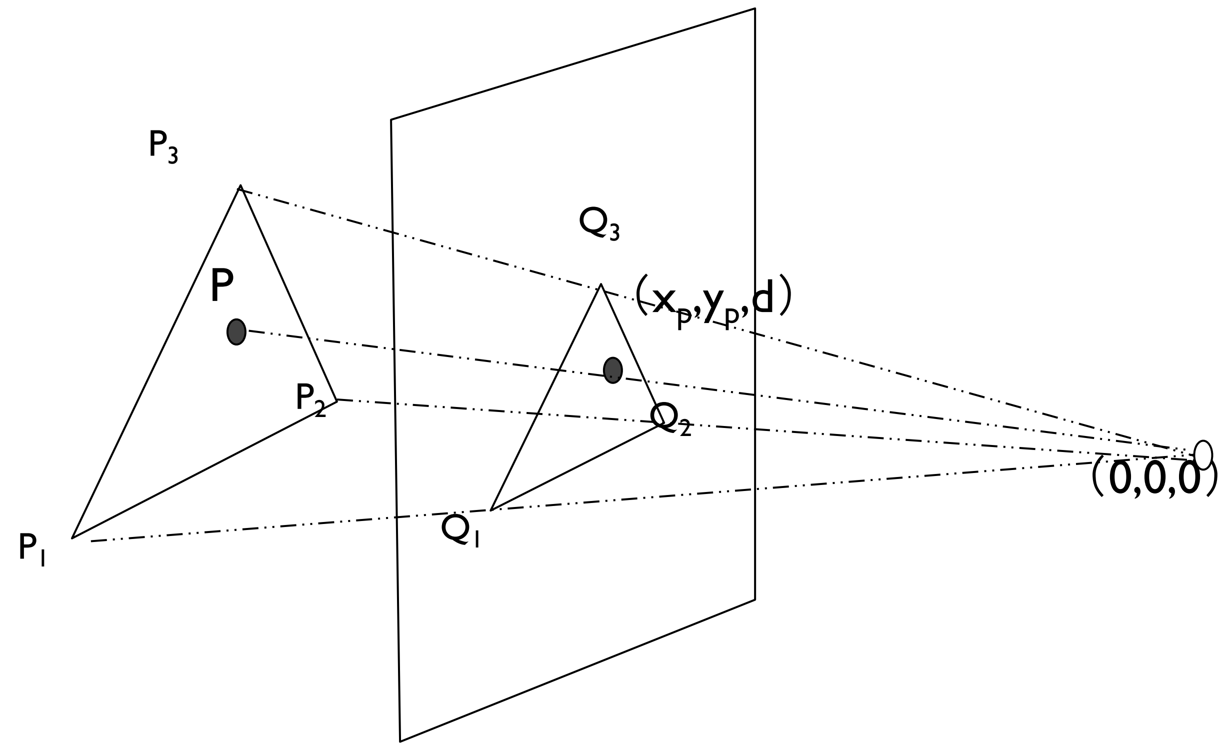

$$Ax + By + Cz + D = 0$$

$$(x, y, z) \rightarrow (x, y, d)$$

$$(x, y, z) \rightarrow (x_p, y_p, d)$$

$$\begin{cases} \dfrac{x_p}{x} = \dfrac{d}{z} \\ \dfrac{y_p}{y} = \dfrac{d}{z} \end{cases}$$

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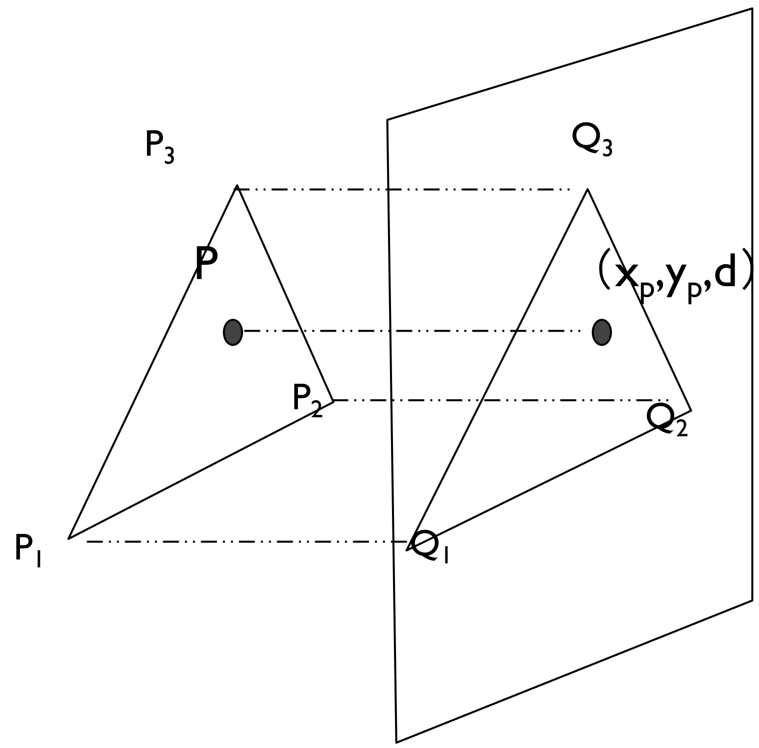


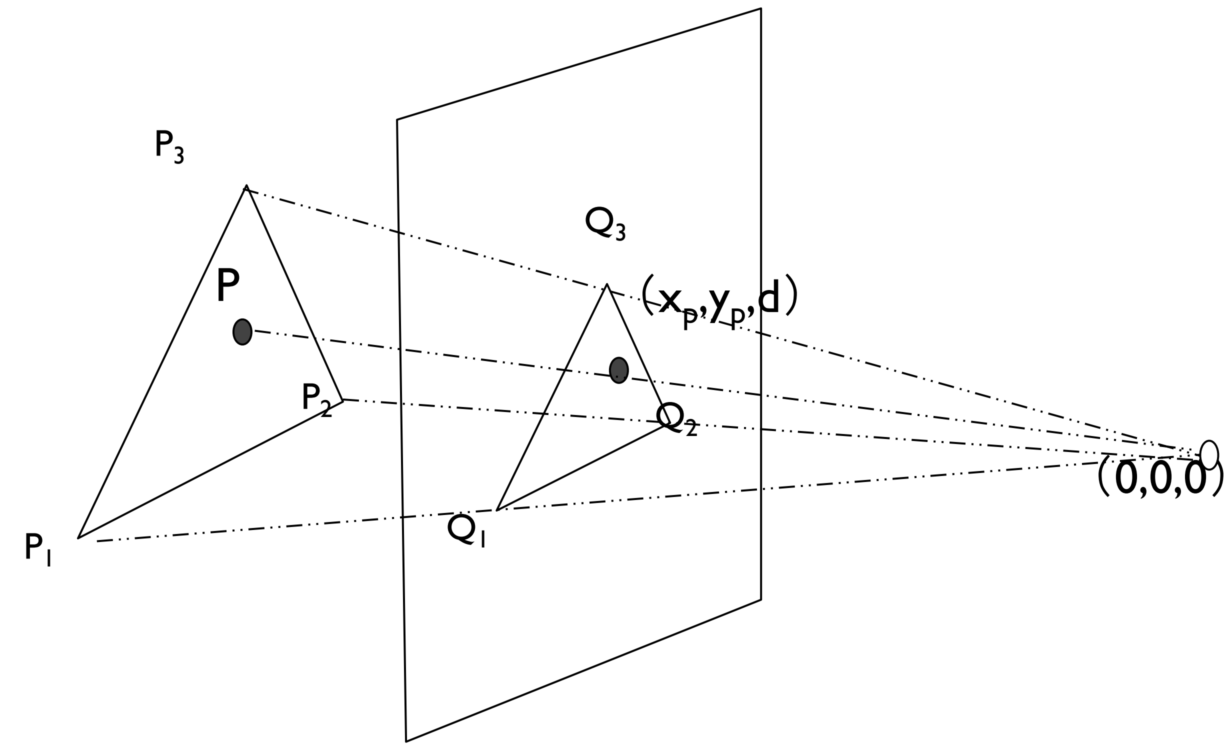


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$$(x, y, z) \rightarrow (x, y, d)$$

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$$\downarrow$$

$$(x_p, y_p, d)$$

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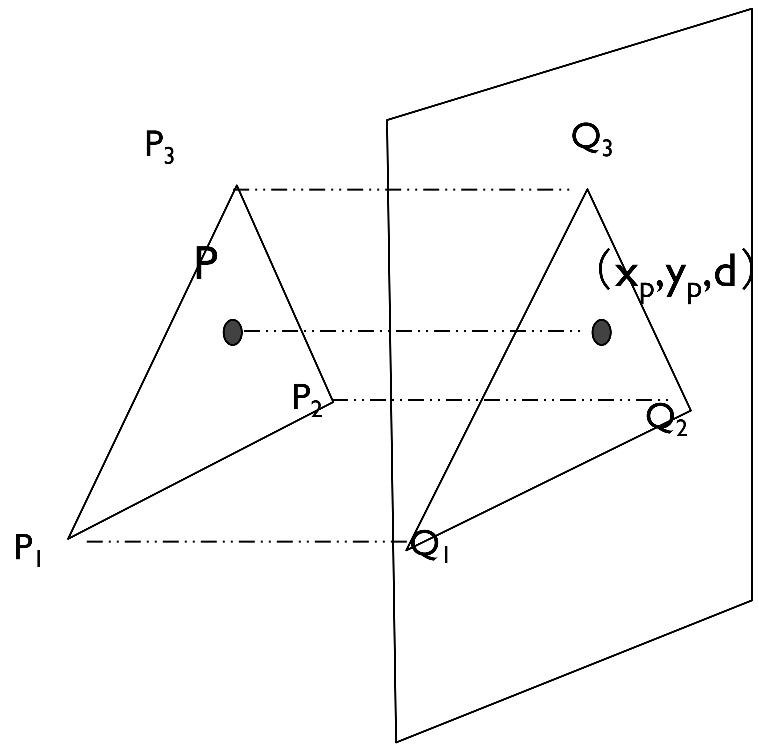
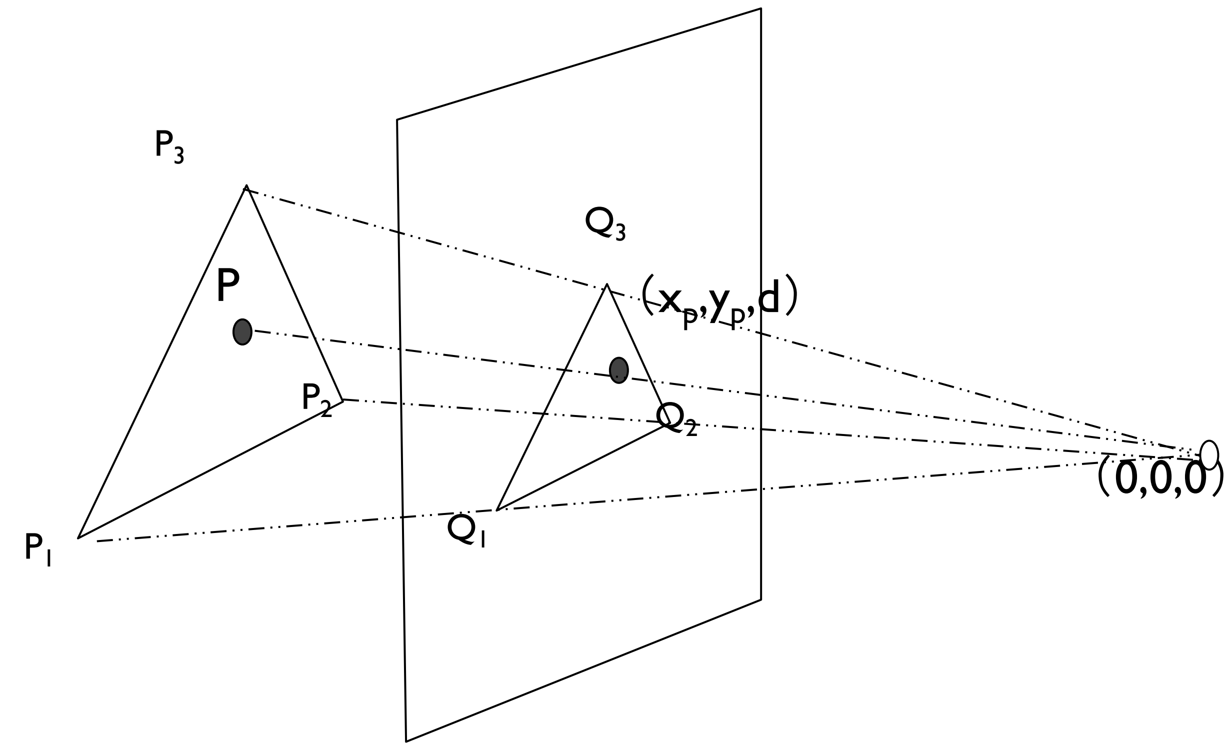



