Computer Graphics 2014

2. 2D Graphics Algorithms

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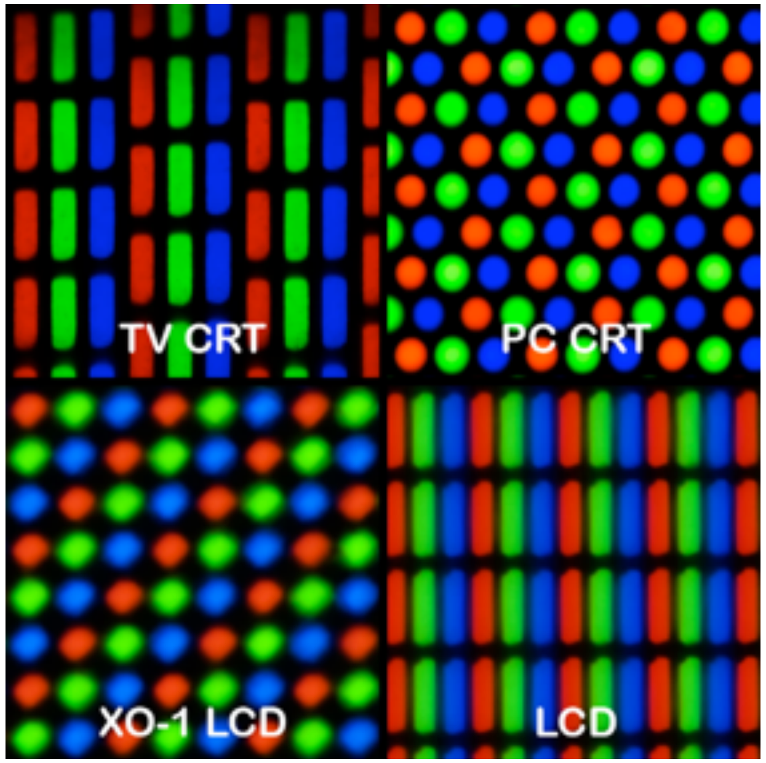
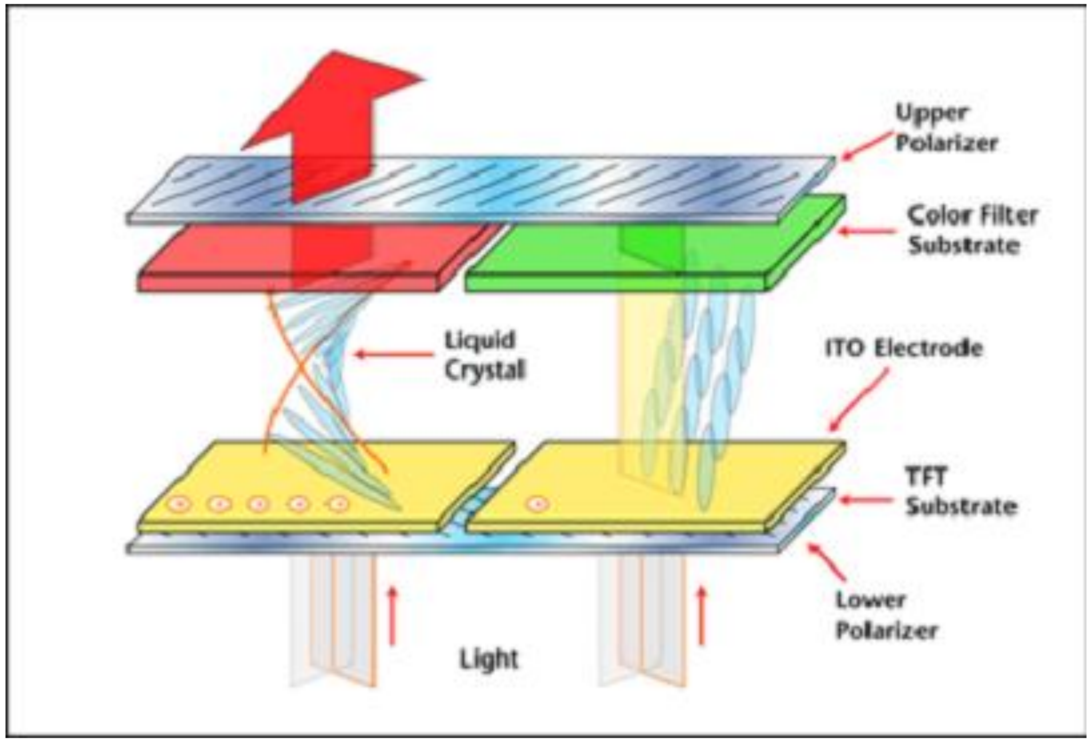
Screen


TV CRT
PC CRT
XO-1 LCD
LCD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Color Filter
Substrate
Liquid
Crystal
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TFT
Substrate
Lower
Polarizer
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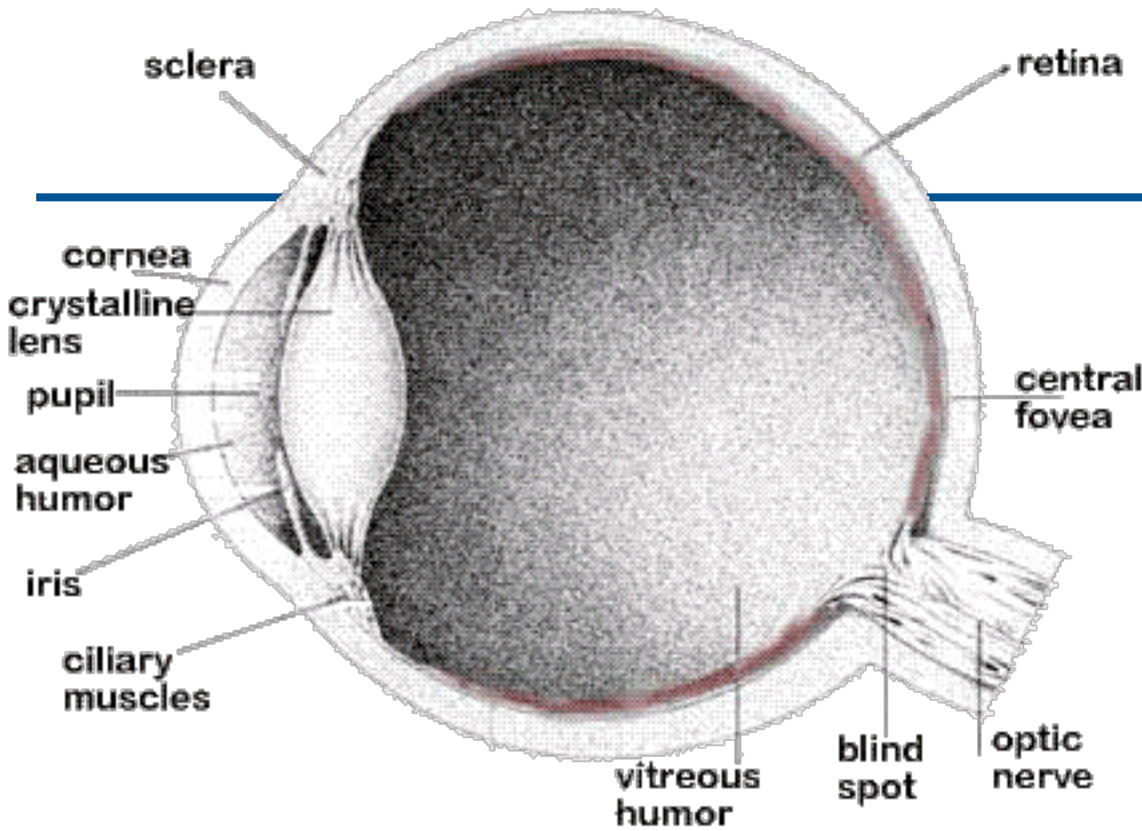
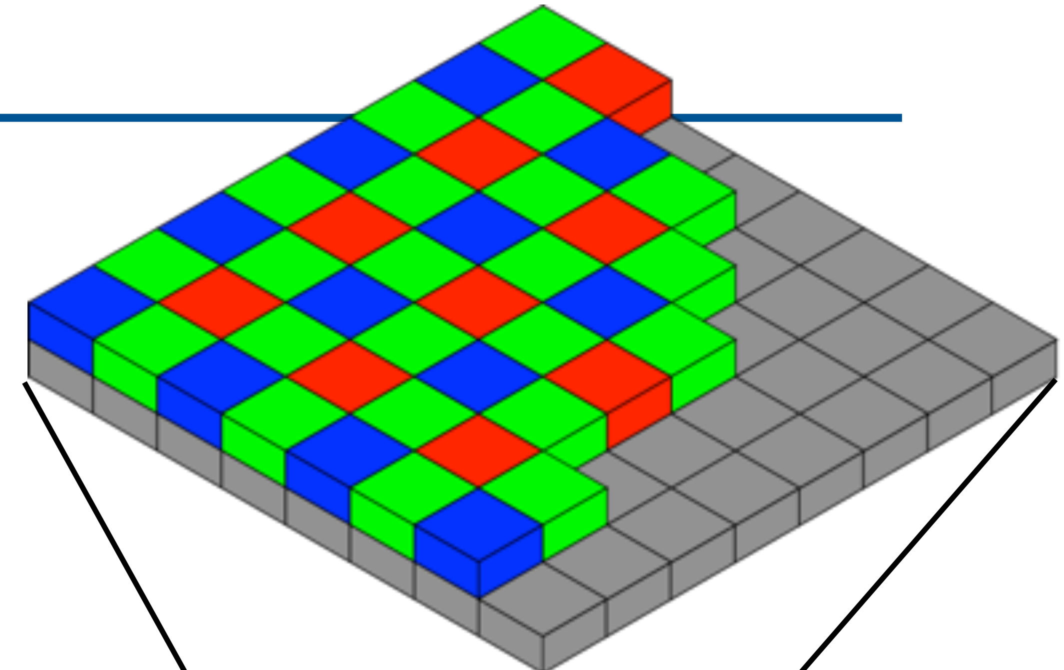
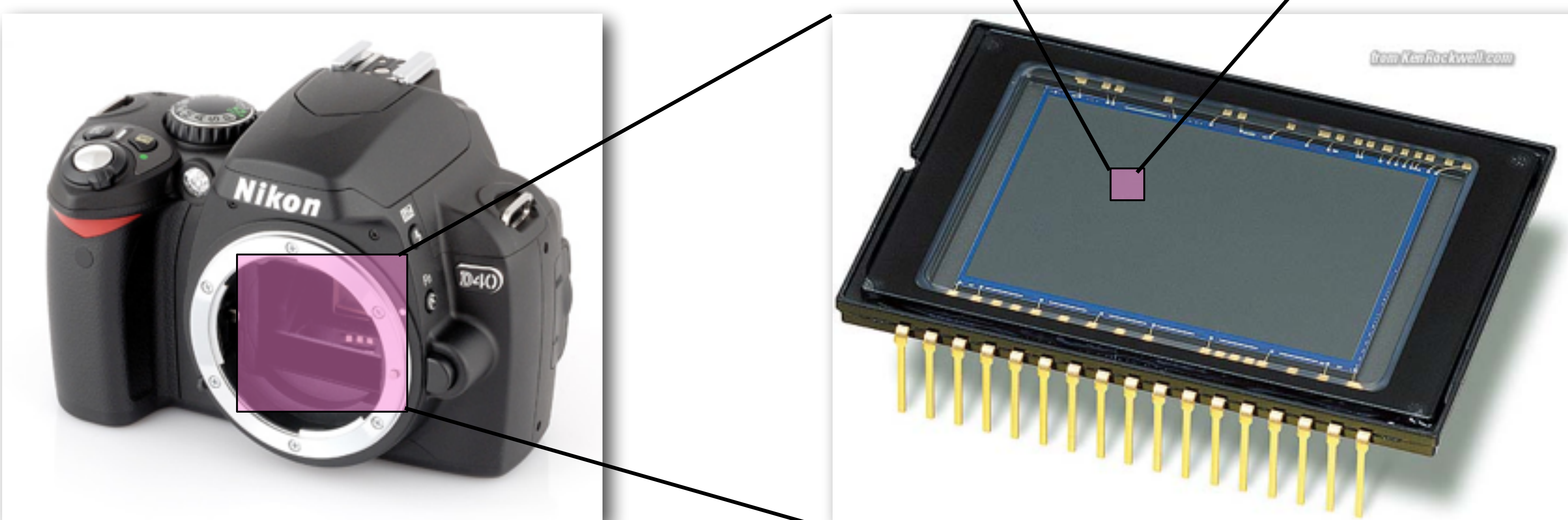




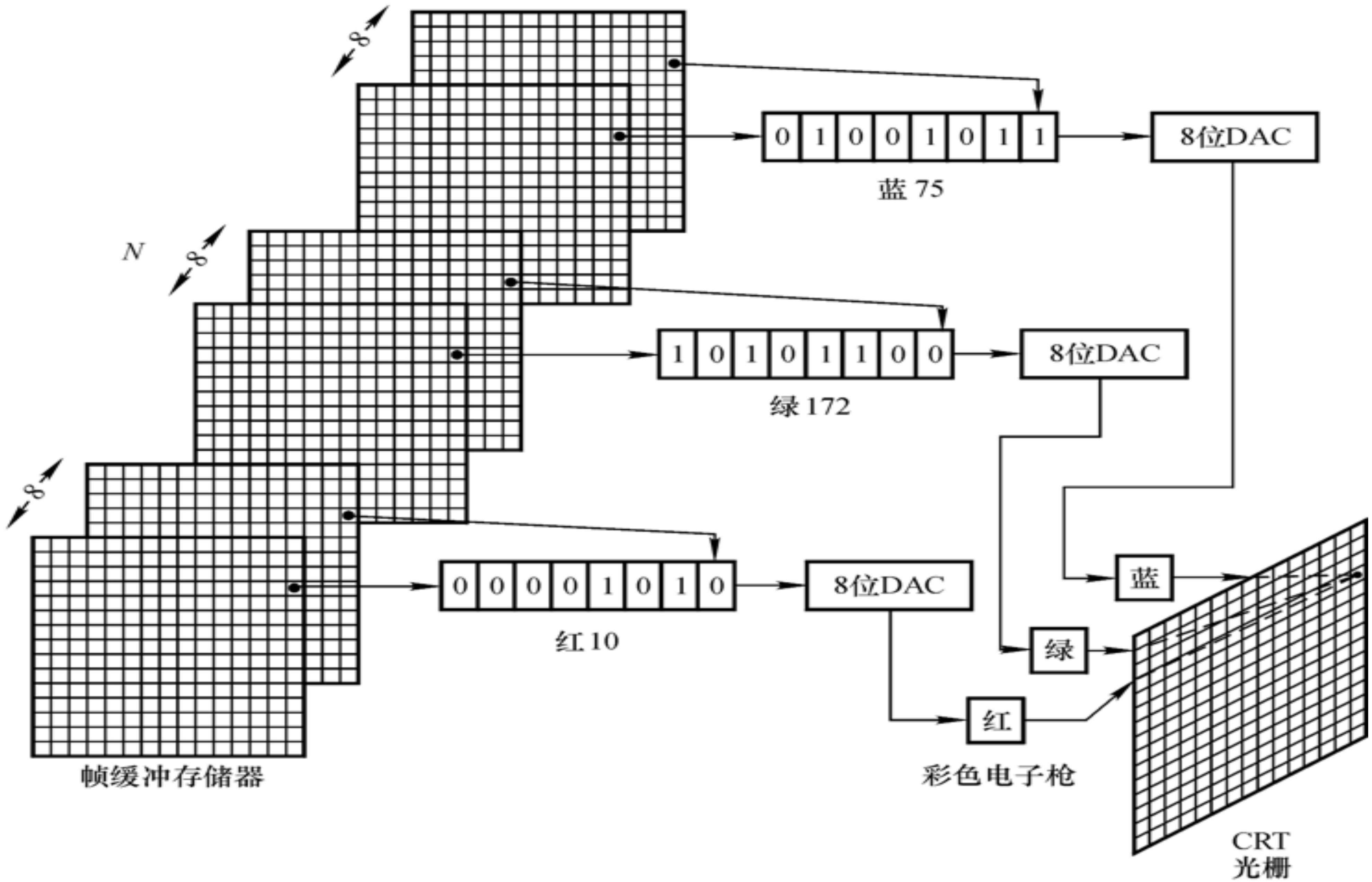


Nikon D40 Sensors

Rasterization

- The task of displaying a world modeled using primitives like lines, polygons, filled / patterned areas, etc. can be carried out in two steps
  - determine the pixels through which the primitive is visible, a process called Rasterization or scan conversion
  - determine the color value to be assigned to each such pixel.