$$Ax + By + Cz + D = 0$$

$$(x, y, z) \rightarrow (x, y, d)$$

$$(x, y, z) \rightarrow (x_p, y_p, z)$$

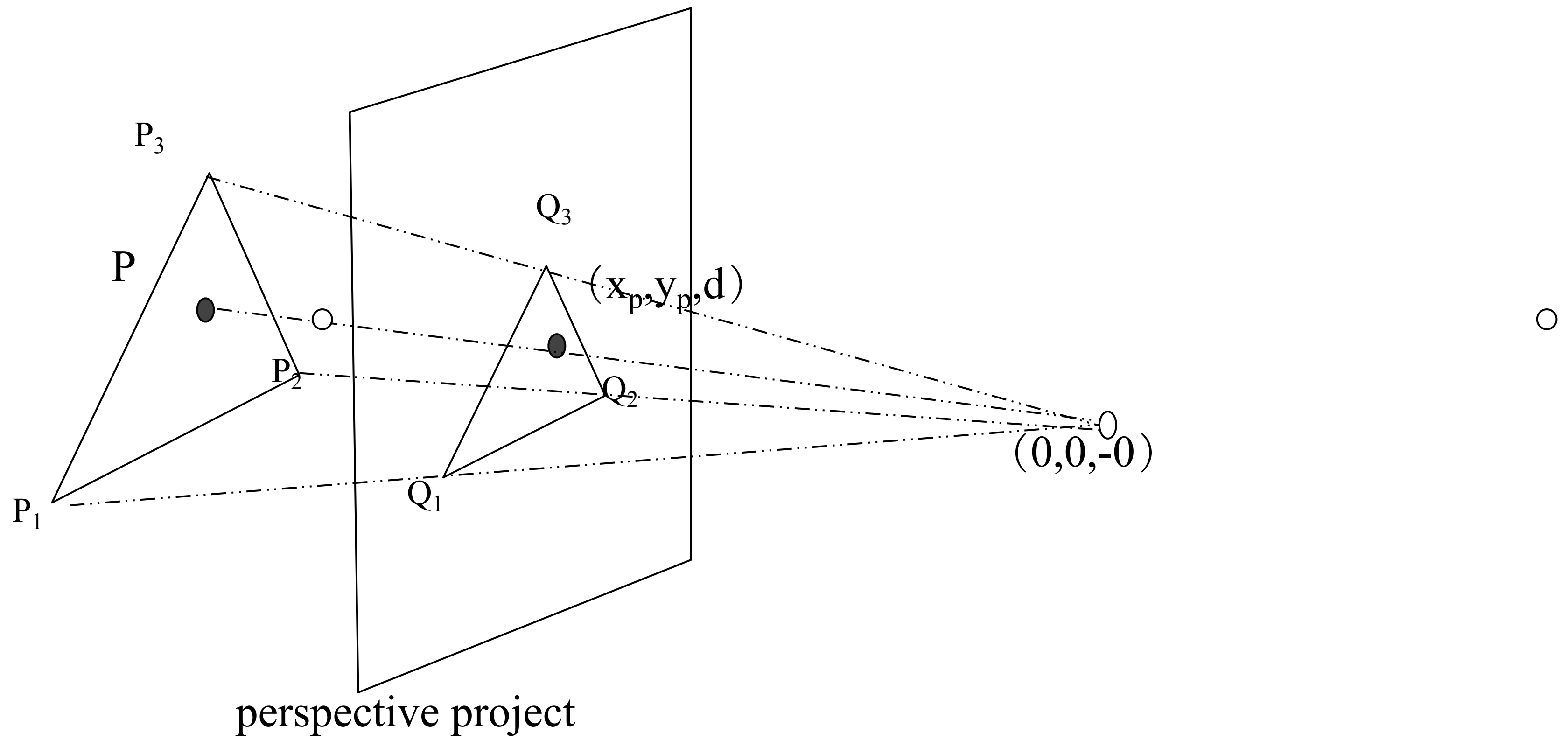
Orthographic project → 

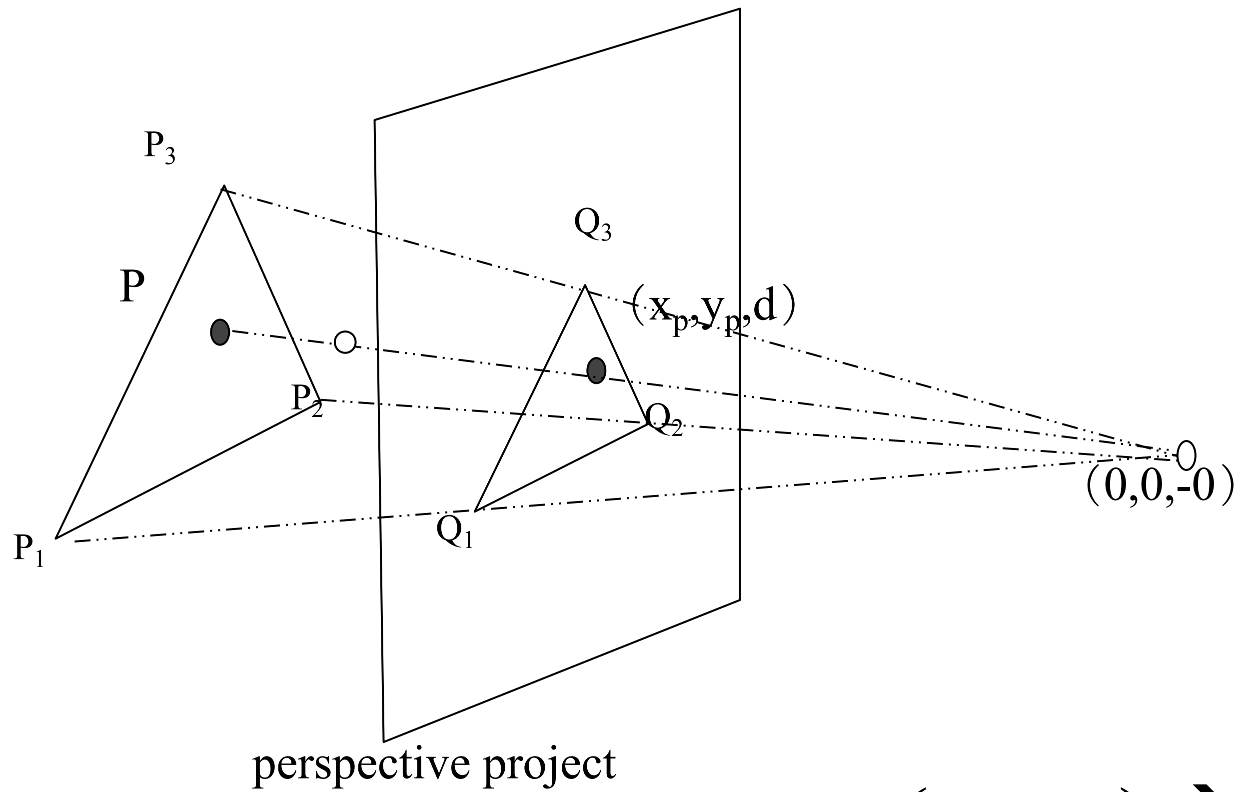
$$\downarrow$$

$$(x_p, y_p, d)$$

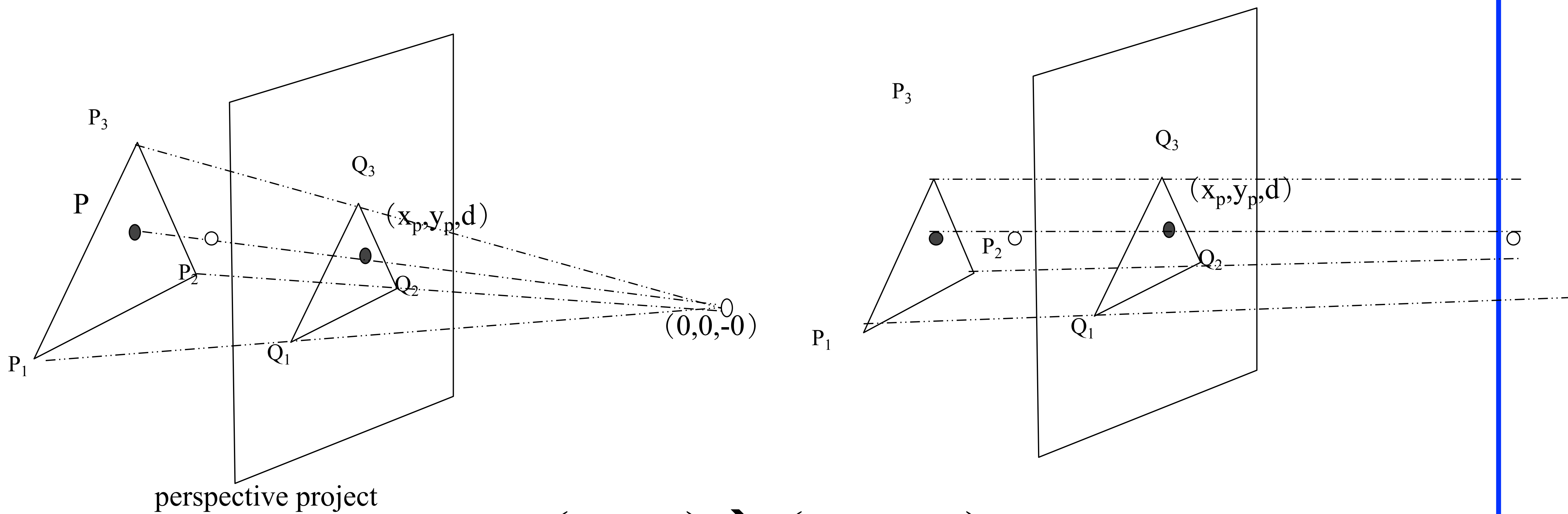
$$(x, y, z) \rightarrow (x_p, y_p, d)$$

$$\begin{cases} \dfrac{x_p}{x} = \dfrac{d}{z} \\ \dfrac{y_p}{y} = \dfrac{d}{z} \end{cases}$$

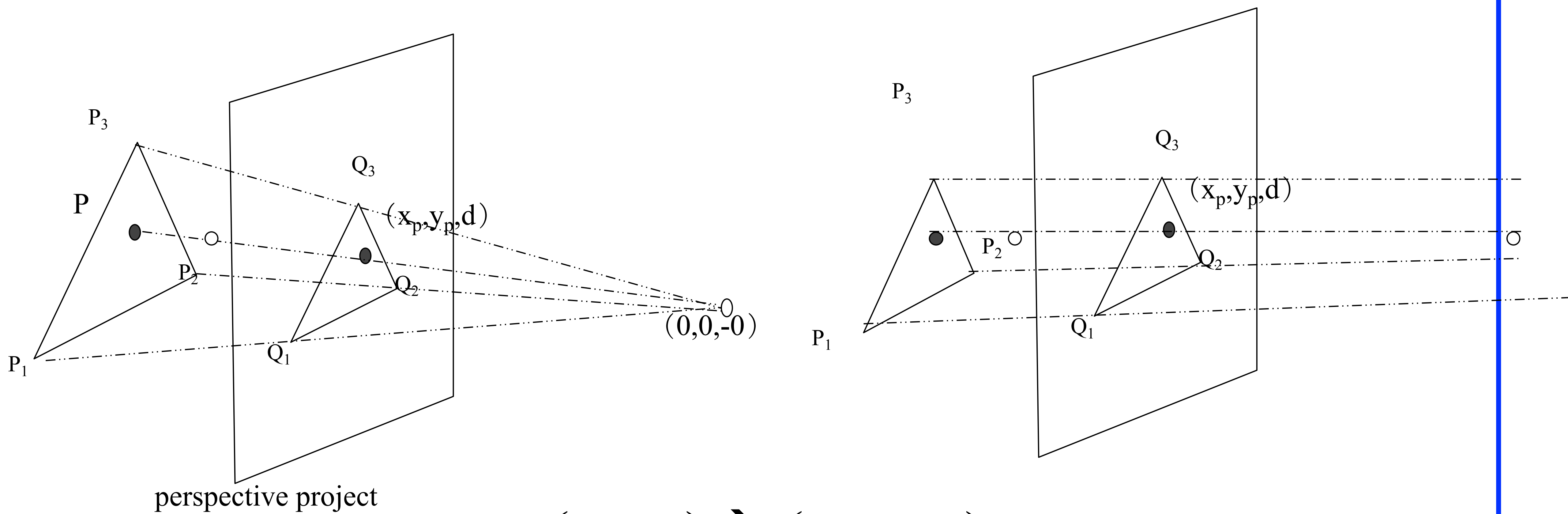

P3
P
Q3
(xp,yp,d)
P2
Q2
(0,0,-0)
Q1
P1
perspective project




$$(x,y,z) \rightarrow (x_p,y_p,z)$$



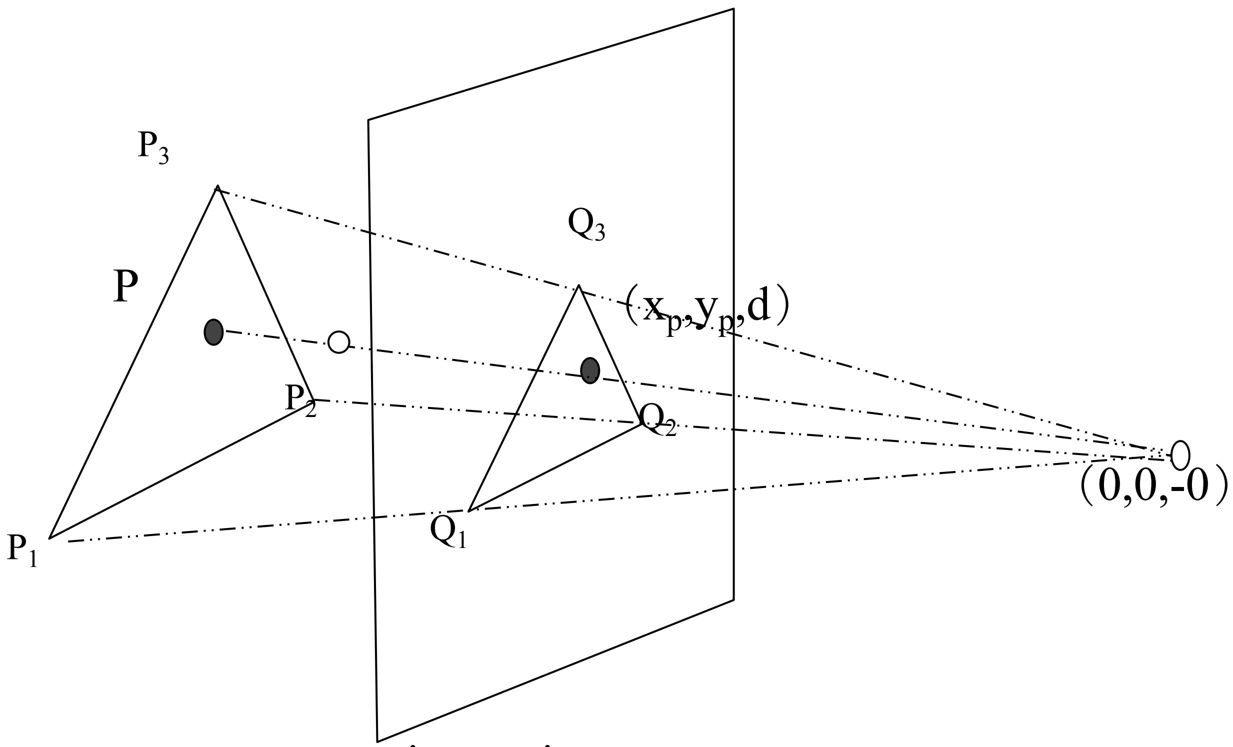

$$(x,y,z) \rightarrow (x_p,y_p,z)$$



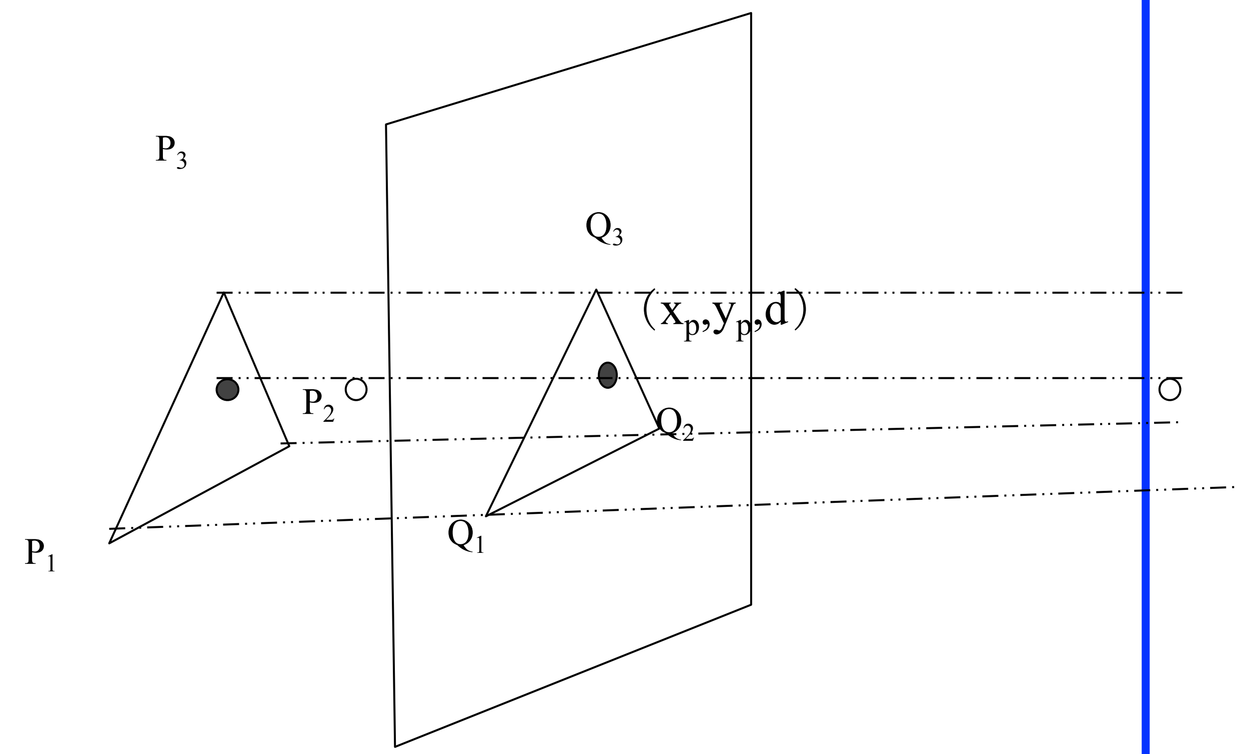

$$(x,y,z) \rightarrow (x_p, y_p, z)$$

$$\downarrow$$

$$(x_p, y_p, d)$$



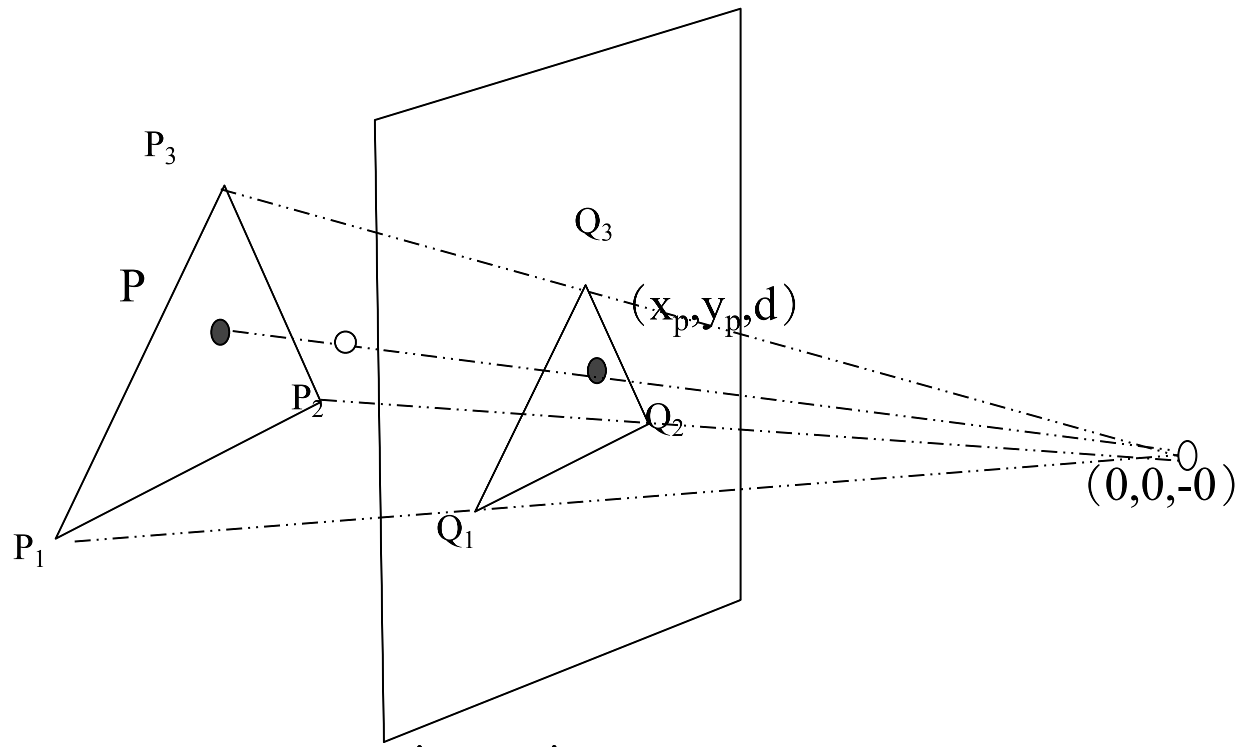

perspective project



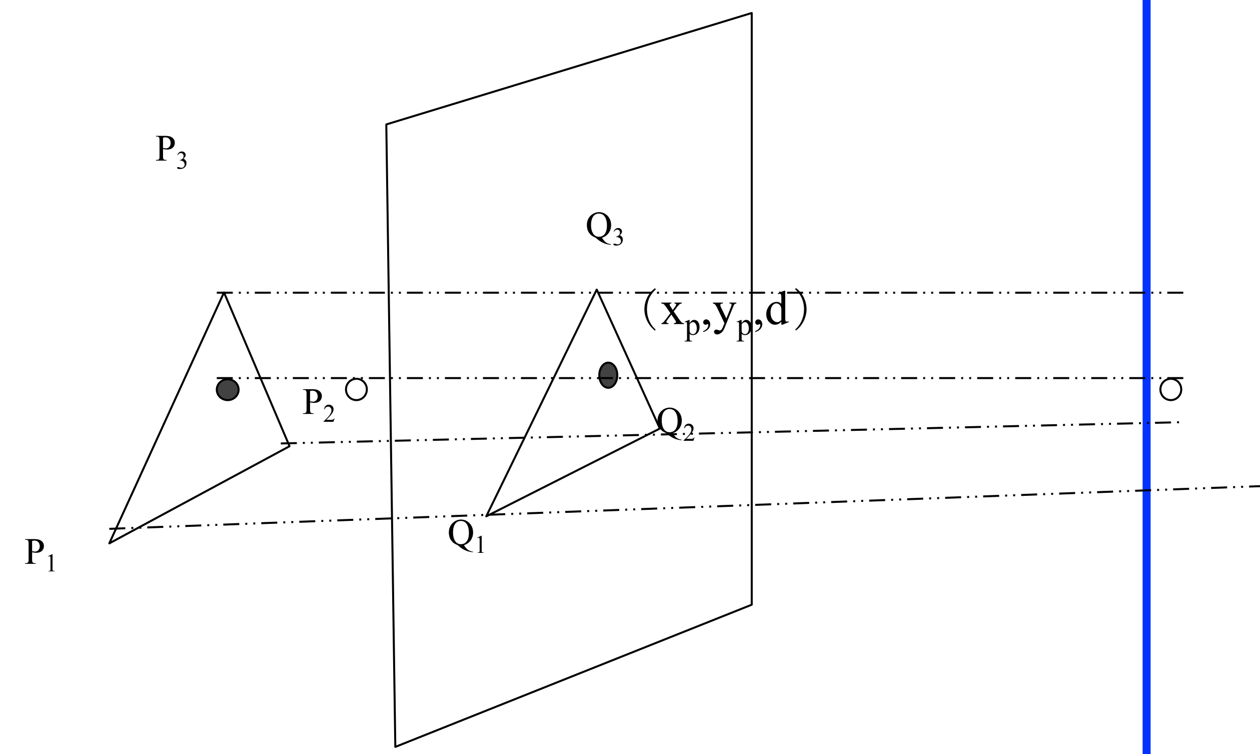

$$(x,y,z) \rightarrow (x_p,y_p,z)$$

Orthographic project → ↓

$$(x_p,y_p,d)$$




perspective project




$(x,y,z) \rightarrow (x_p,y_p,z)$

Perspective Transformation

Orthographic project →

↓

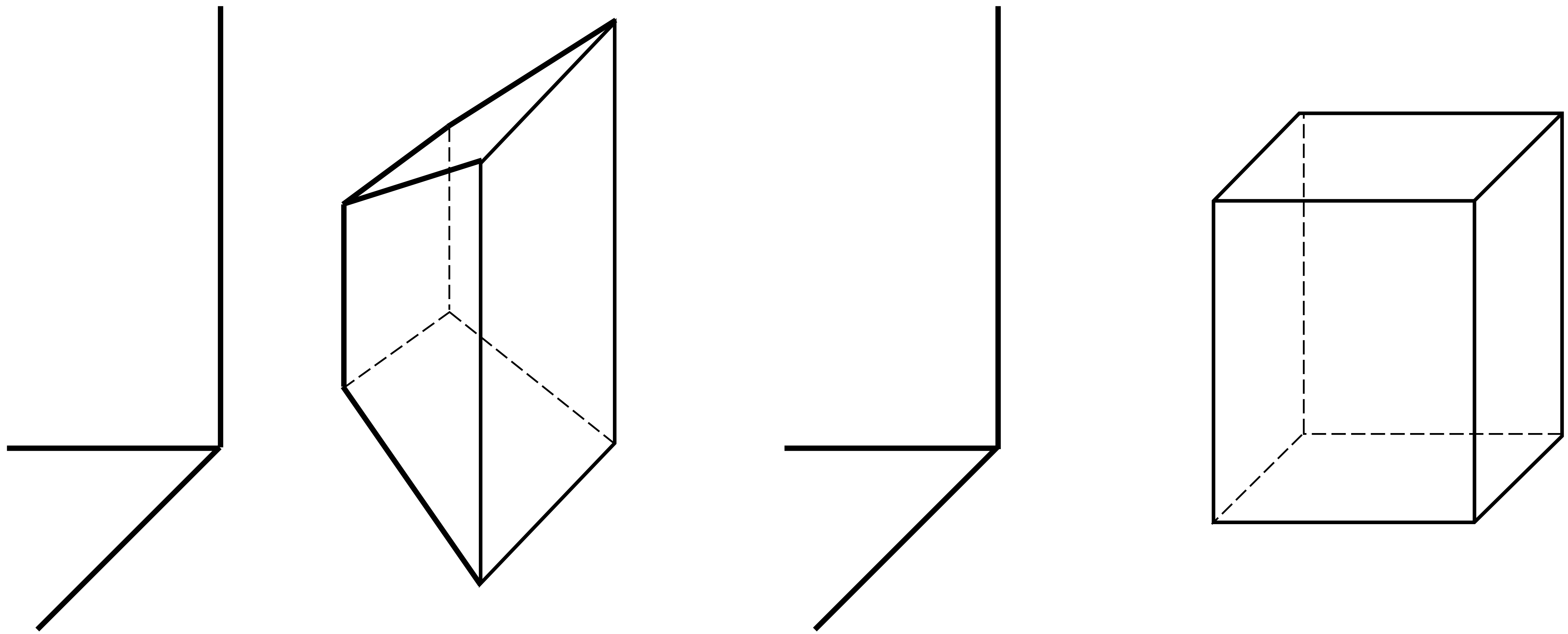
$(x_p,y_p,d)$

# Perspective Transformation...

- We need to apply a perspective transformation to the view volume and transform it into a rectangular parallel-piped one