Rasterization



Raster Graphics Packages

- The efficiency of these steps forms the main criteria to determine the performance of a display
- The raster graphics package is typically a collection of efficient algorithms for scan converting (rasterization) of the display primitives
- High performance graphics workstations have most of these algorithms implemented in hardware

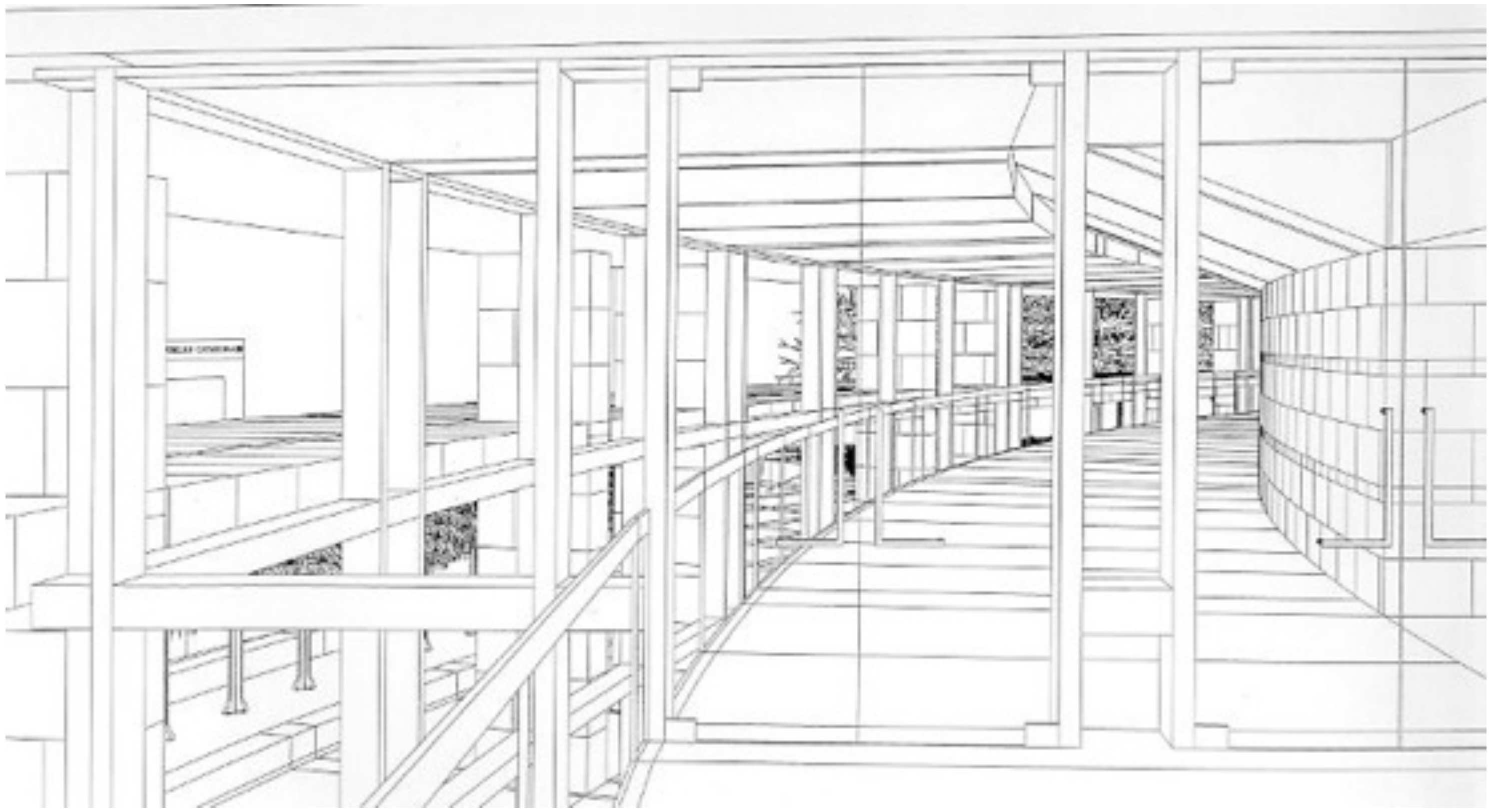
Why Study these Algorithms?

- Some of these algorithms are very good examples of clever algorithmic optimization done to dramatically improve performance using minimal hardware facilities

- Mobile graphics
- Inspiration

Scan Converting a Line Segment

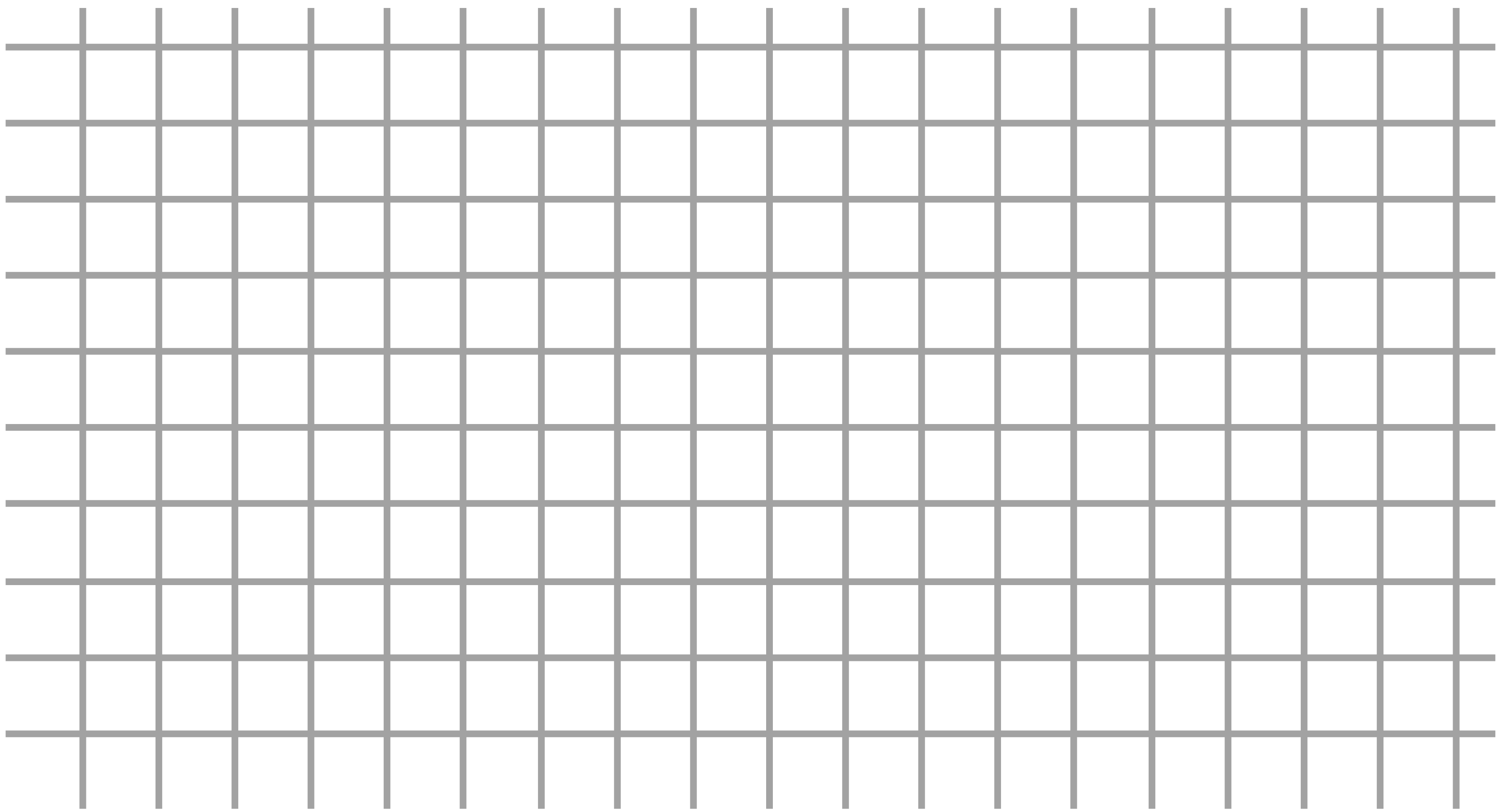
- The line is a powerful element used since the days of Euclid to model the edges in the world.



Given a line segment defined by its endpoints determine the pixels and color which best model the line segment.

Scan converting lines

start from (x_1, y_1) end at (x_2, y_2)



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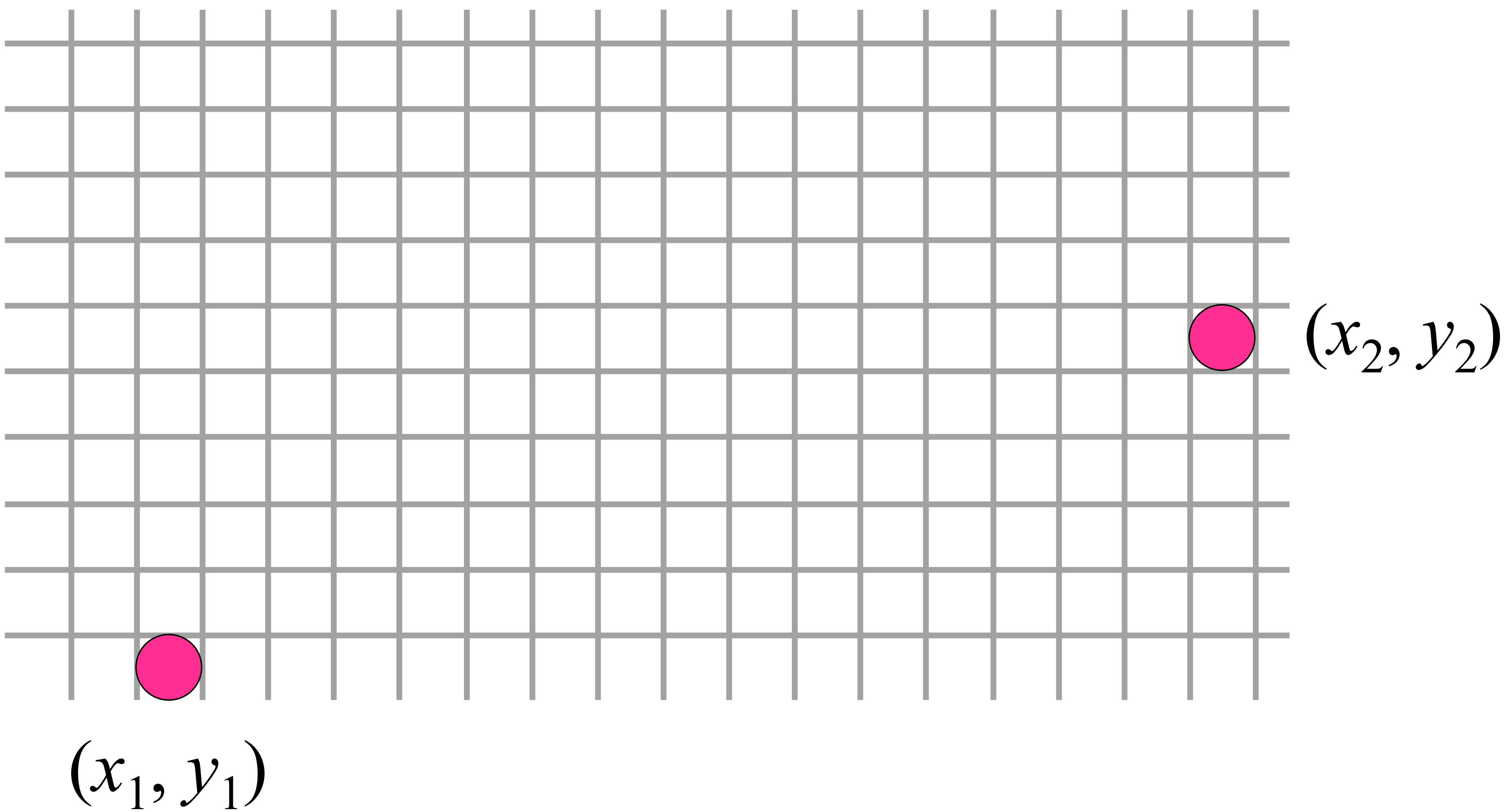

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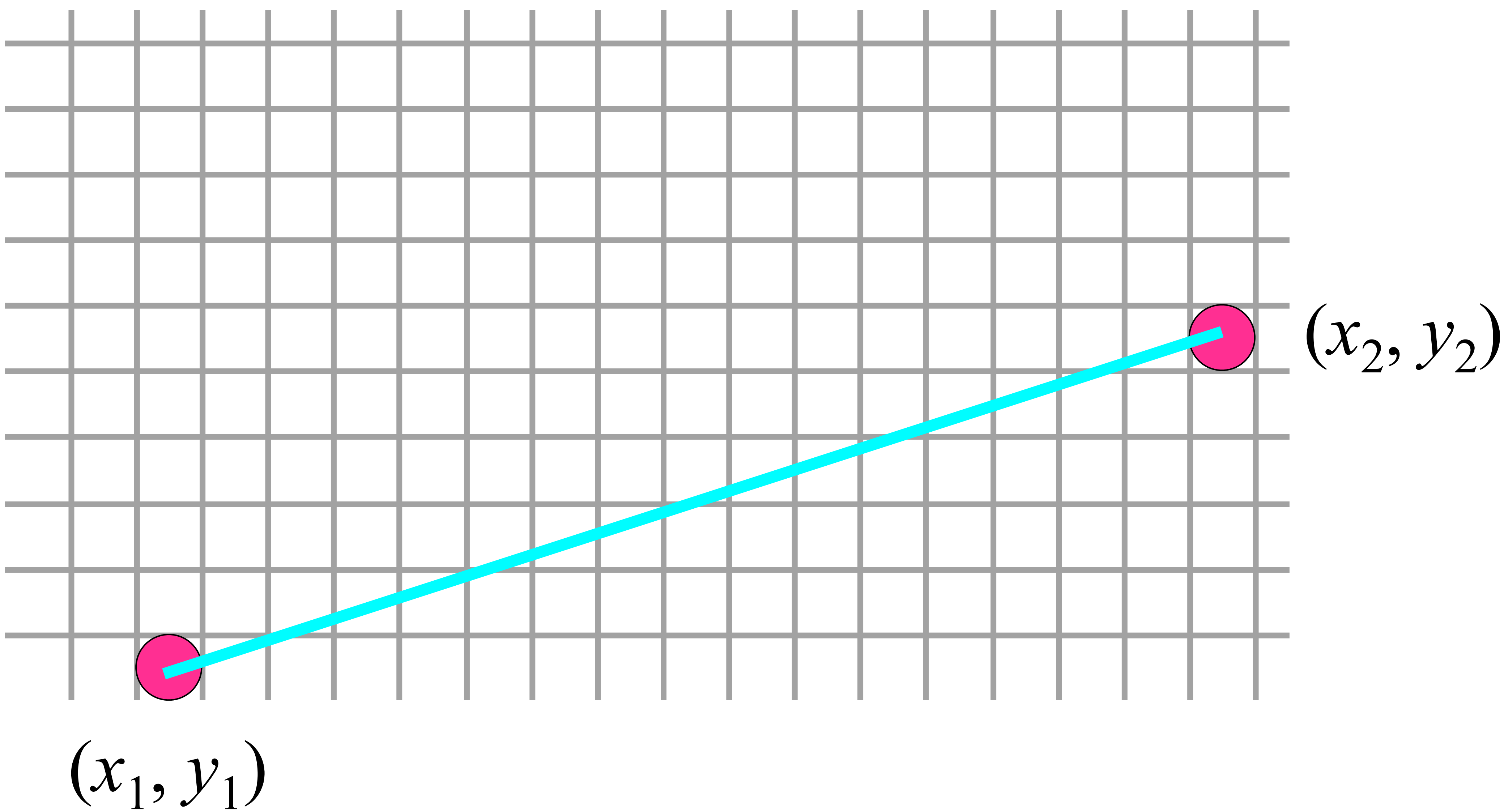