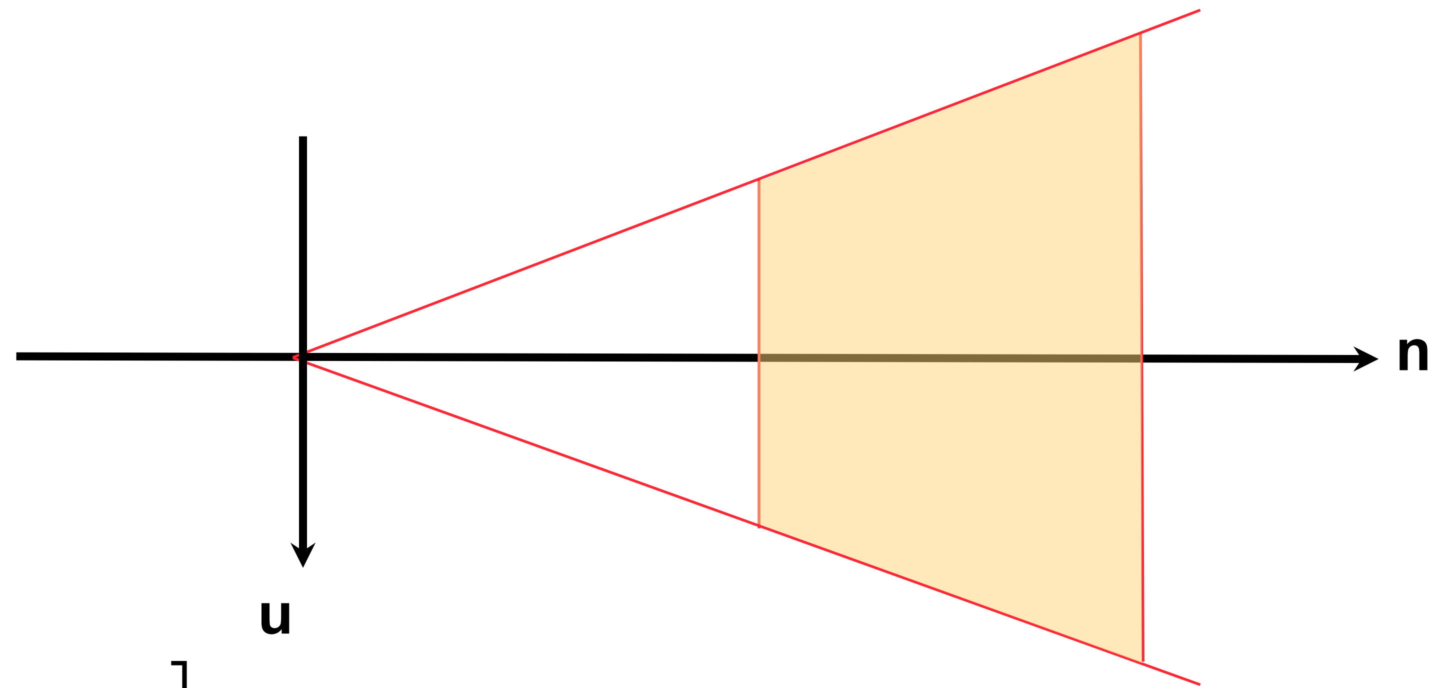
- This makes the final 3D view volume of a perspective view the same as that of a parallel view, just before projection

# Perspective Transformation

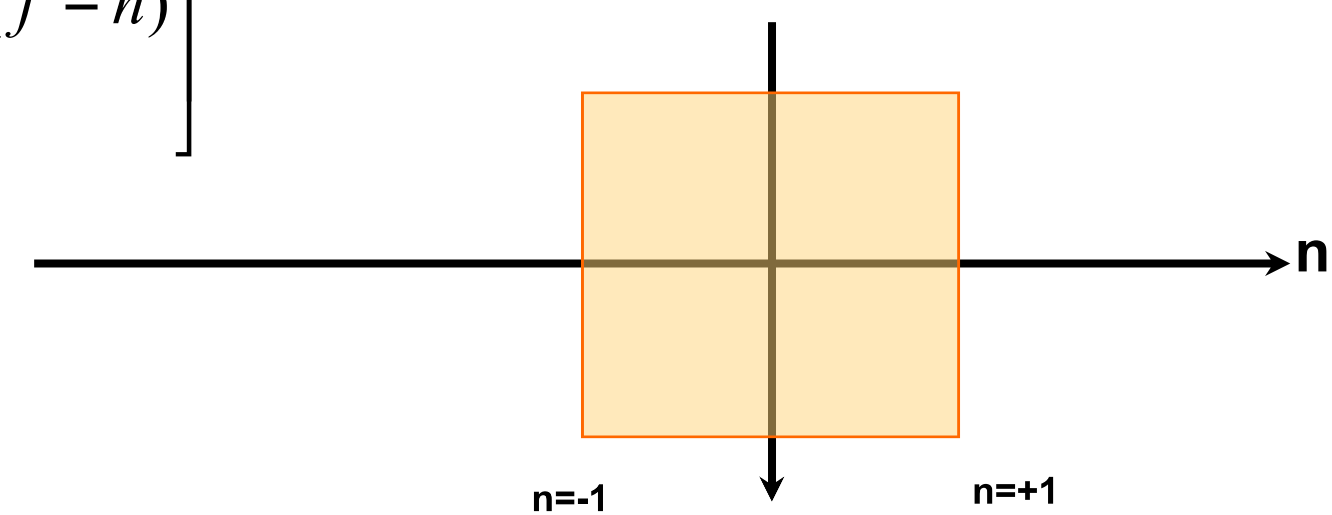


- A perspective transformation preserves relative depth, straight lines and planes

# Perspective Transformation




$$\mathbf{M}_P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & (f+n)/(f-n) & -2f*n/(f-n) \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

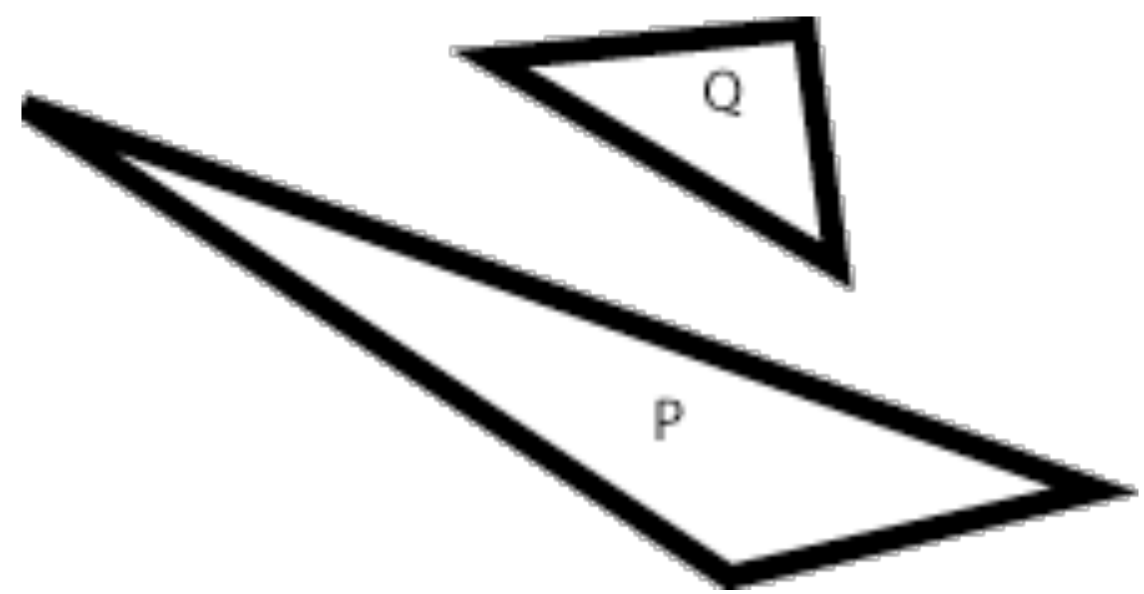


# A-buffer

- Accumulation buffer
  - used in Lucasfilm REYES
  - not only store depth but also other data
  - support transparent surfaces

# Depth-sorting

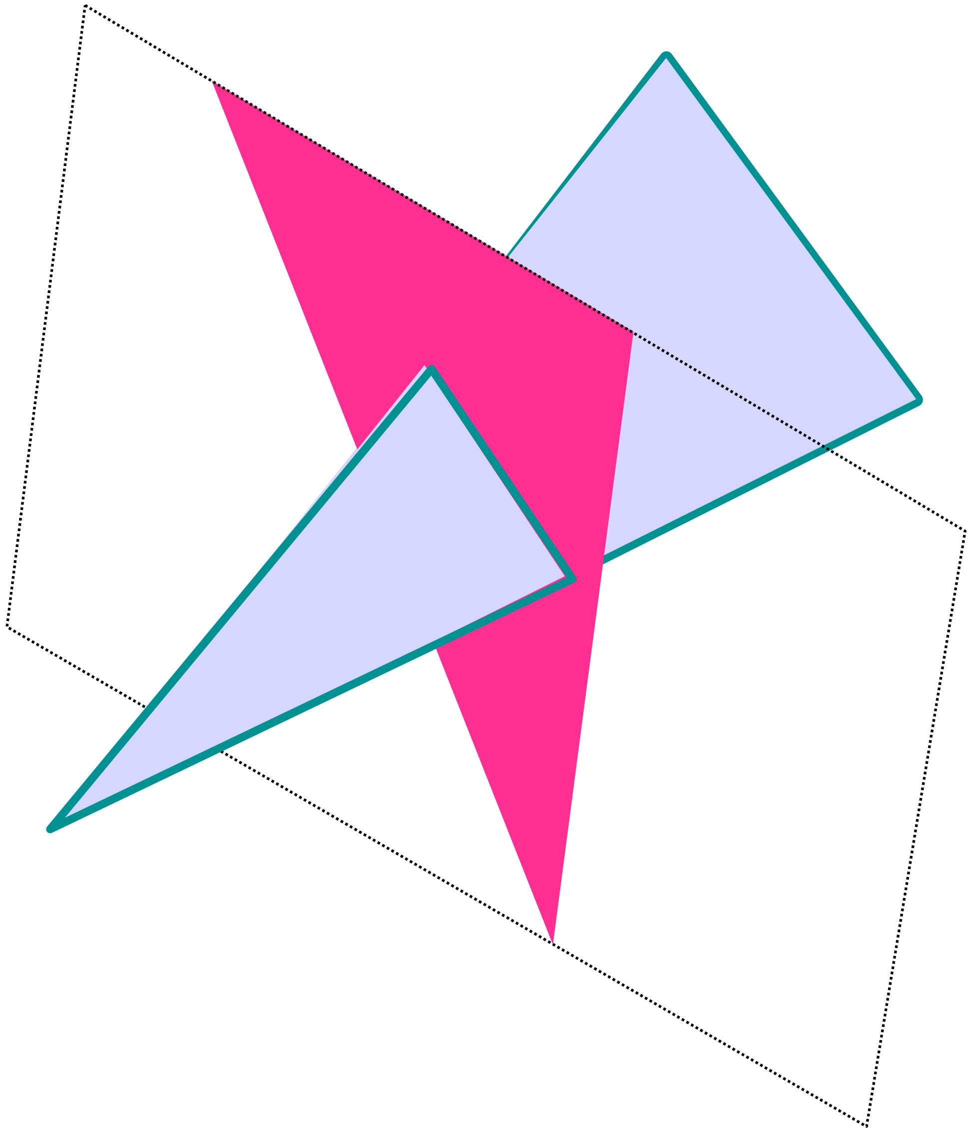
- space-image space hybrid method
  - space or image space:
    - sort surface by depth
  - image space:
    - do scan conversion from deepest surfaces