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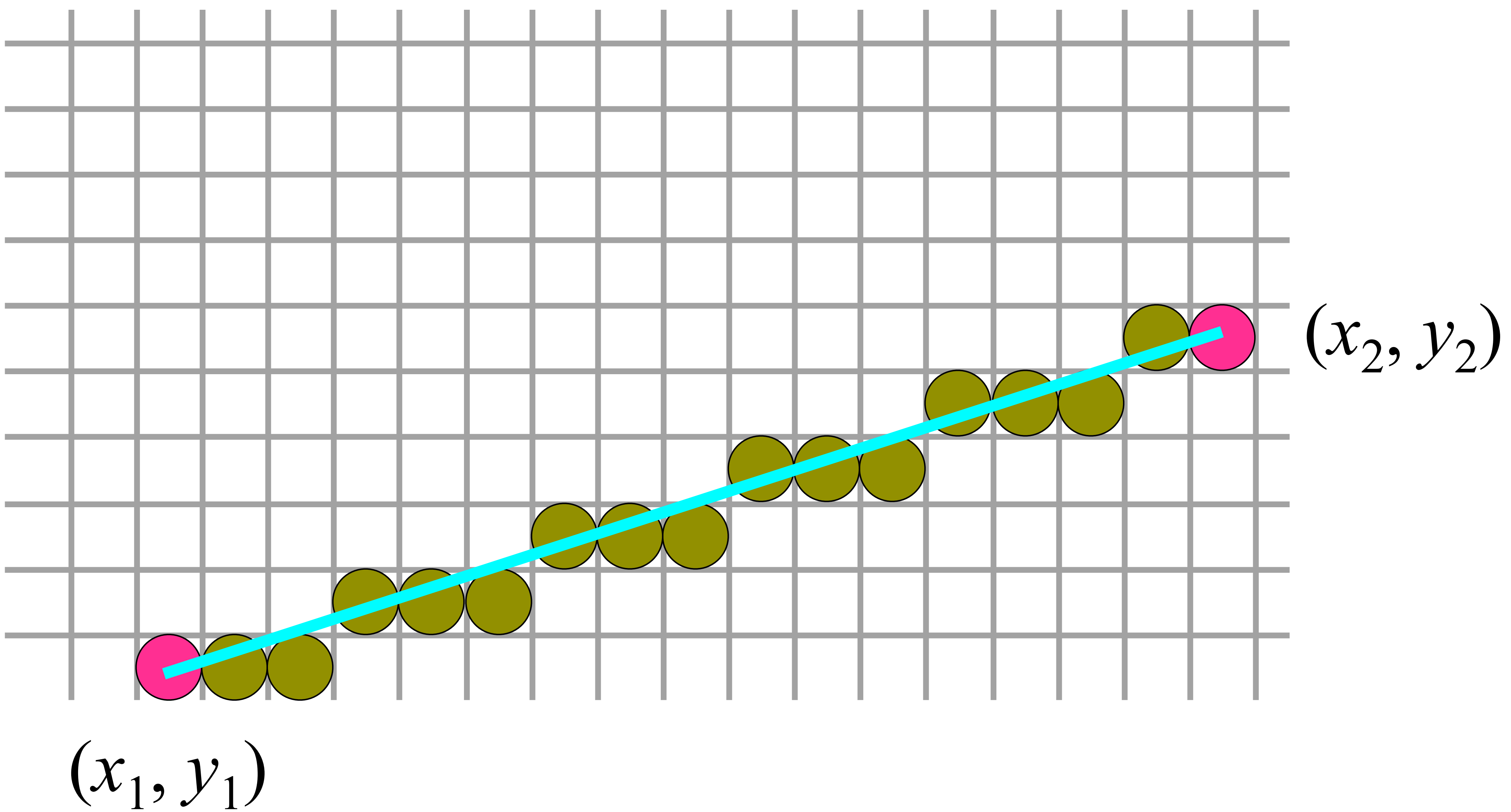
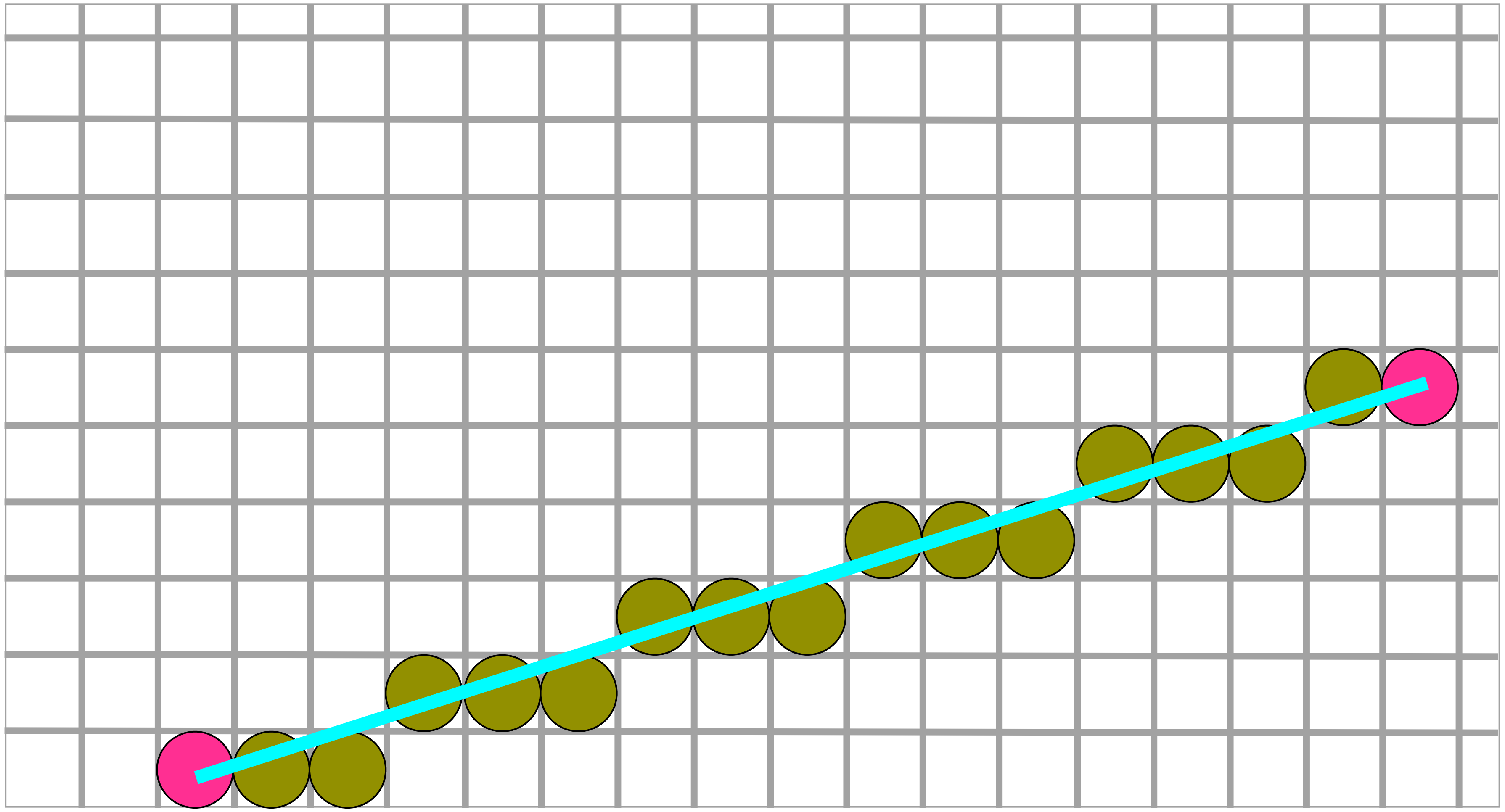


Scan converting lines

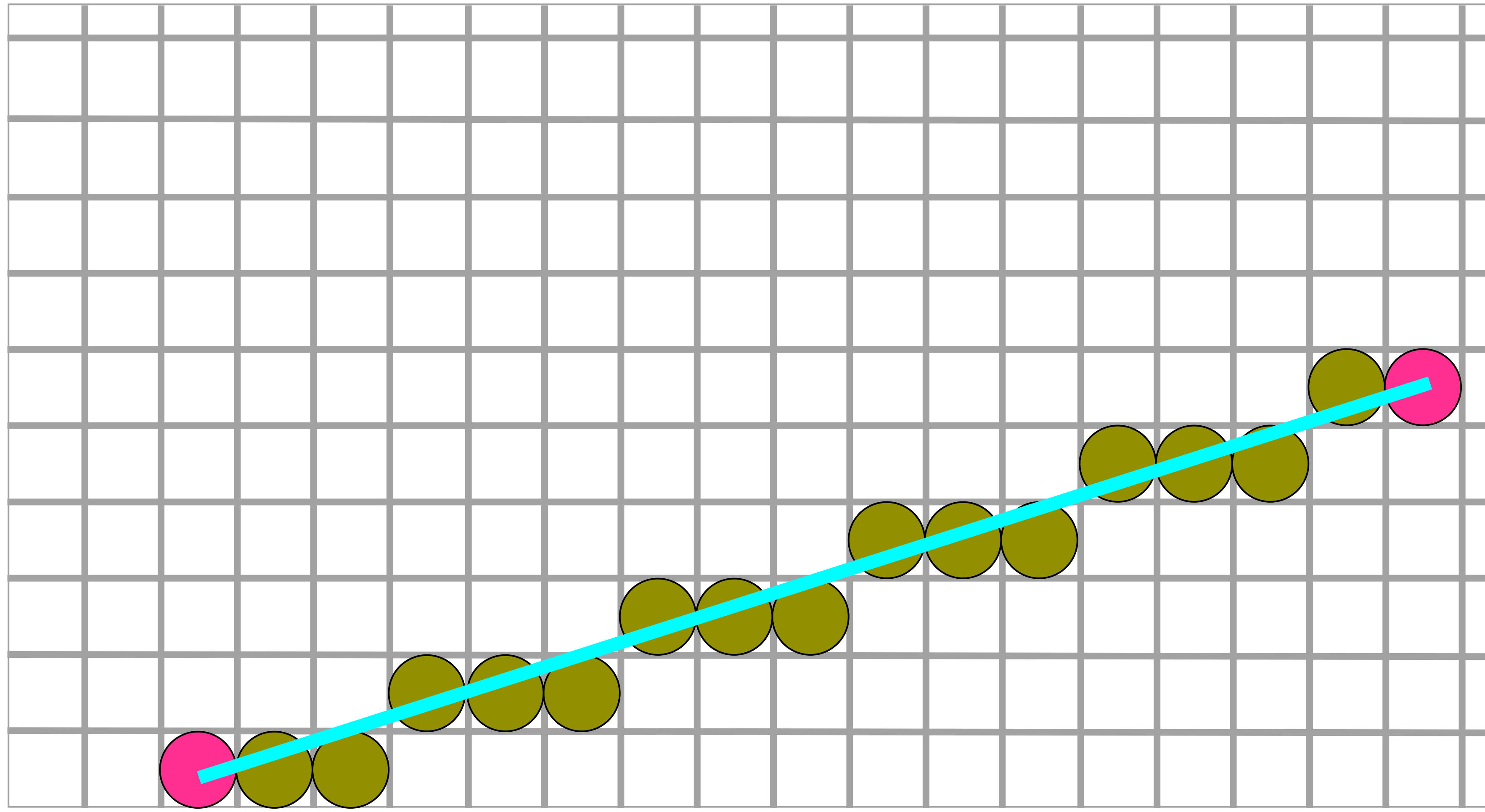
- Requirements
  - chosen pixels should lie as close to the ideal line as possible
  - the sequence of pixels should be as straight as possible
  - all lines should appear to be of constant brightness independent of their length and orientation
  - should start and end accurately
  - should be drawn as rapidly as possible
  - should be possible to draw lines with different width and line styles

Question 1: How ?

$(x_1, y_1), (x_2, y_2)$



Question 1: How ?



$(x_1, y_1), (x_2, y_2)$

$y=mx+b$

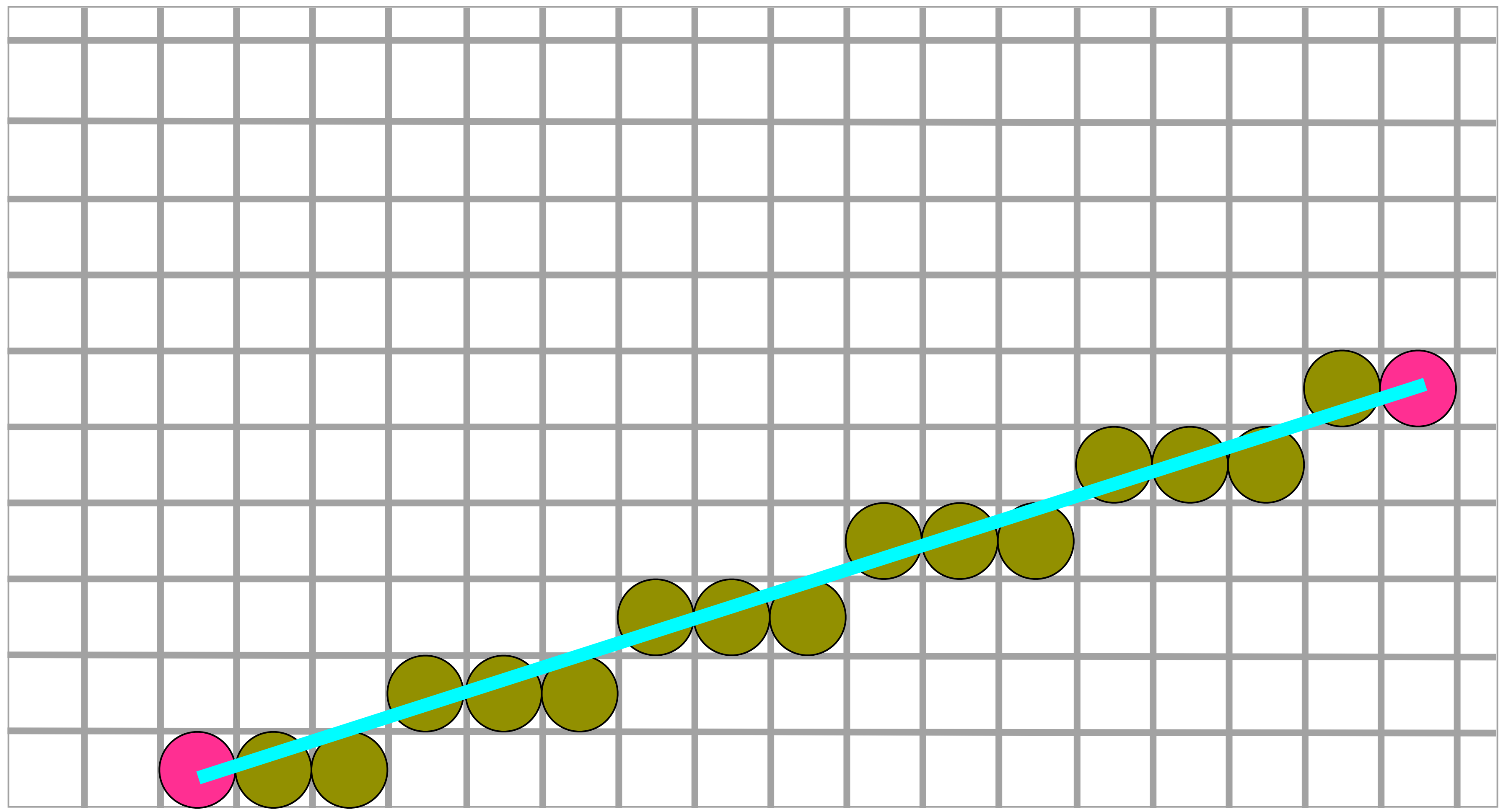
$x_1+1 \Rightarrow y=?$, rounding

$x_1+2 \Rightarrow y=?$, rounding

$x_1+i \Rightarrow y=?$, rounding

Question 1: How ?