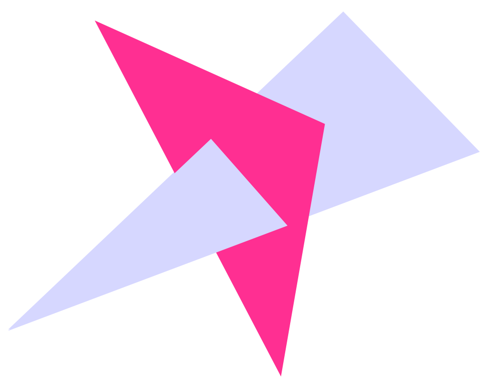
# Binary Space Partitioning Trees

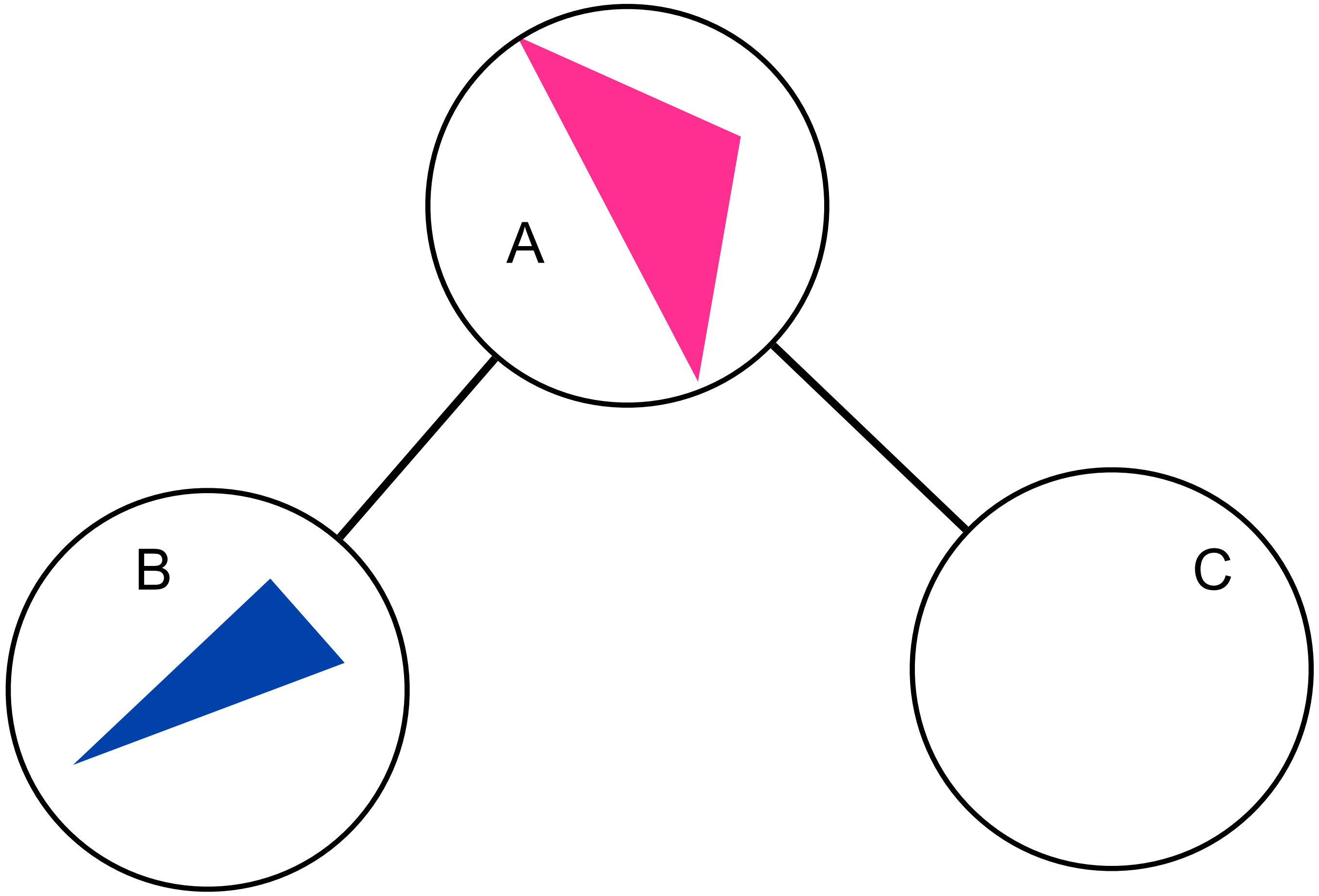
- BSP Tree
  - Very efficient for a static group of 3D polygons as seen from an arbitrary viewpoint
  - Correct order for Painter's algorithm is determined by a suitable traversal of the binary tree of polygons

# BSP Tree

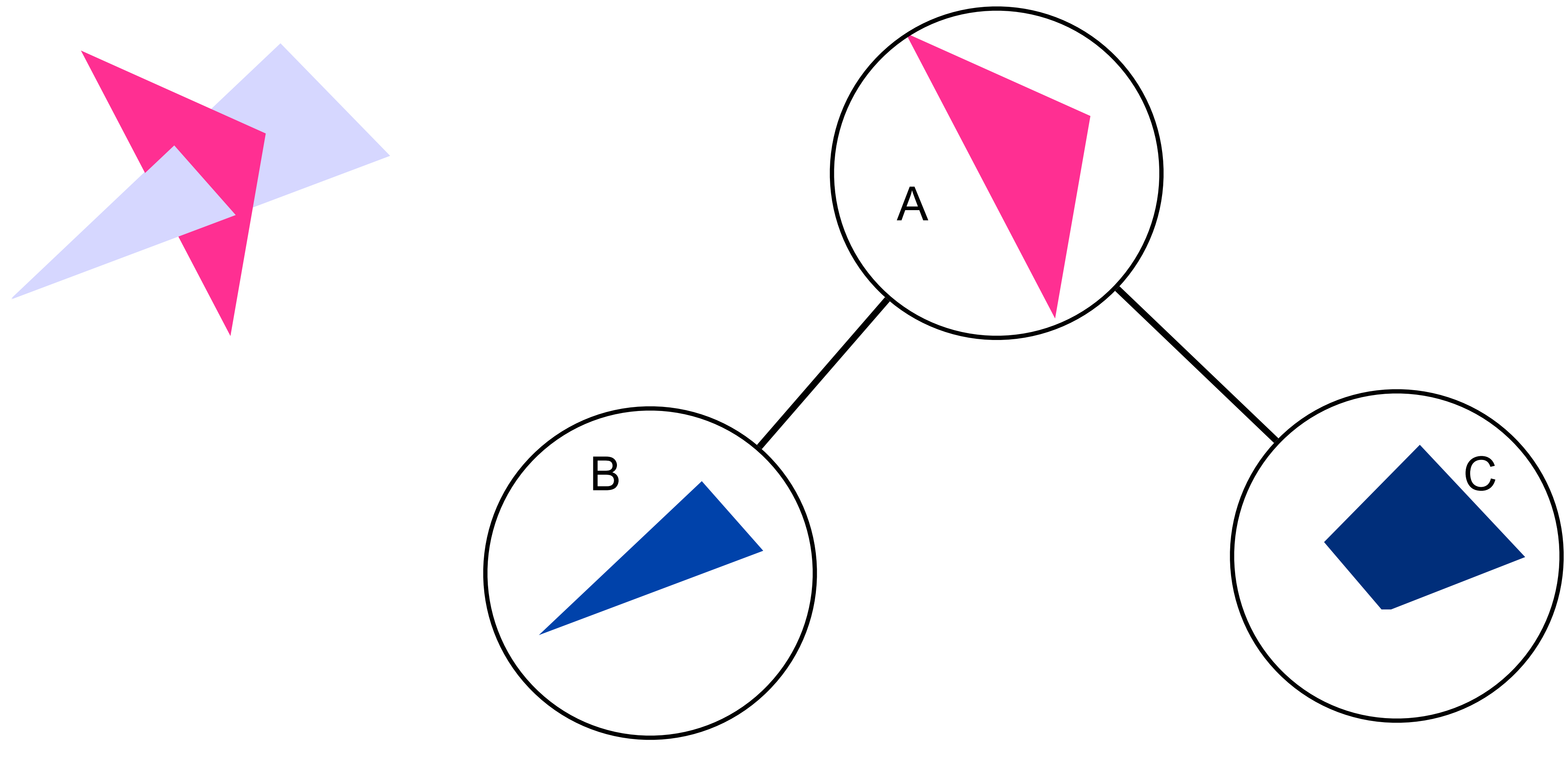


# BSP Tree





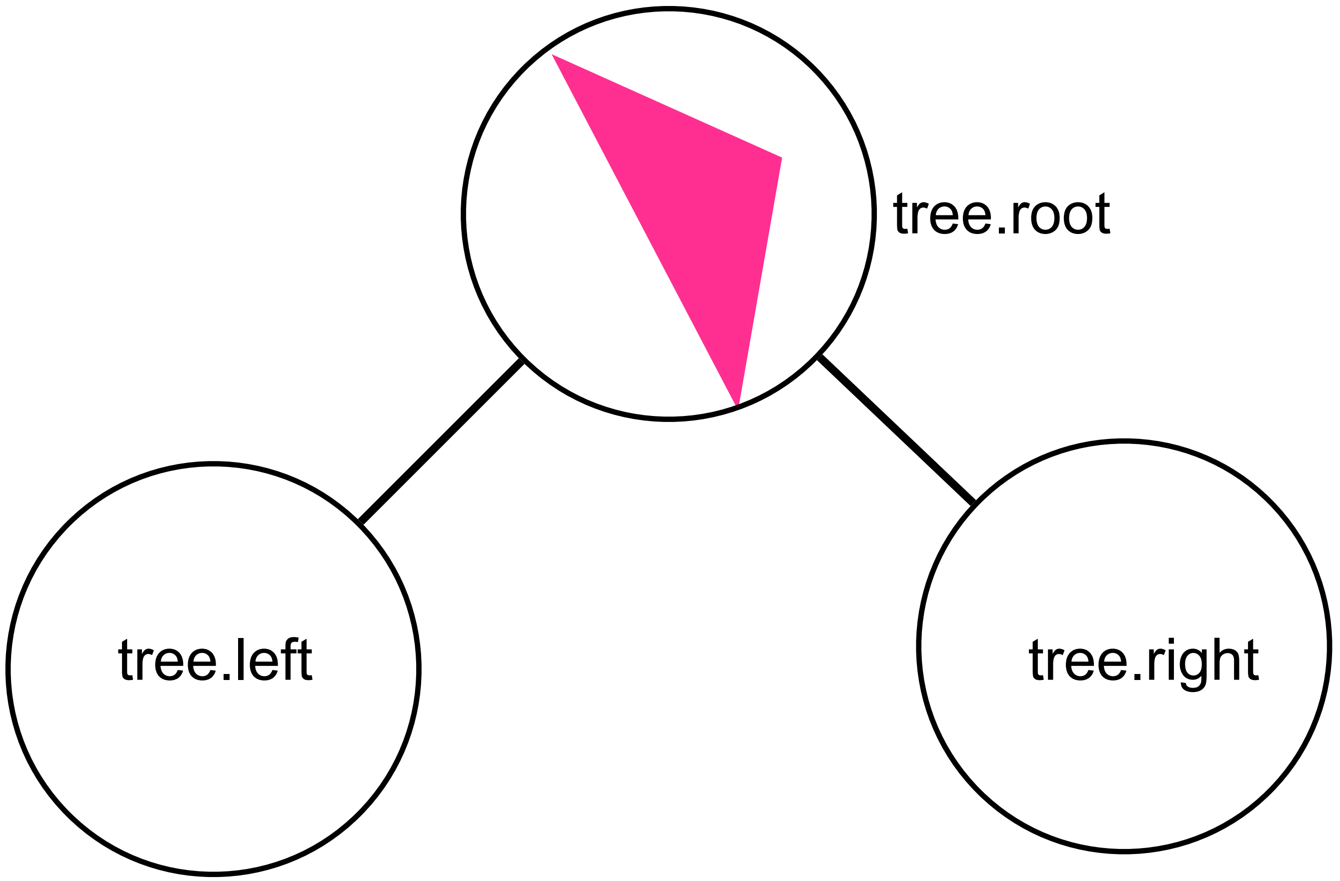
# BSP Tree



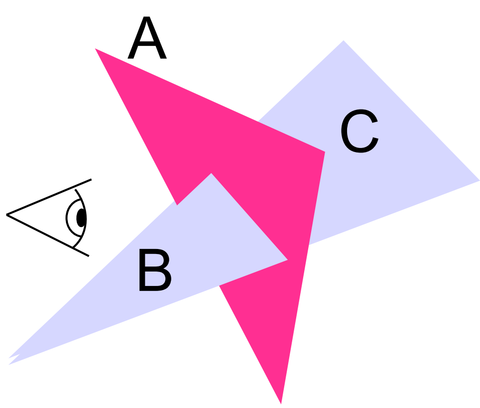
# BSP Tree

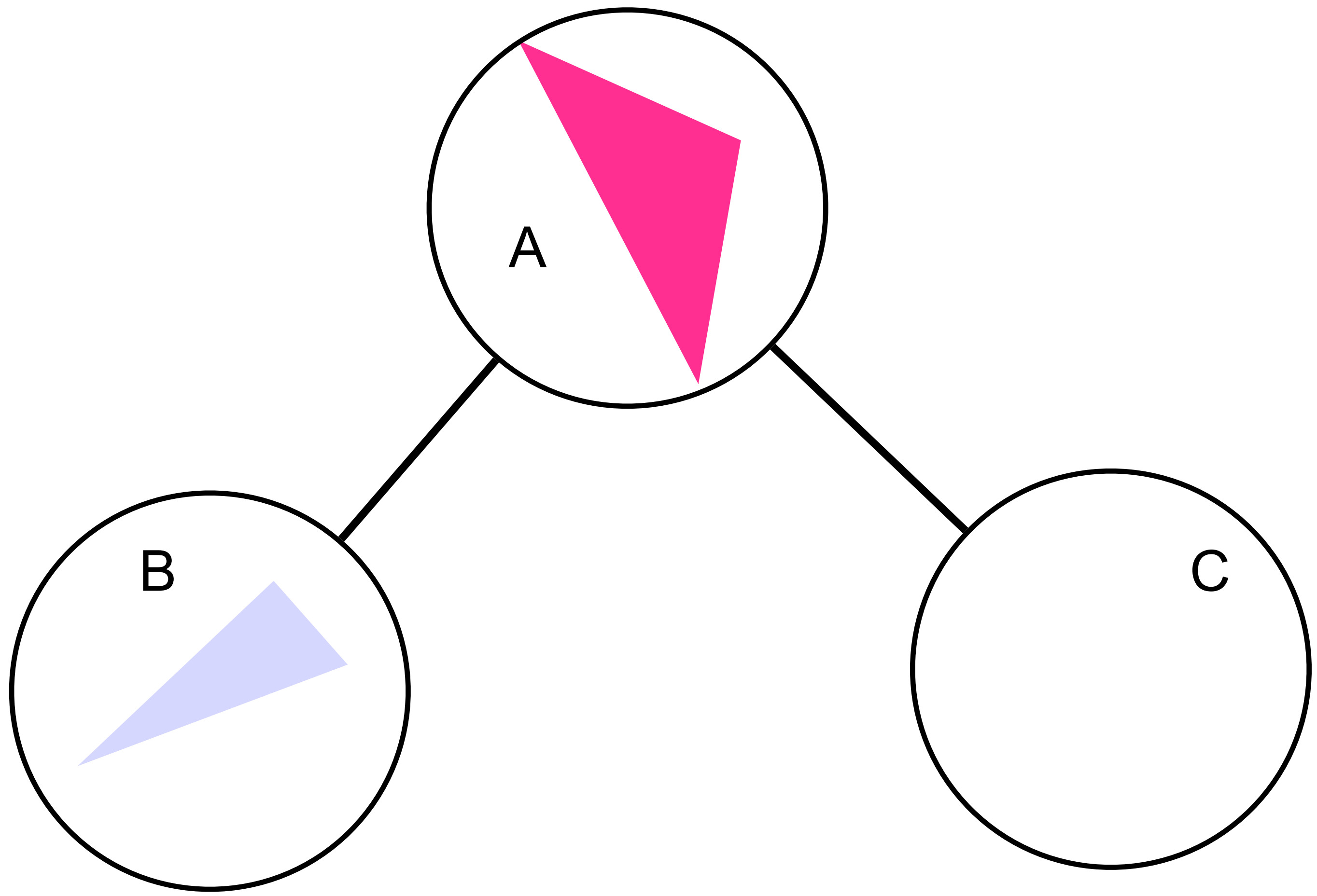
**<u>Draw BSP Tree</u>**

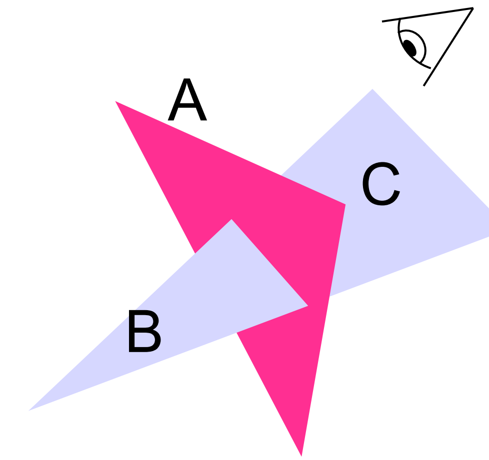
**function** *draw*(bsptree *tree*, point *eye*)
if *tree*.*empty* then
    return
if $f_{\text{tree.root}}(eye) < 0$
    draw (*tree.right*)
    rasterize(*tree.root*)
    draw(*tree.left*)
else
    draw (*tree.left*)
    rasterize(*tree.root*)
    draw(*tree.right*)



# BSP Tree



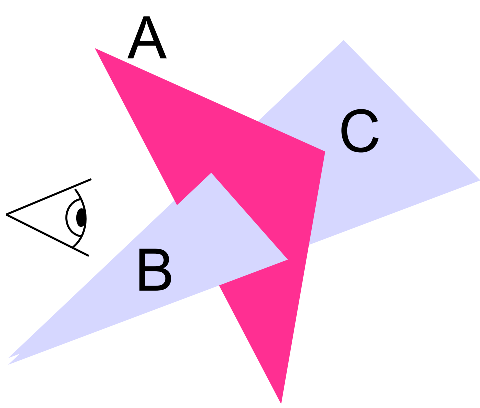



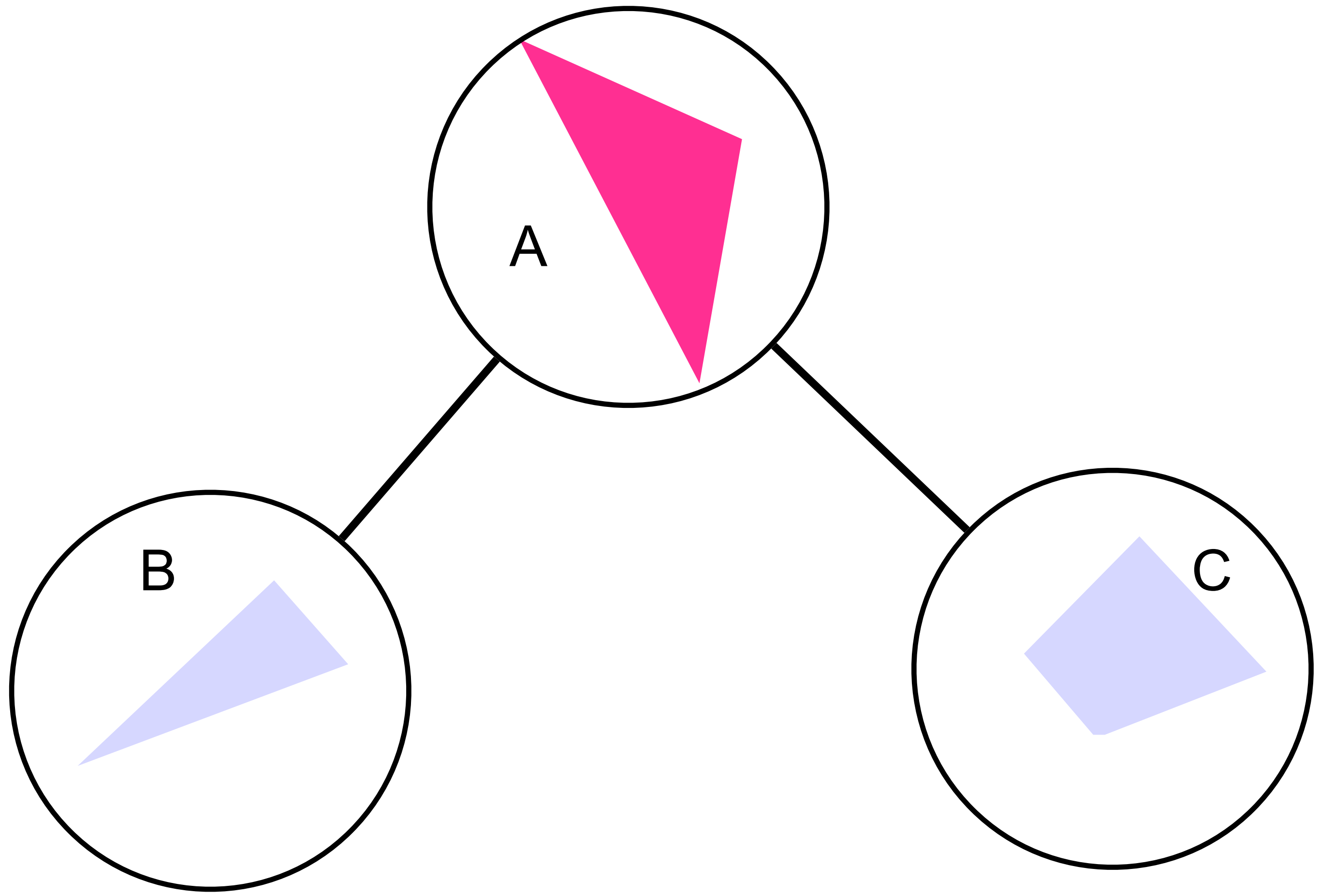



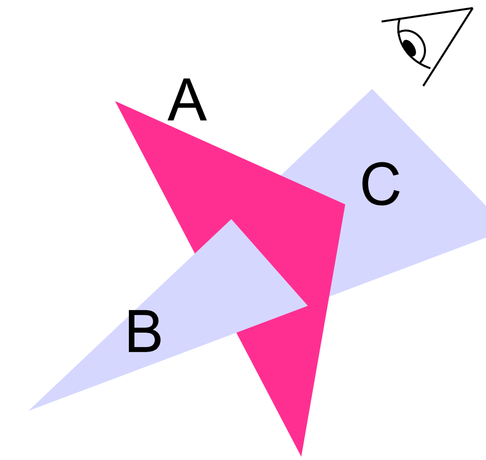

rasterize(C)
rasterize(A)
rasterize(B)

rasterize(B)
rasterize(A)
rasterize(C)

# BSP Tree



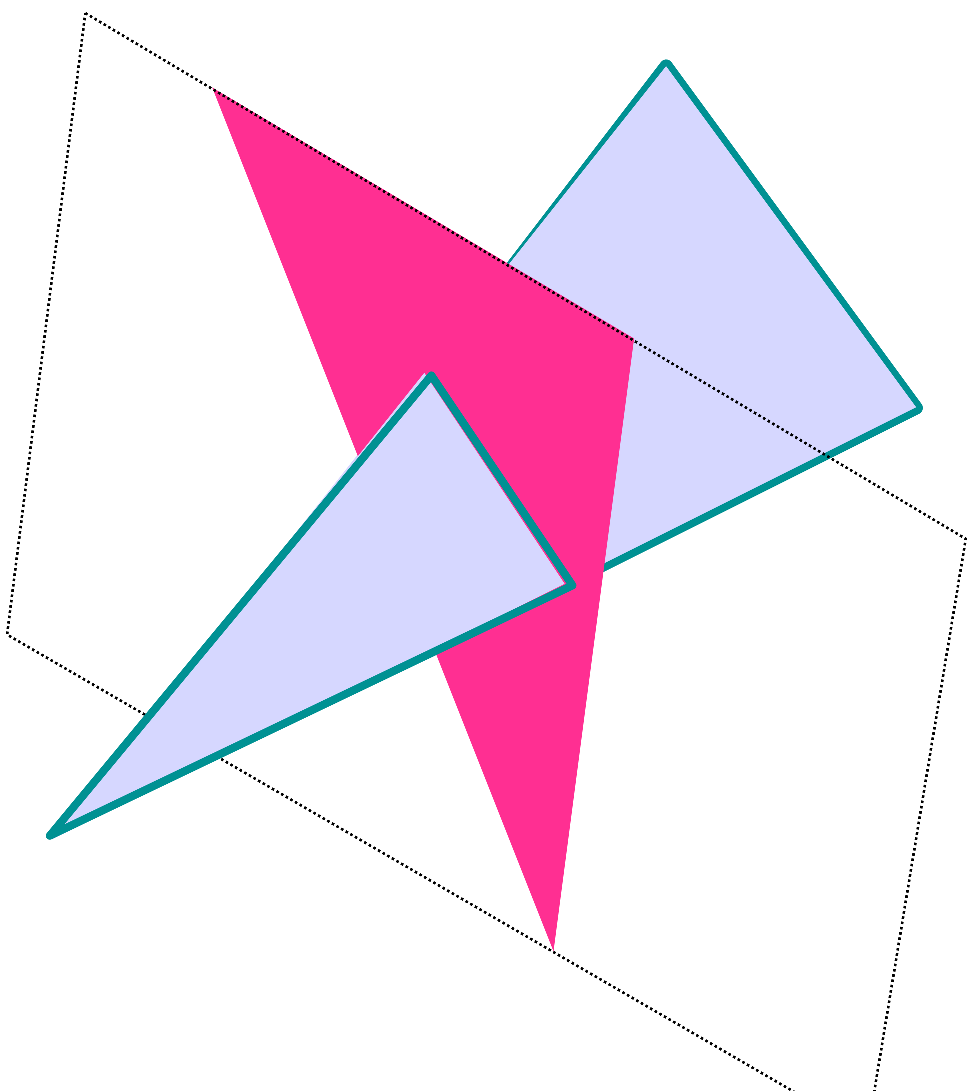

rasterize(C)
rasterize(A)
rasterize(B)

rasterize(B)
rasterize(A)
rasterize(C)