$(x_1, y_1), (x_2, y_2)$



$y = mx + b$

$x_1 + 1 \Rightarrow y = ?$, rounding

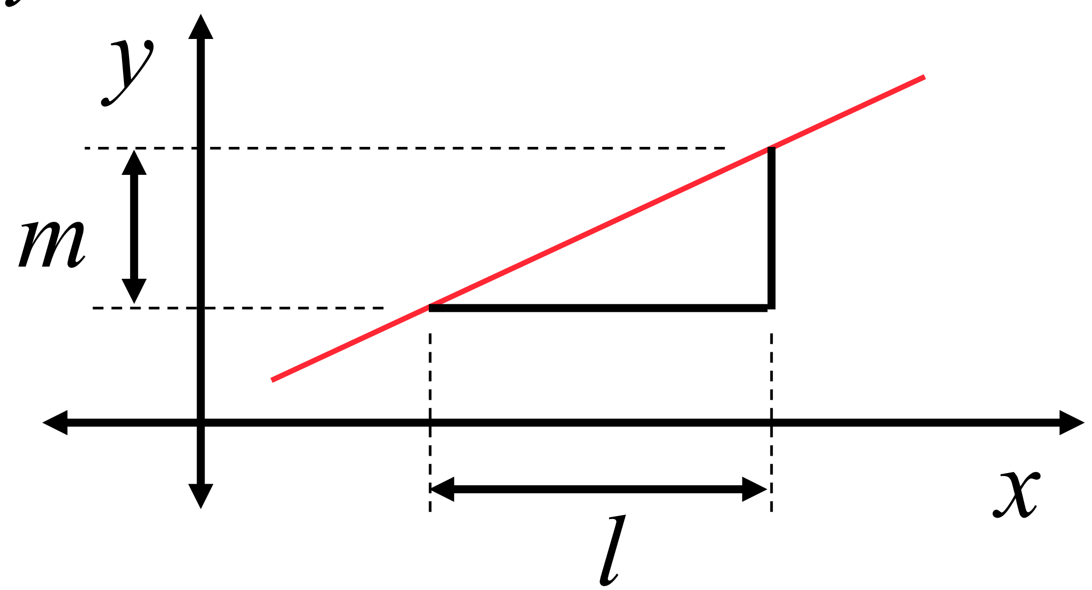
$x_1 + 2 \Rightarrow y = ?$, rounding

$x_1 + i \Rightarrow y = ?$, rounding

Question 2: How to speed up?

Equation of a Line

- Equation of a line is $y - m \cdot x + c = 0$
- For a line segment joining points
- $\boldsymbol{P}(x_1, y_1)$ and $\boldsymbol{Q}(x_2, y_2)$ $\quad$ *slope* $\quad m = \dfrac{y2 - y1}{x2 - x1} = \dfrac{\Delta y}{\Delta x}$
- Slope m means that for every unit increment in x the increment in y is m units



Digital Differential Analyzer (DDA)

- We consider the line in the first octant. Other cases can be easily derived.
- Uses differential equation of the line

$$y_i = mx_i + c$$

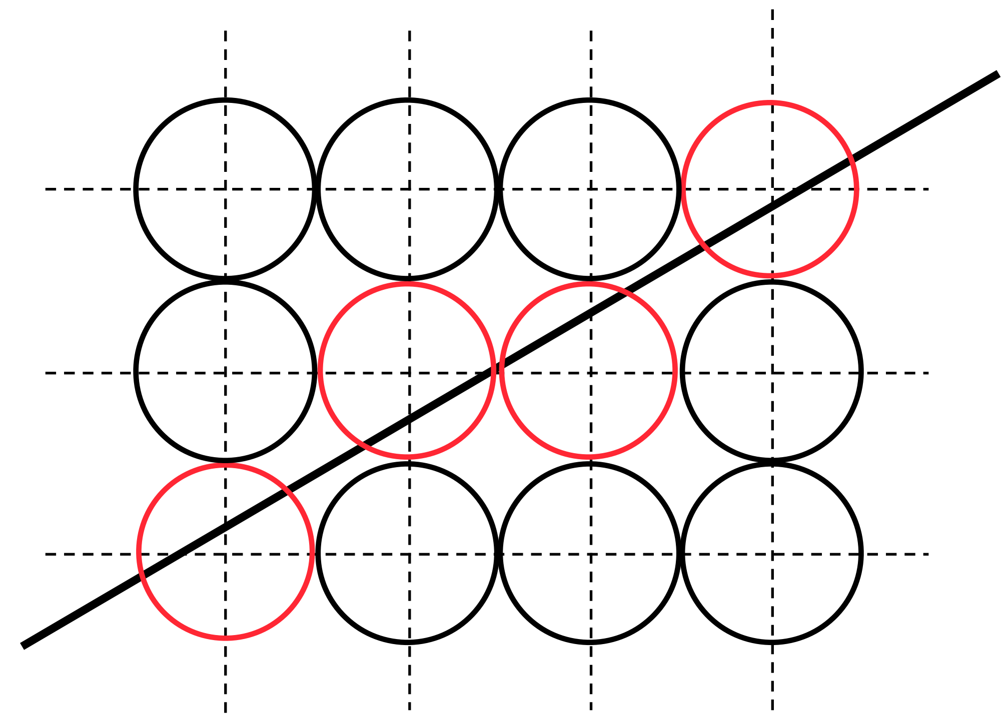
$$where, \quad m = \frac{y2 - y1}{x2 - x1}$$

- Incrementing X-coordinate by 1

$$x_i = x_{i\_prev} + 1$$

$$y_i = y_{i\_prev} + \mathrm{m}$$

- Illuminate the pixel $[x_i, round(y_i)]$



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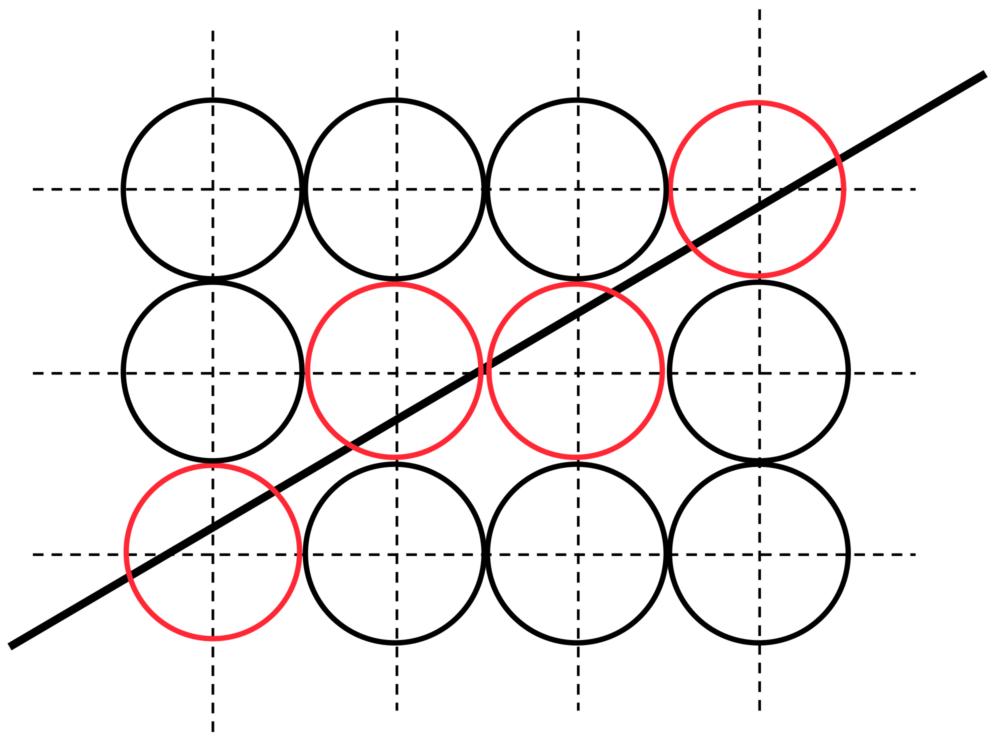

Discussion1: What technique makes it fast?

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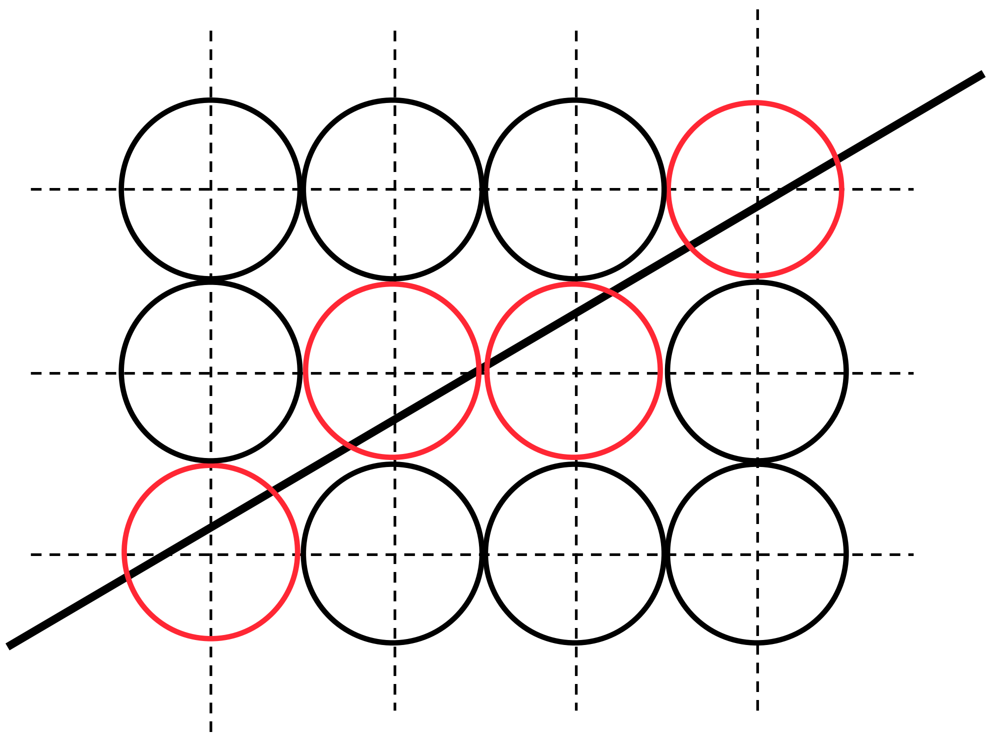

Discussion1: What technique makes it fast?

Discussion2: Is there any problem in the algorithm? How to avoid it?

If $\triangle x < \triangle y$

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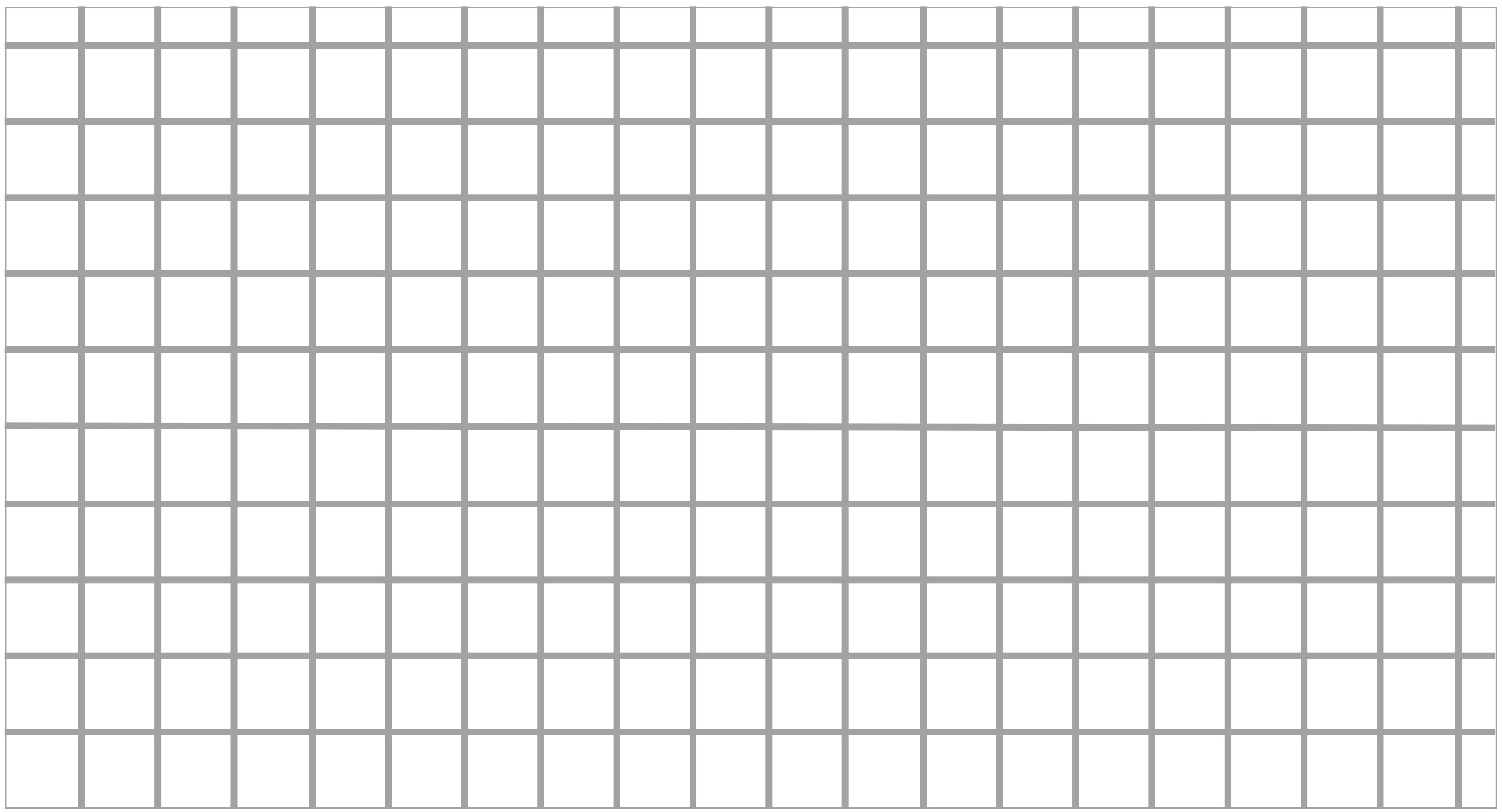