# BSP Tree

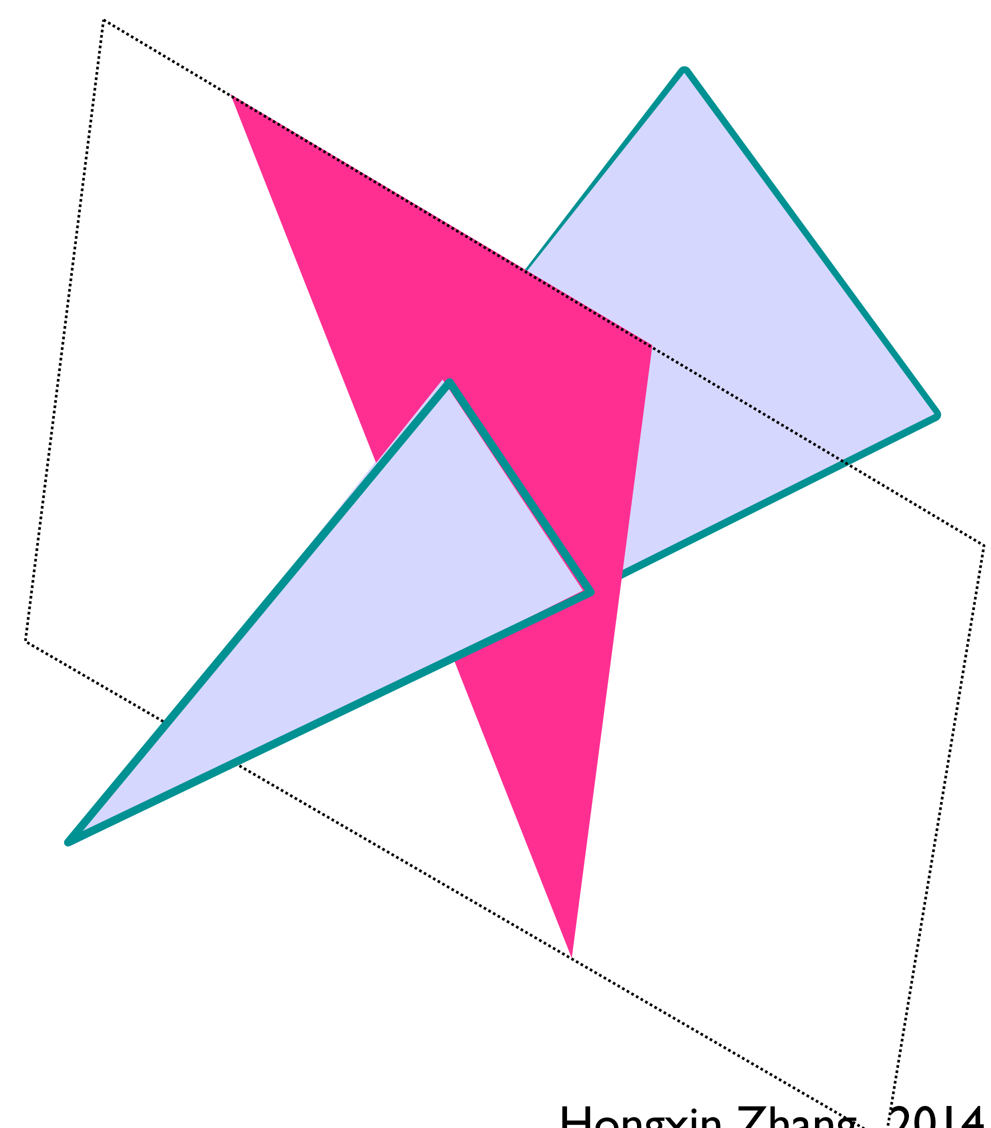
- Code works for any view
- Tree can be pre-computed
- Requires evaluation of

$$f_{\textit{plane of the triangle}}(\text{eye})$$



# BSP Tree Construction

- The binary tree is constructed using the following principle:
  - For each polygon, we can divide the set of other polygons into two groups
  - One group contains those lying in front of the plane of the given polygon
  - The other group contains those in the back
  - The polygons intersecting the plane of the given polygon are split by that plane



# BSP Tree

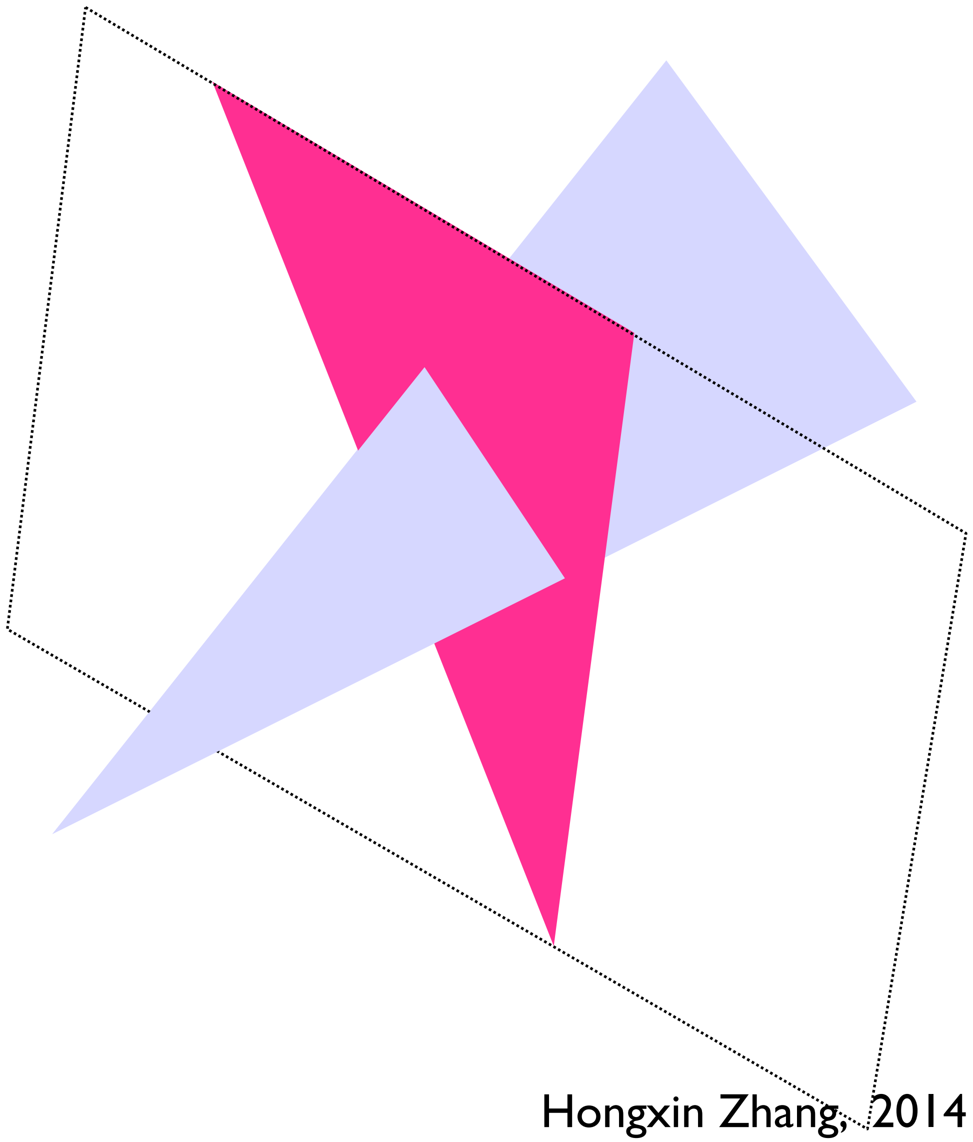
- Split Triangle:

  How to?

# BSP Tree

- Split Triangle:

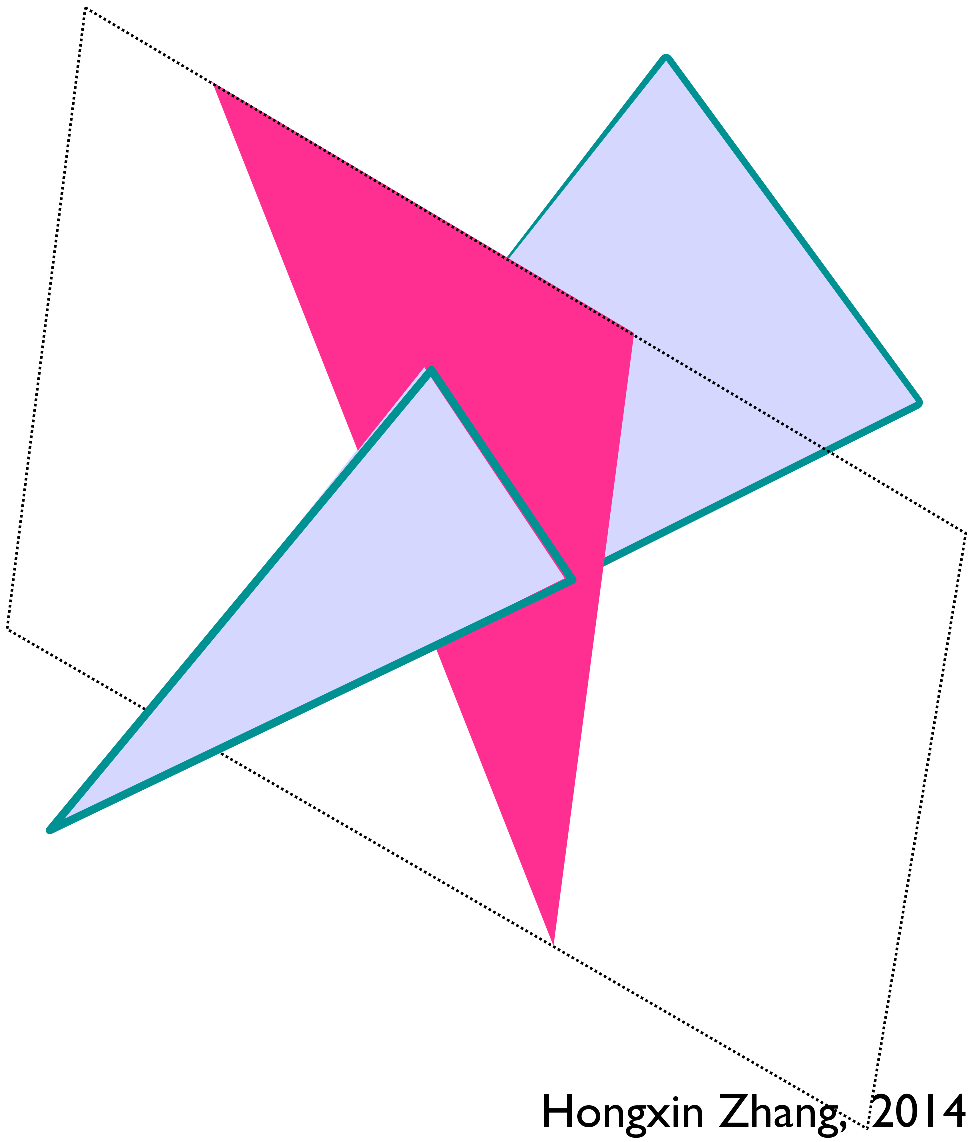
  How to?



# BSP Tree

- Split Triangle:

  How to?



# Summary: BSP Trees

- Pros:
  - Simple, elegant scheme
  - Only writes to frame-buffer (i.e., painters algorithm)
    - Thus very popular for video games (but getting less so)
- Cons:
  - Computationally intense preprocess stage restricts algorithm to static scenes
  - Worst-case time to construct tree: $O(n^3)$
  - Splitting increases polygon count
    - Again, $O(n^3)$ worst case

Z-buffer

Scan-line

Warnock：

A divide and conquer

**Computational expensive of clipping**

**Z-buffer**

**Scan-line**

**Warnock：**

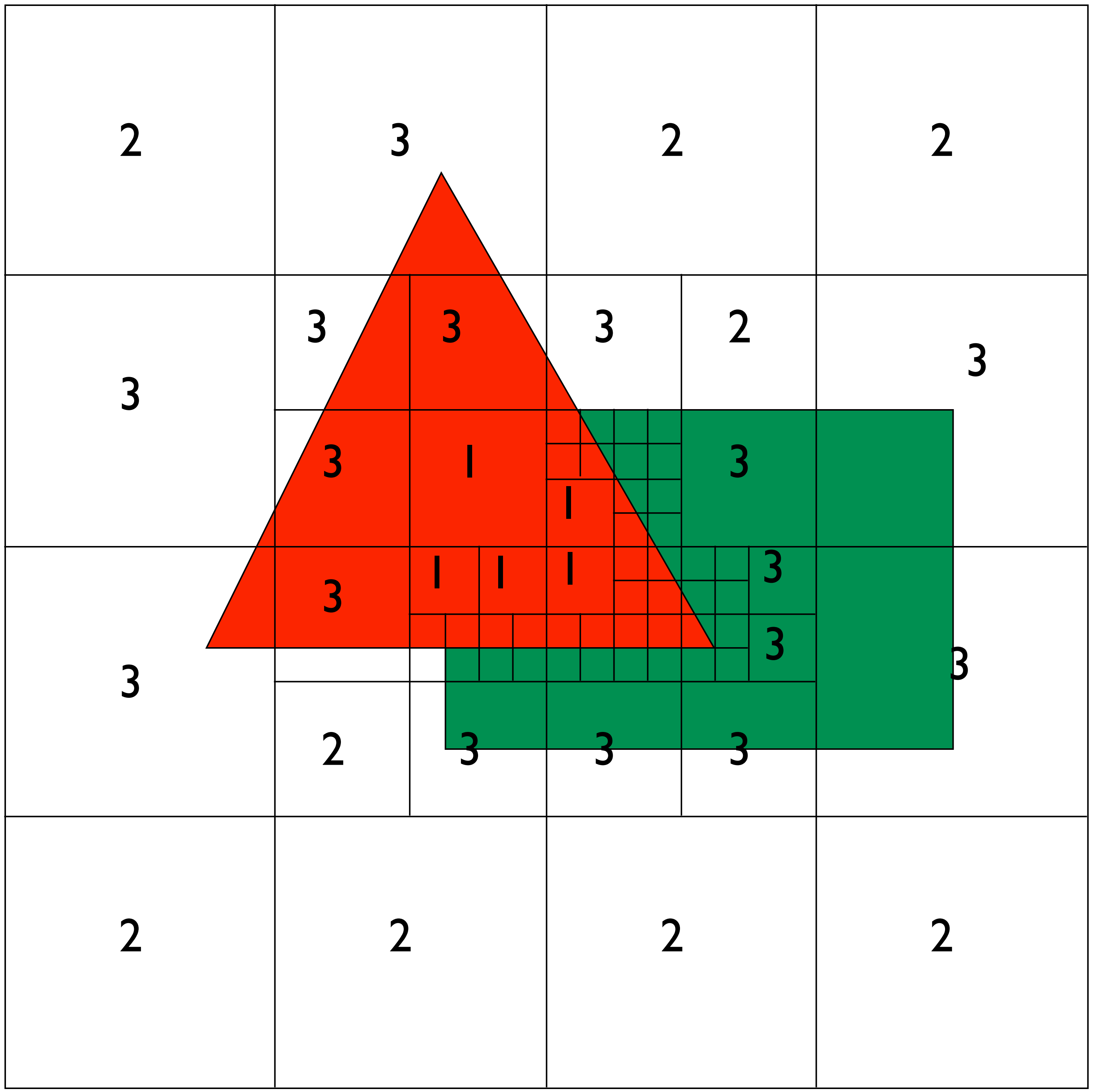
**A divide and conquer**

# Warnock's Area Subdivision (Image Precision)

- Start with whole image
- If one of the easy cases is satisfied, draw what's in front
  - front polygon covers the whole window or
  - there is at most one polygon in the window.
- Otherwise, subdivide region into 4 windows and recurse
- If region is single pixel, choose surface with smallest depth

- Advantages:
  - No over-rendering
  - Anti-aliases well - just recurse deeper to get sub-pixel information
- Disadvantage:
  - Tests are quite complex and slow

# Warnock's Algorithm




- Regions labeled with case used to classify them:

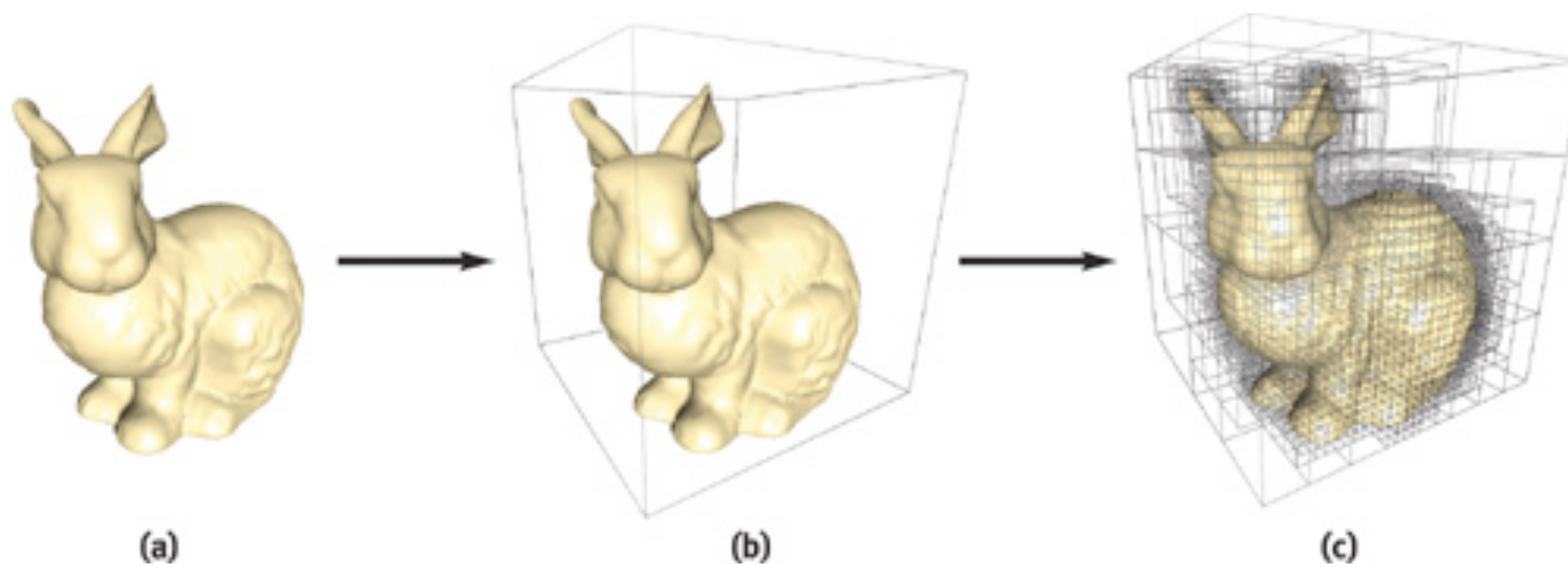
  One polygon in front

  Empty

  One polygon inside, surrounding or intersecting

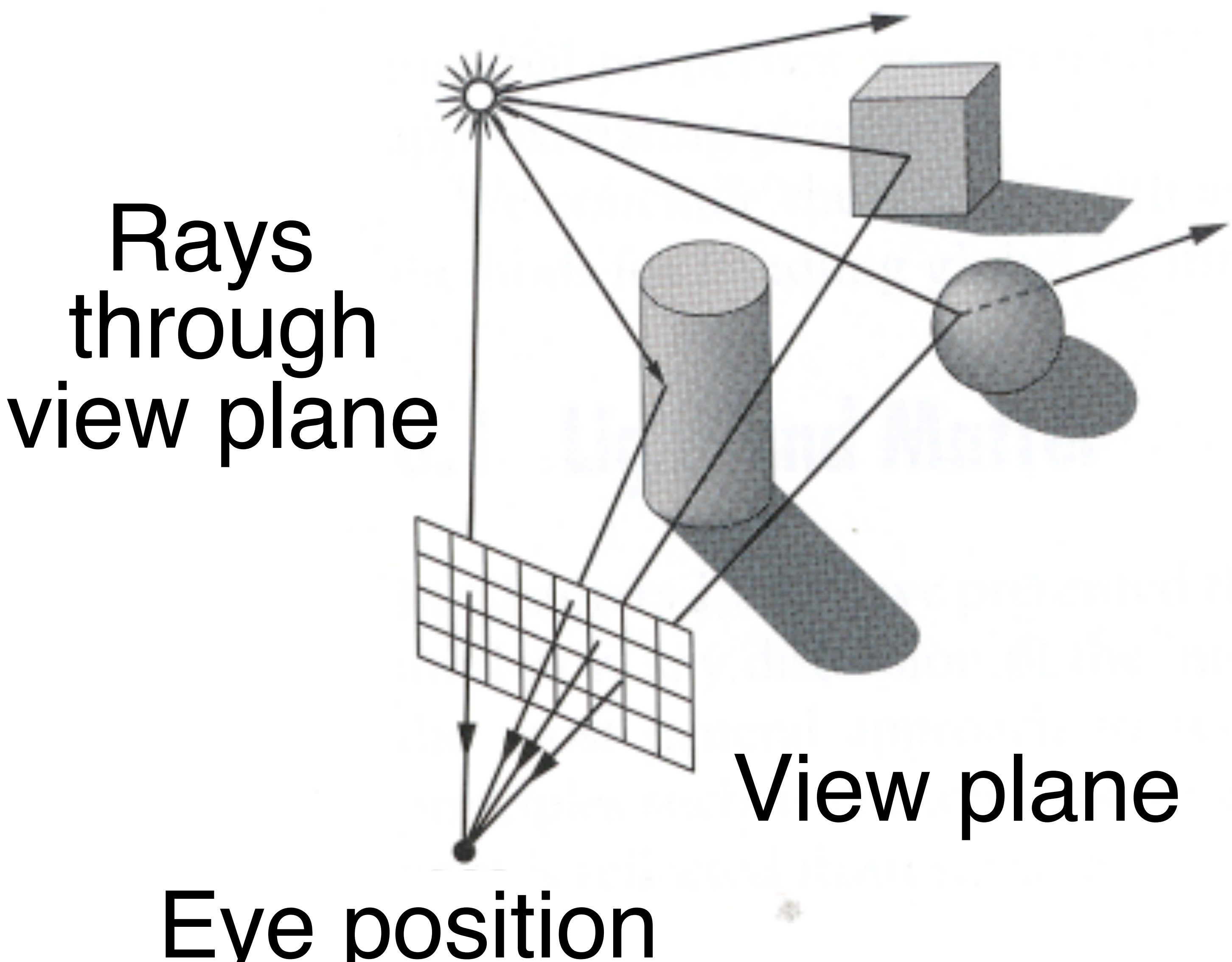
- Small regions not labeled

# Octree



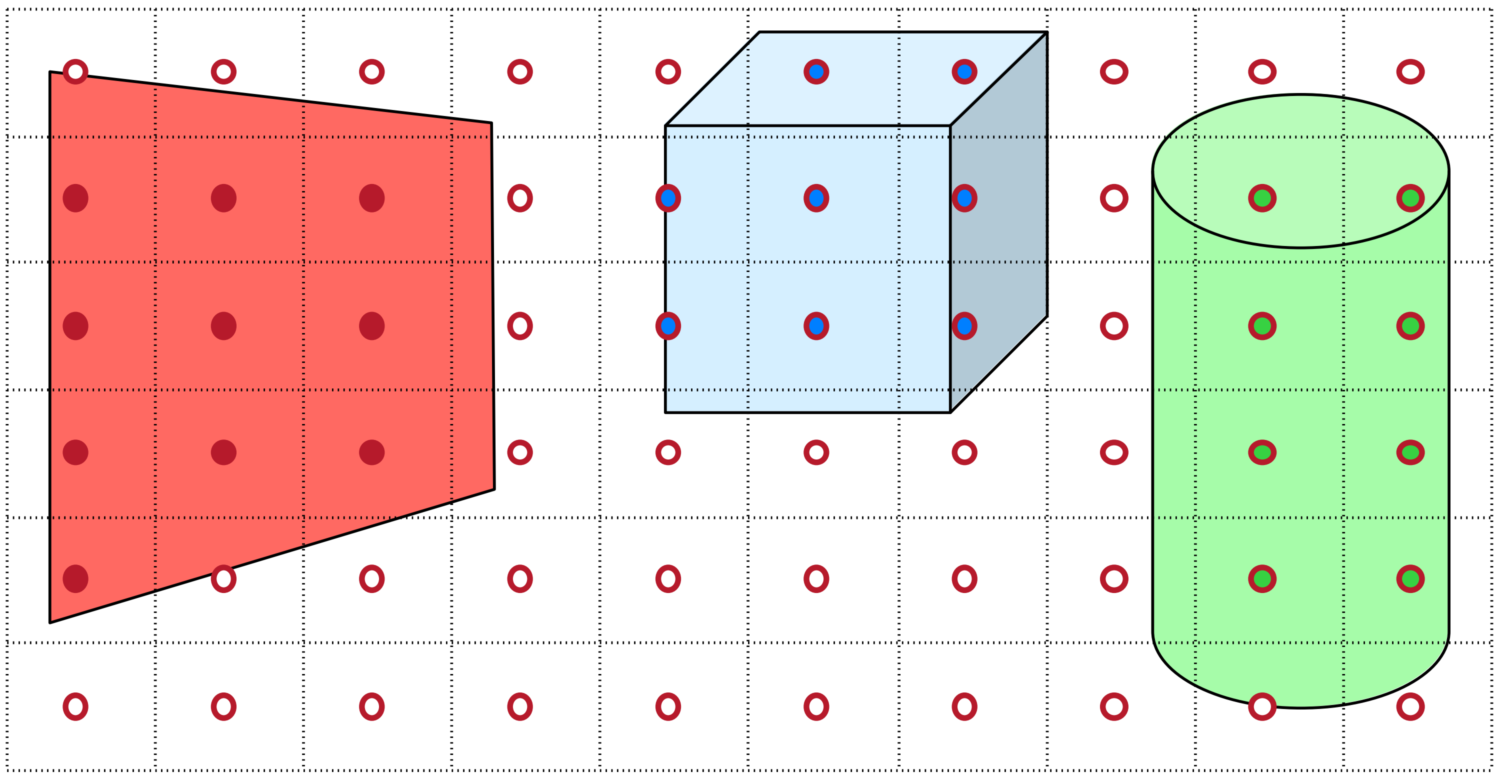

http://en.wikipedia.org/wiki/View_frustum_culling

# ray casting



# Ray Casting

- For each sample …
  - Construct ray from eye position through view plane
  - Find first surface intersected by ray through pixel
  - Compute color sample based on surface radiance



# Thank You