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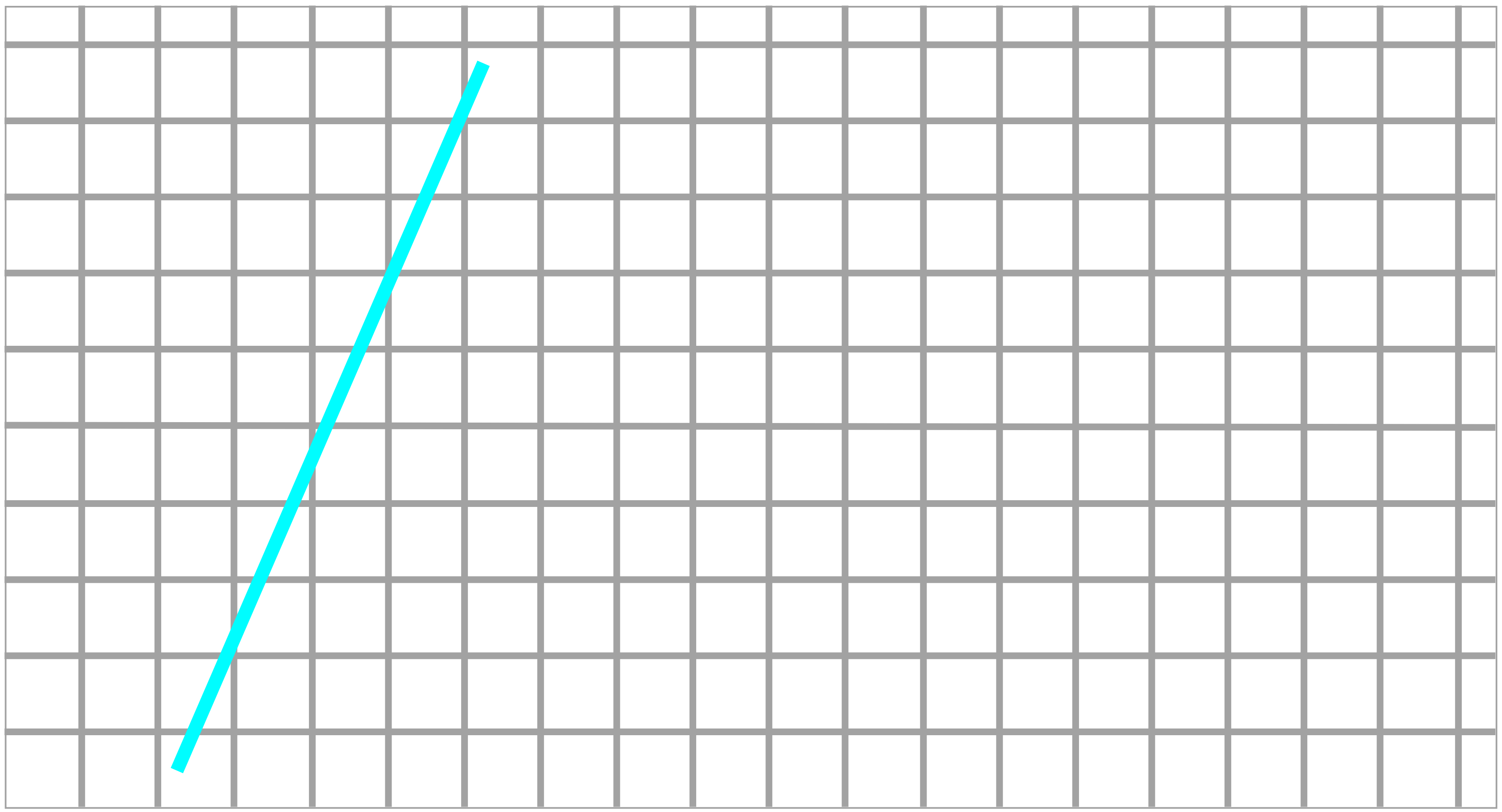

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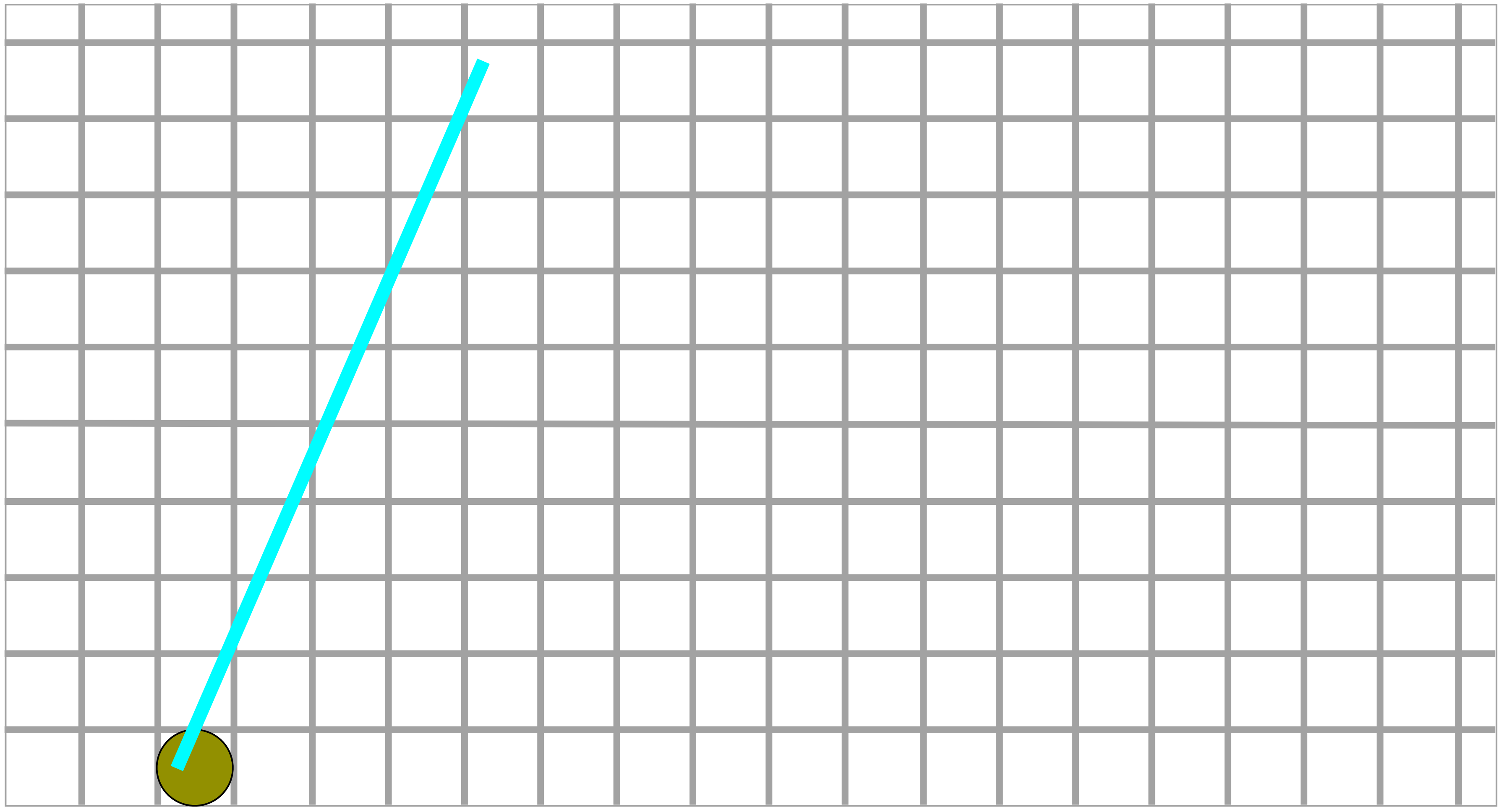

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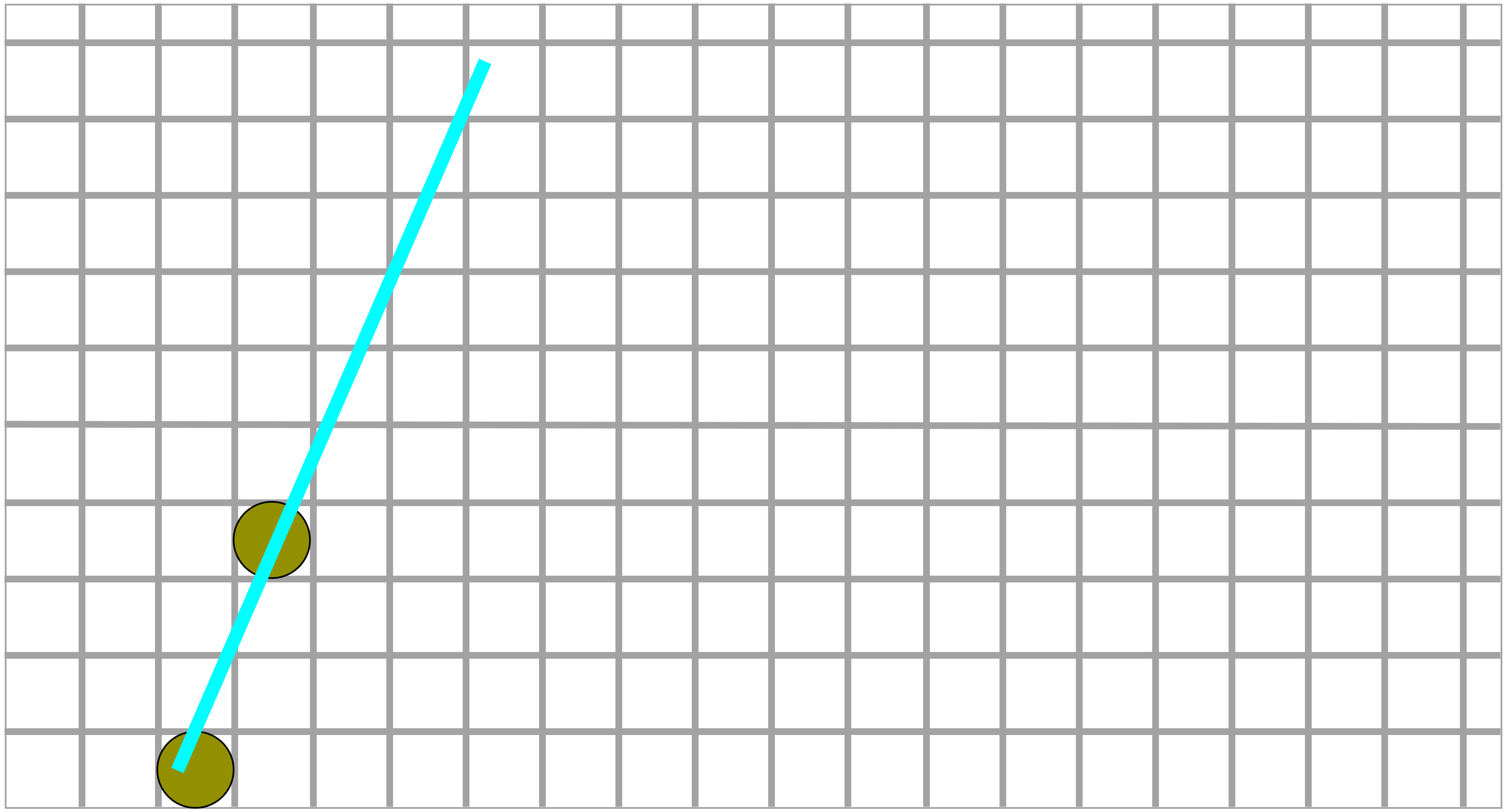

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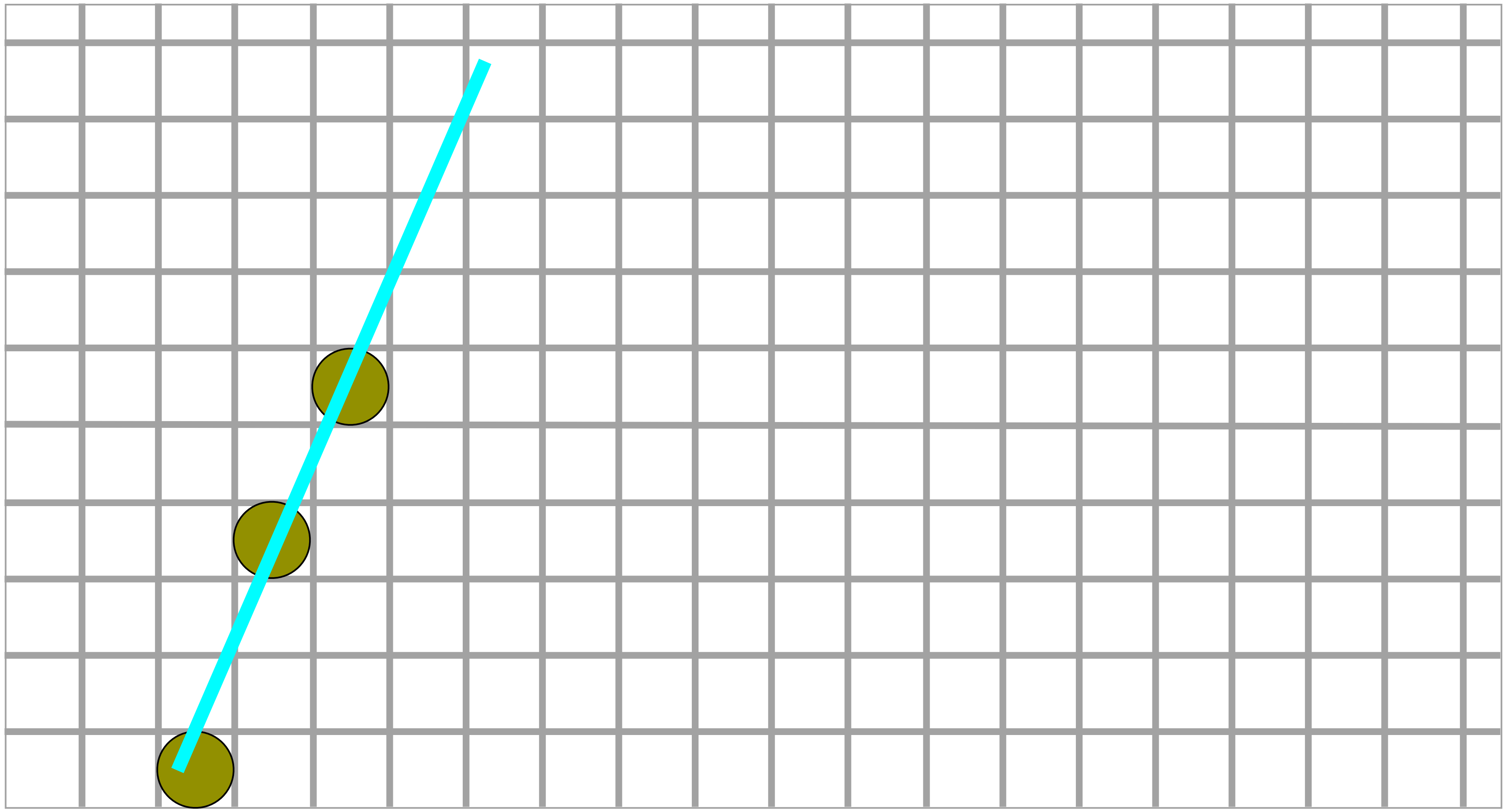

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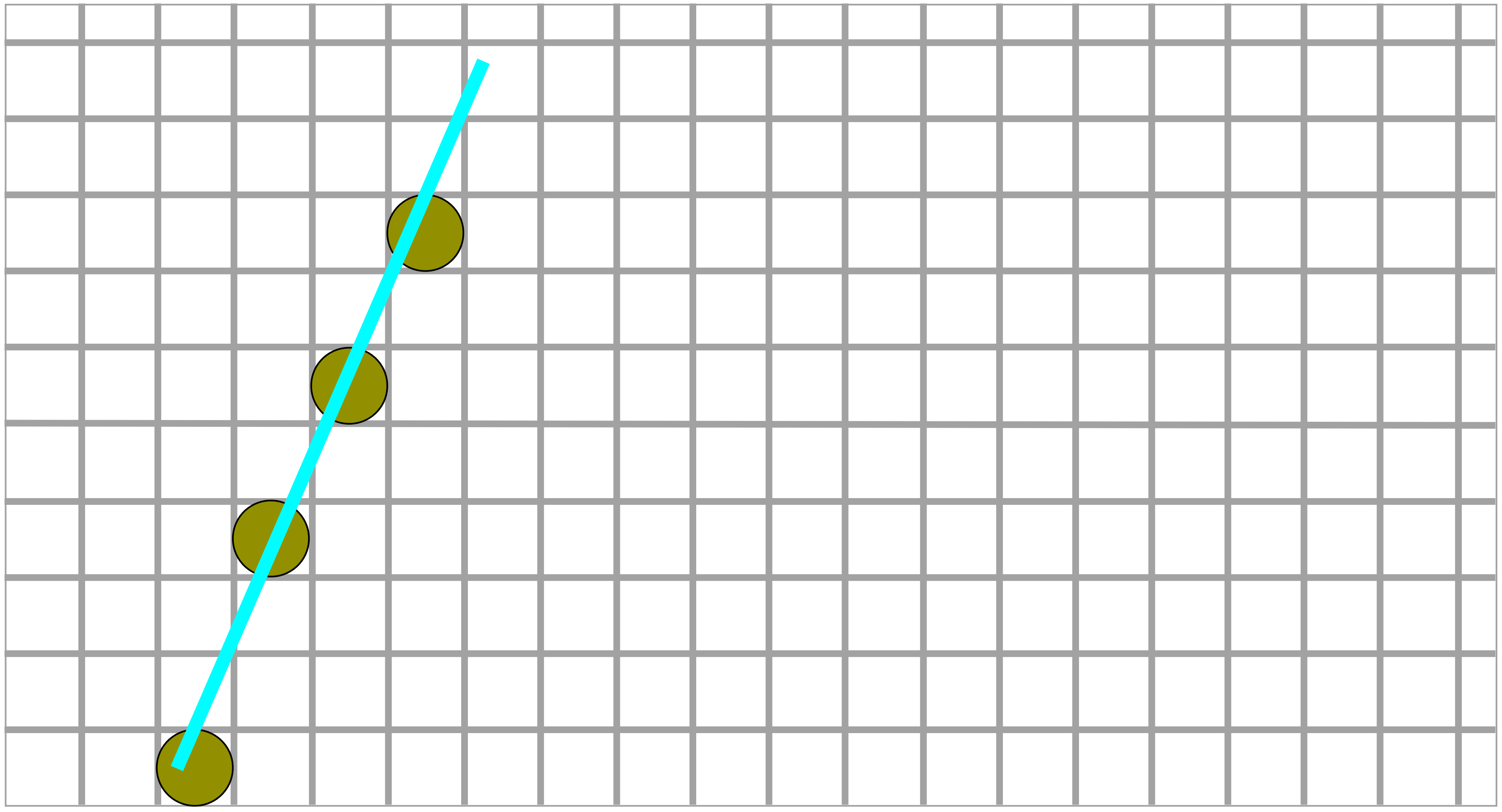

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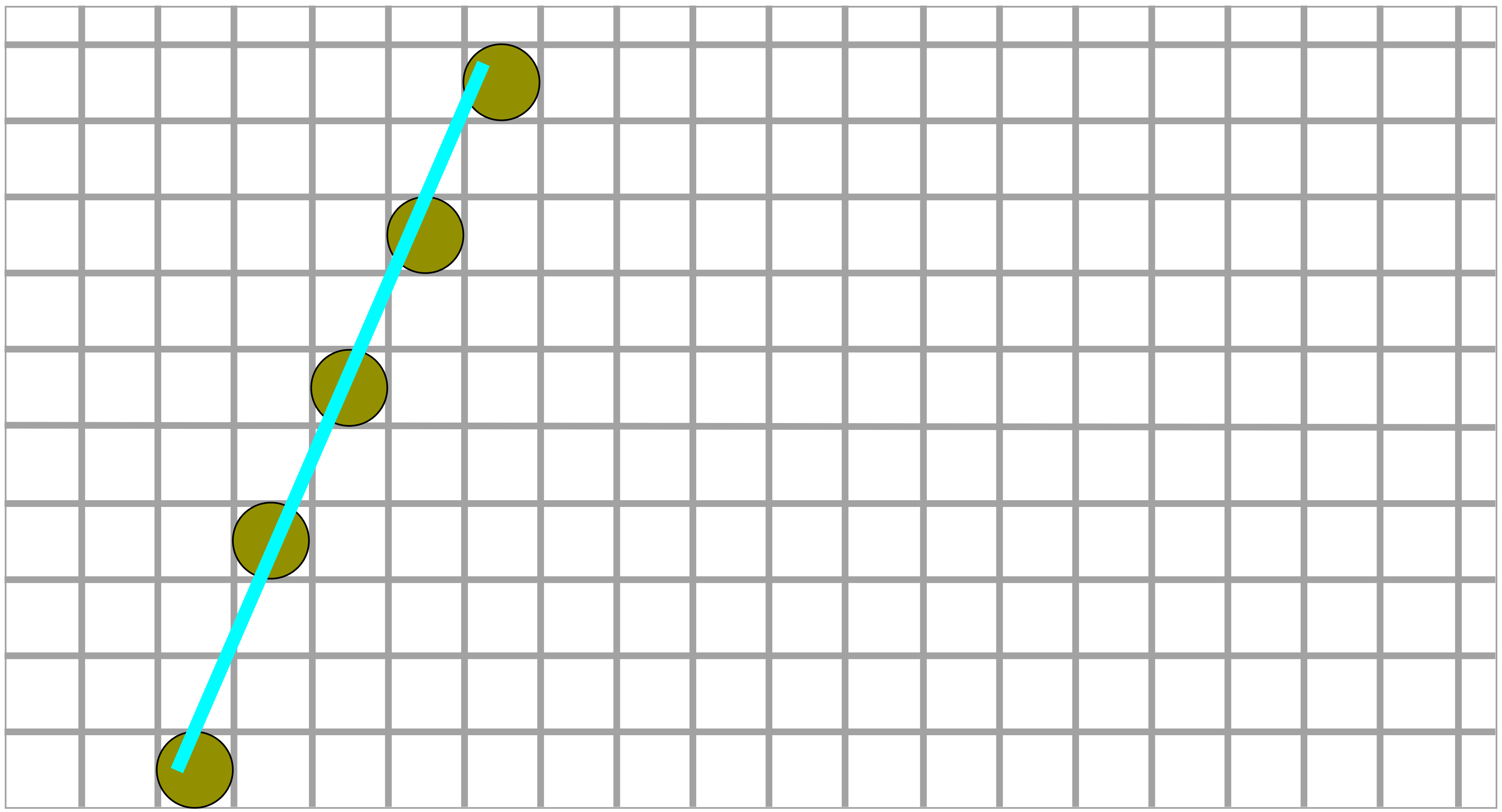

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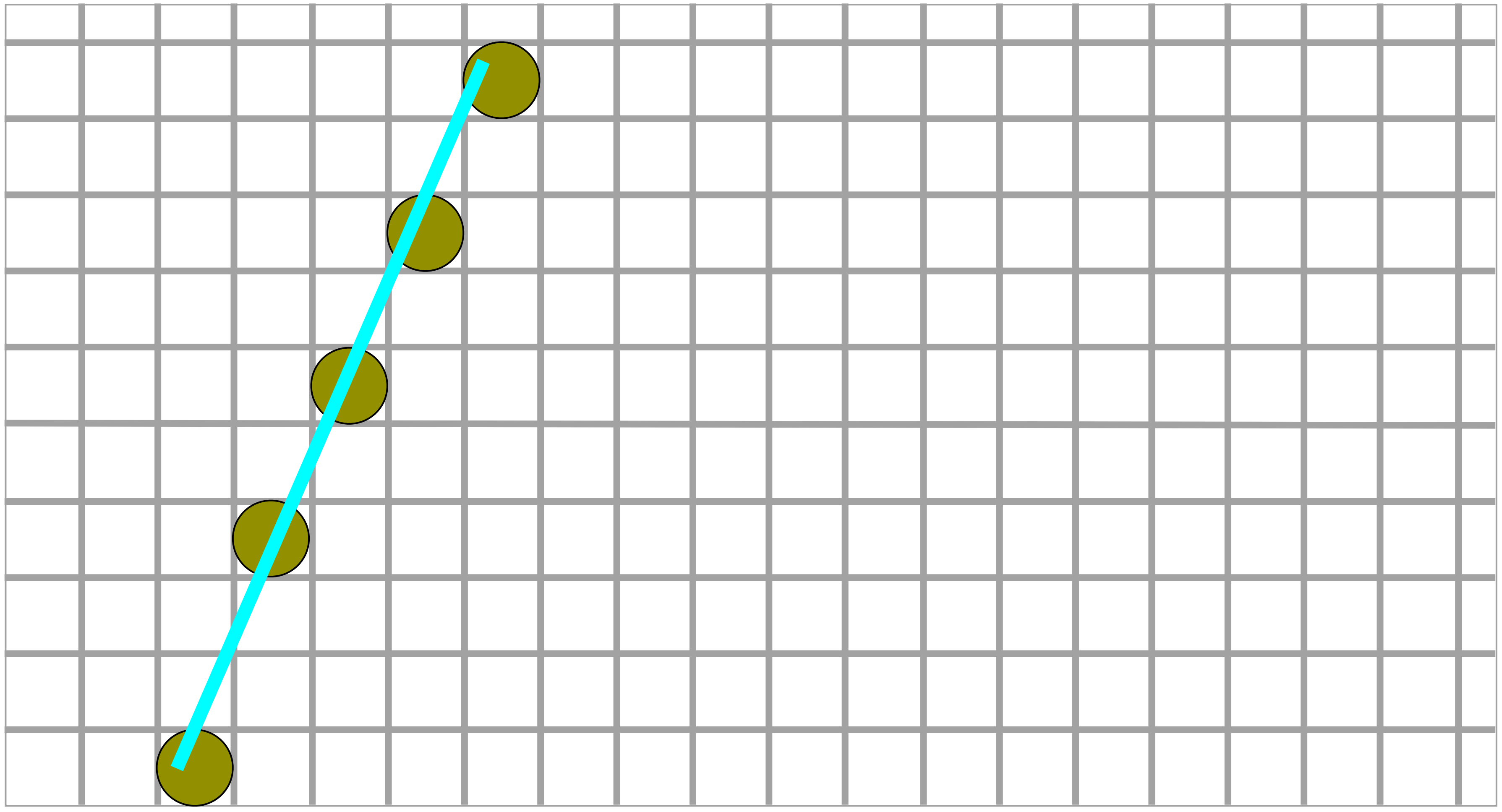

$$y \mathrel{+}= 1;\ x \mathrel{+}= 1/m;$$

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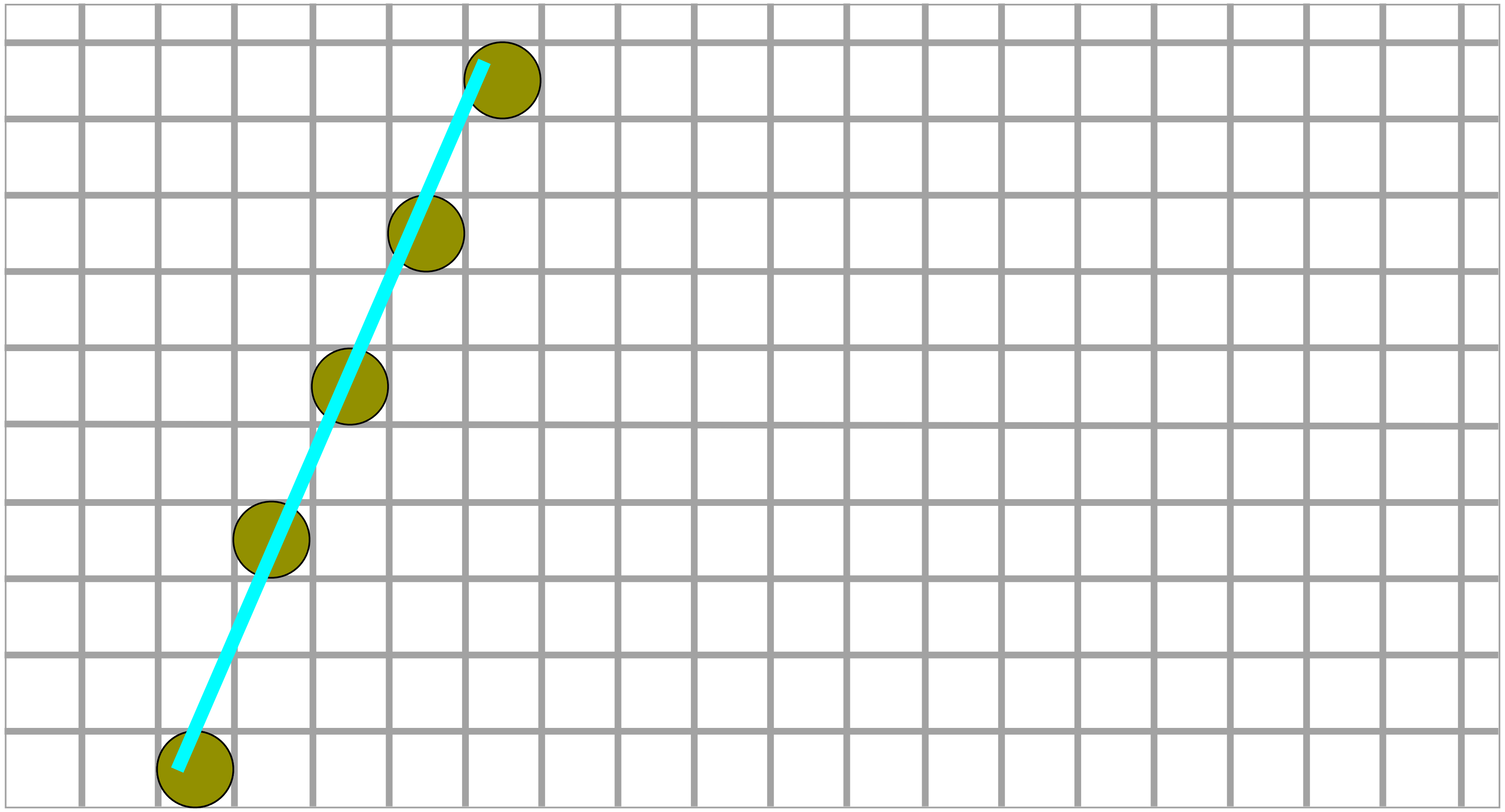

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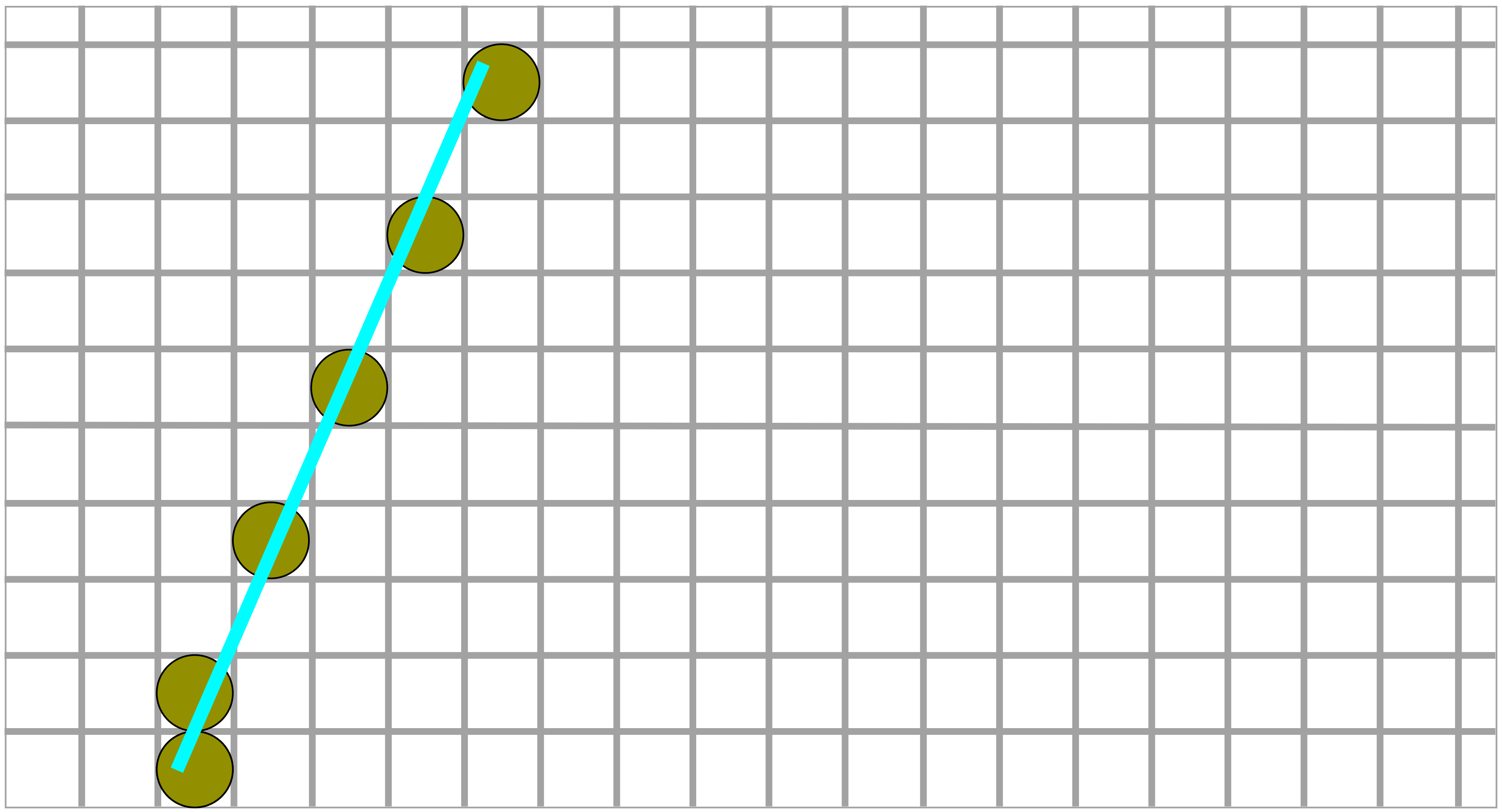

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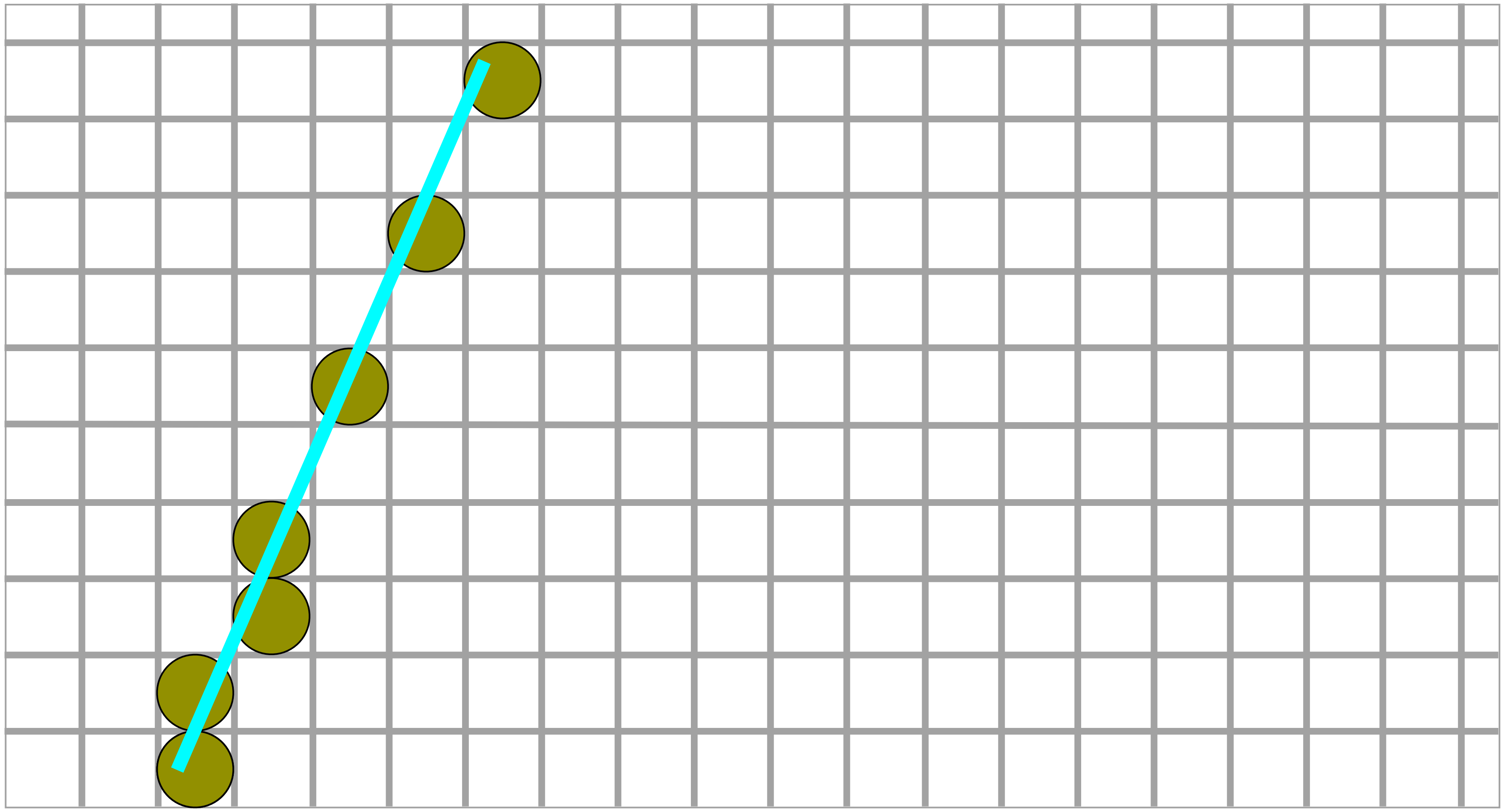

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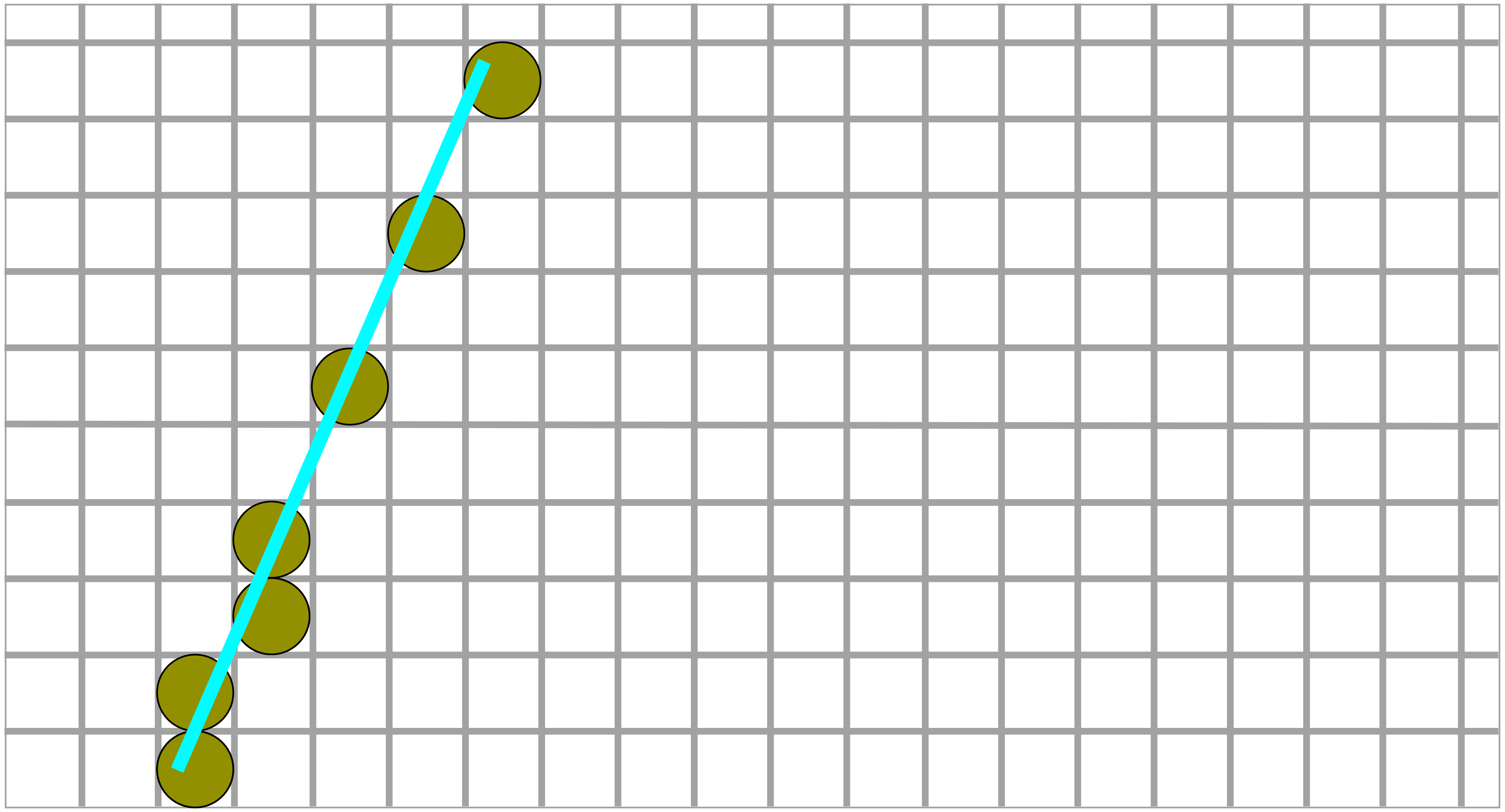

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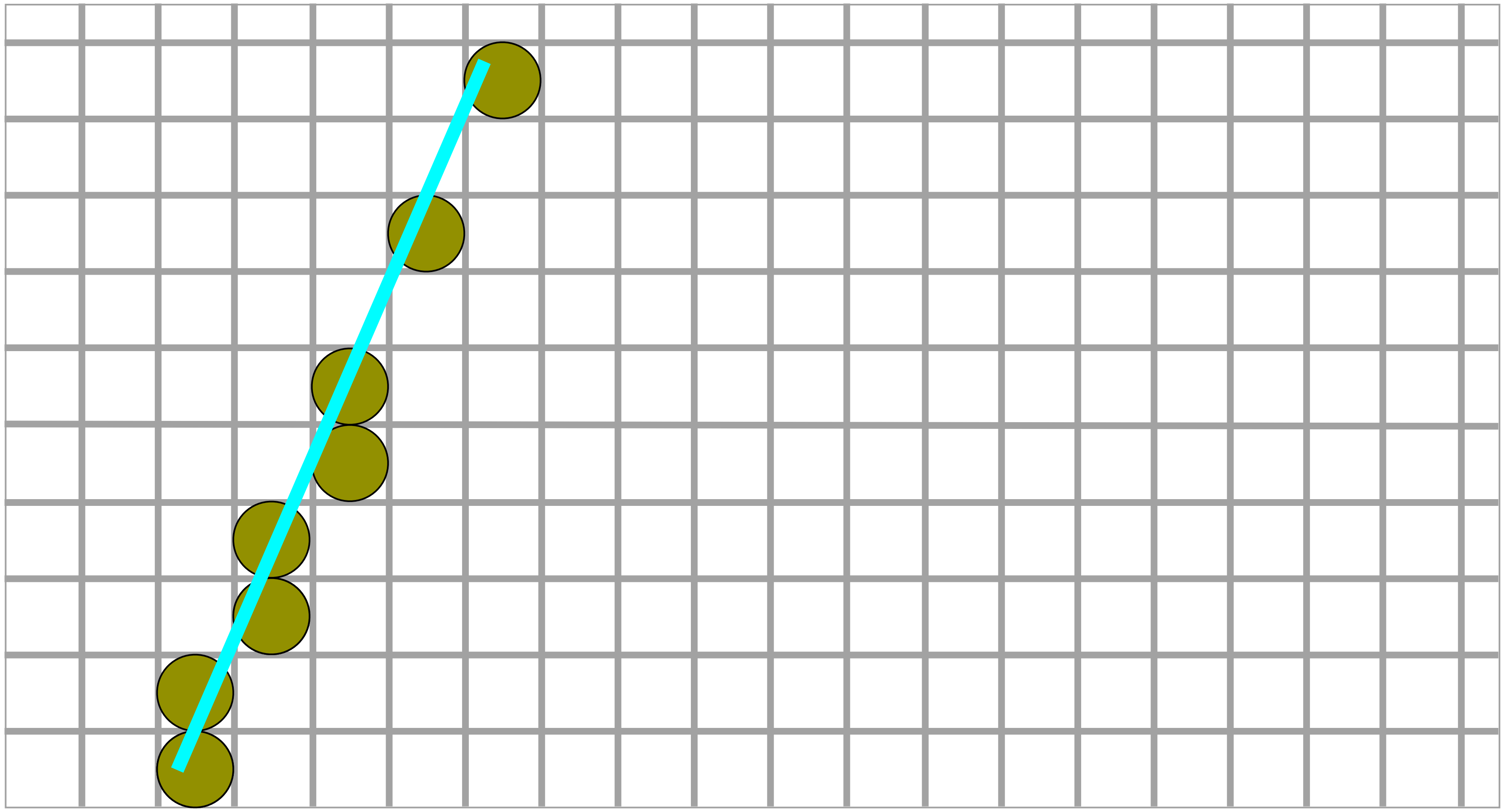

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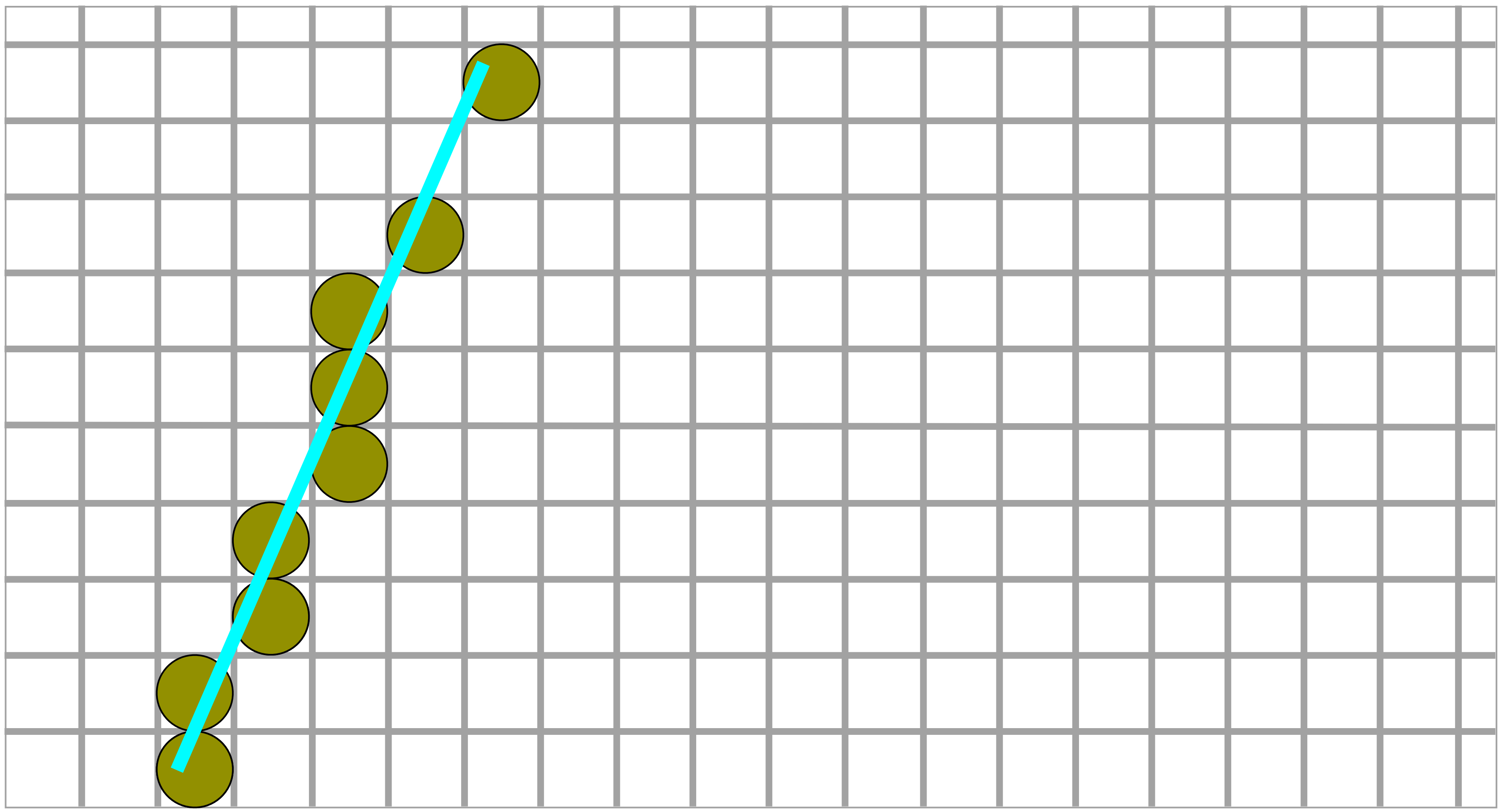

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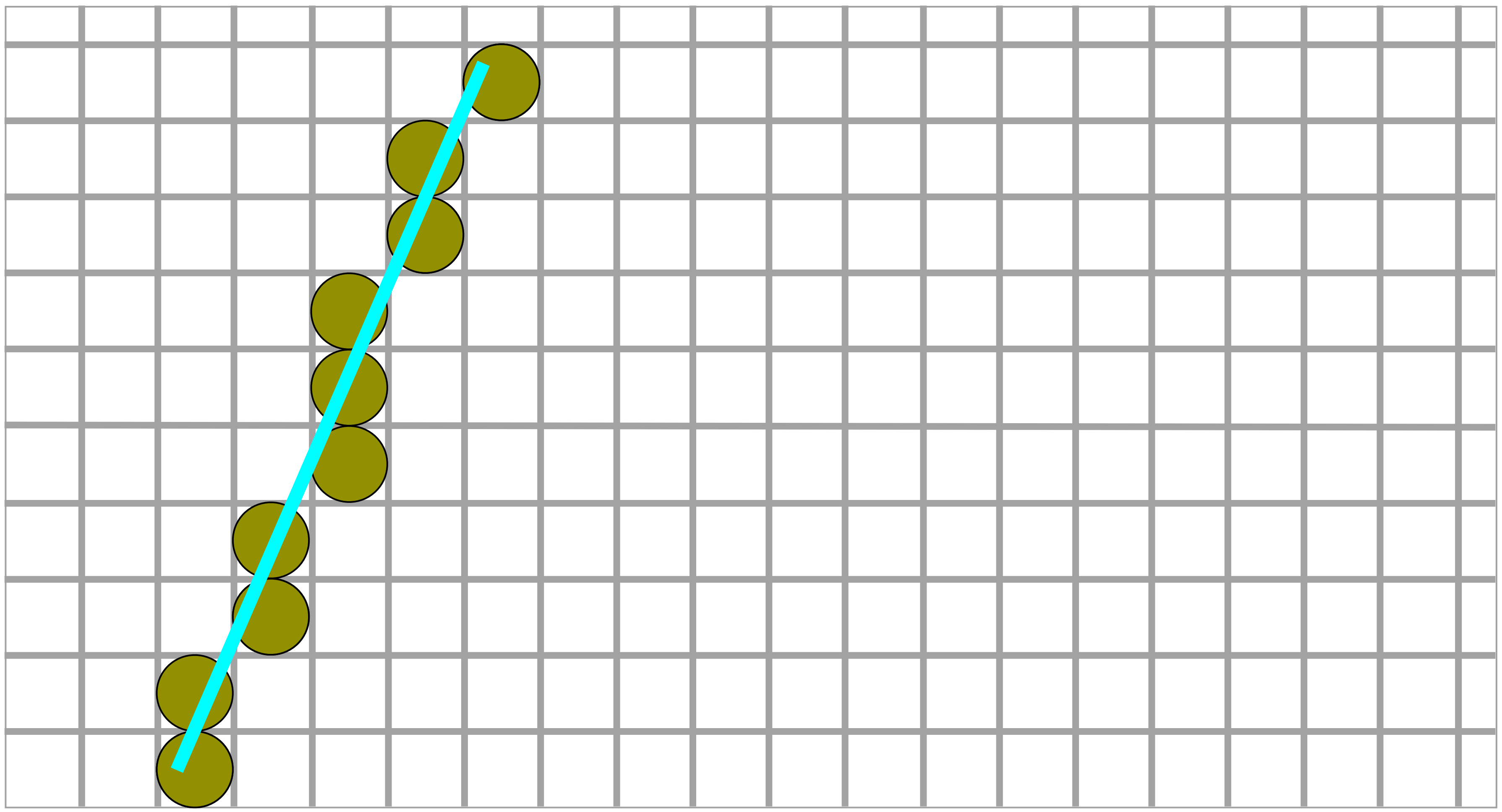

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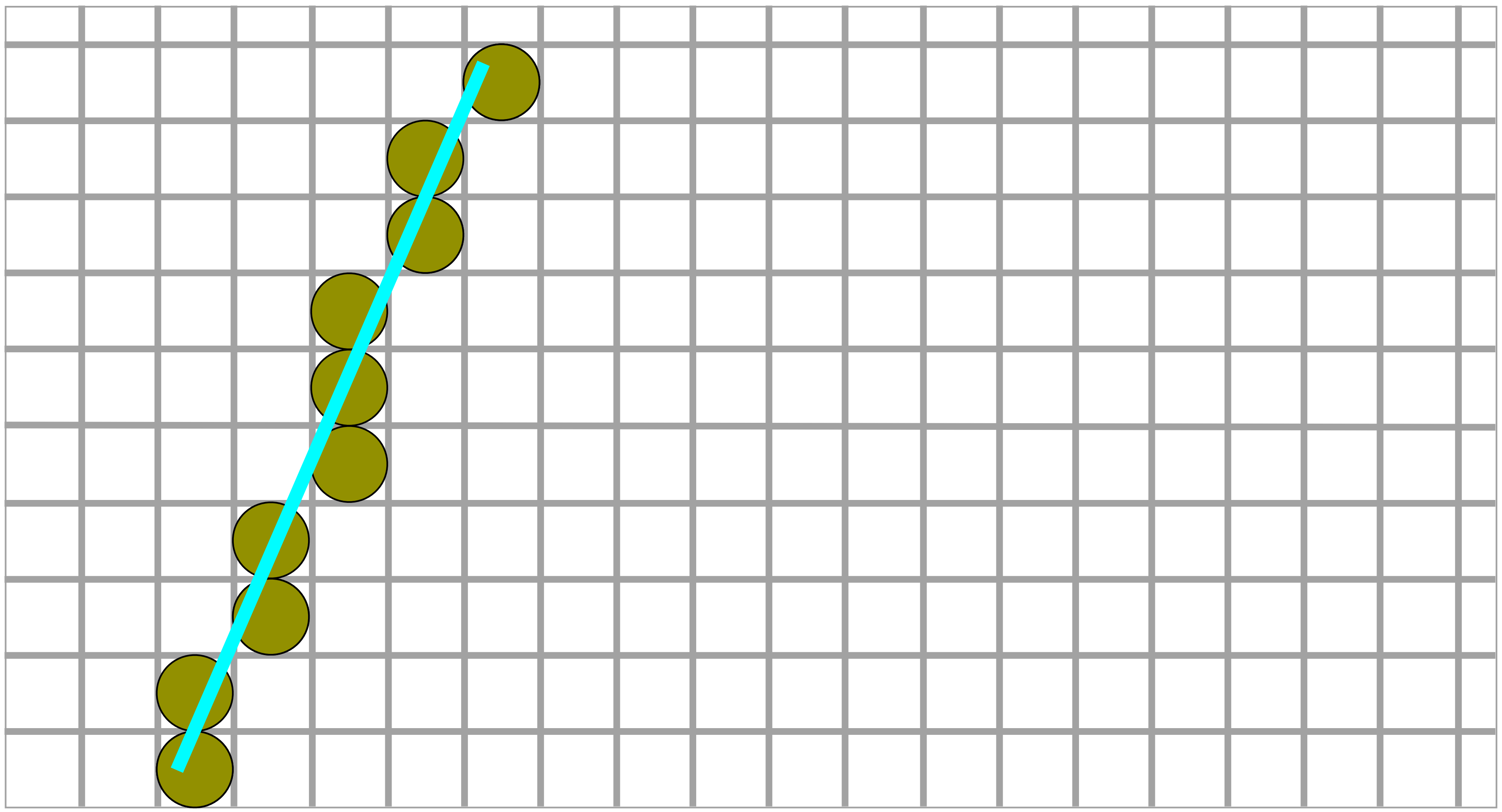

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Divide and conquer!

Digital Differential Analyzer

- Digital Differential Analyzer algorithm (a.k.a. DDA)
- Incremental algorithm: at each step it makes incremental calculations based on the calculations done during the preceding step
- The algorithm uses floating point operations.

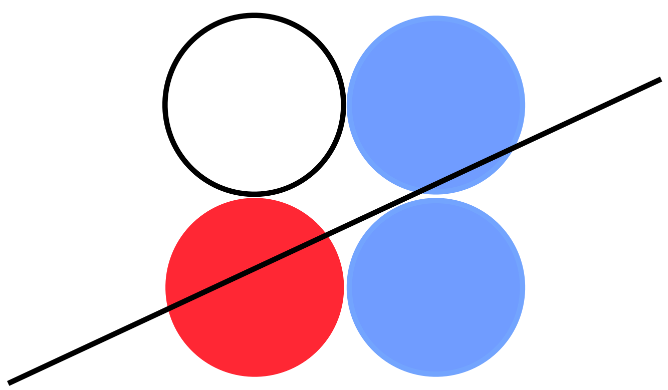
- An algorithm to avoid this problem is first proposed by J. Bresenham of IBM.
- The algorithm is well known as Bresenham's Line Drawing Algorithm.

Bresenham Line Drawing

$$y_i = mx_i + c$$

$$where, \qquad m = \frac{y2 - y1}{x2 - x1}$$

Bresenham Line Drawing



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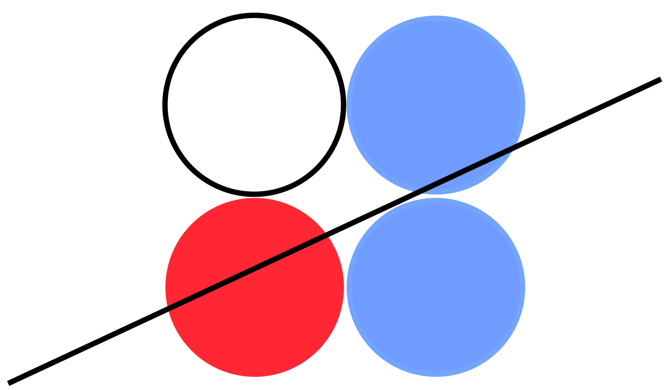

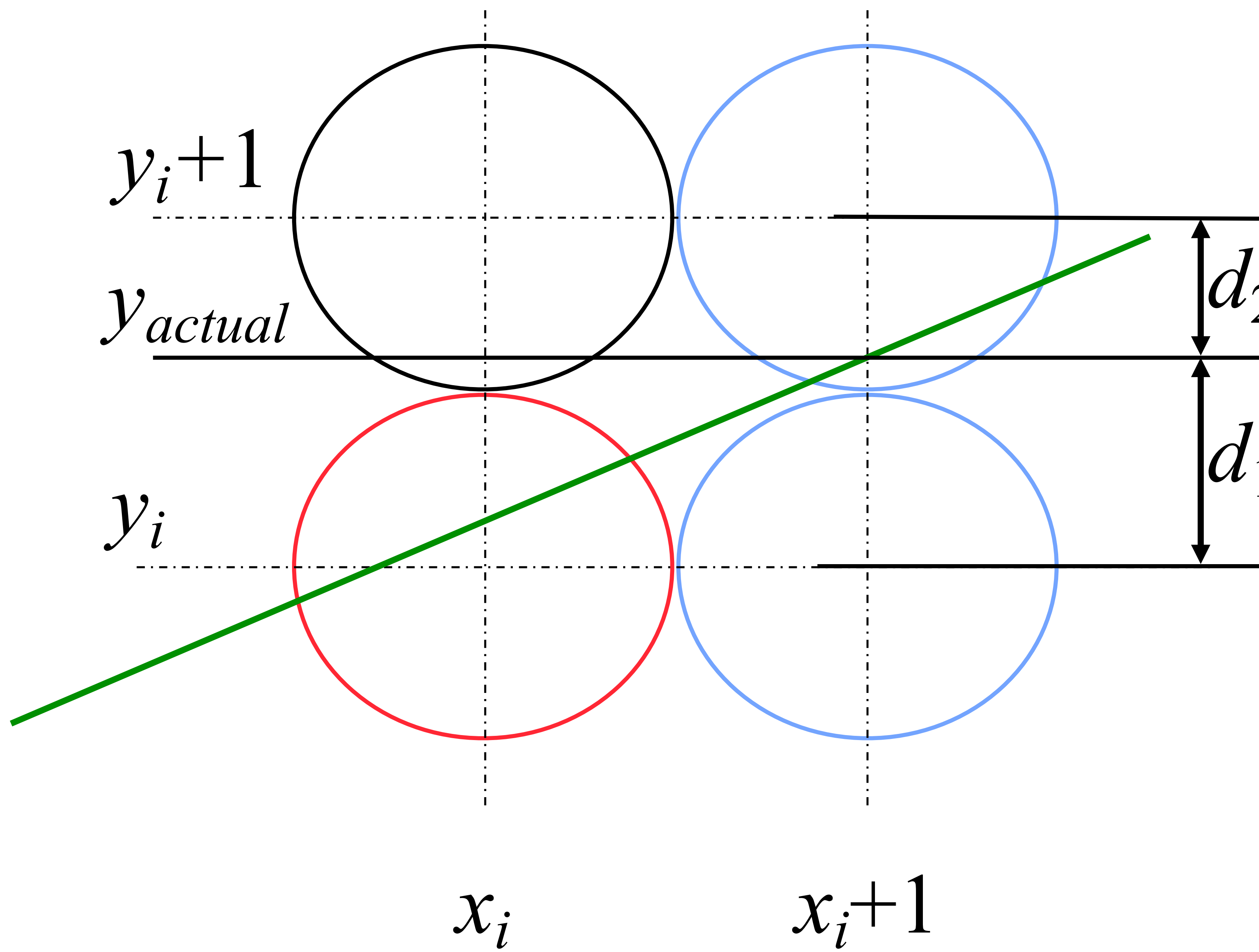

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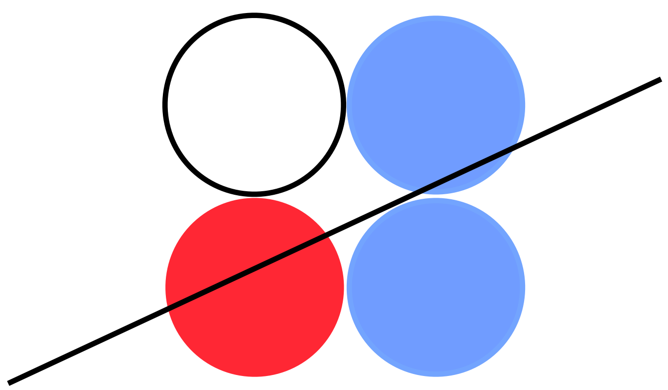
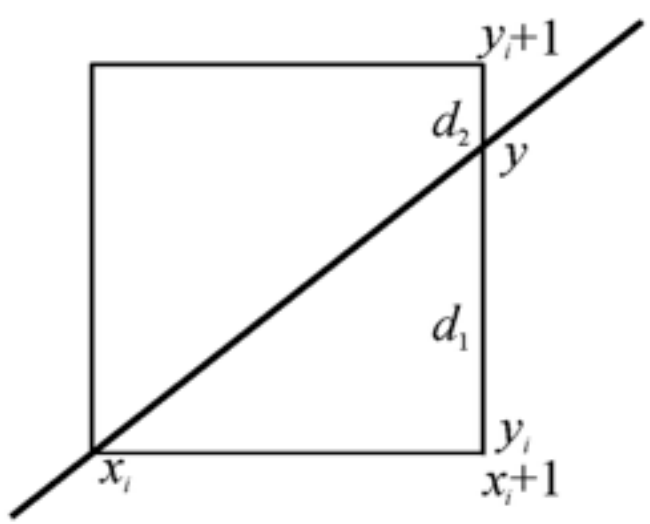
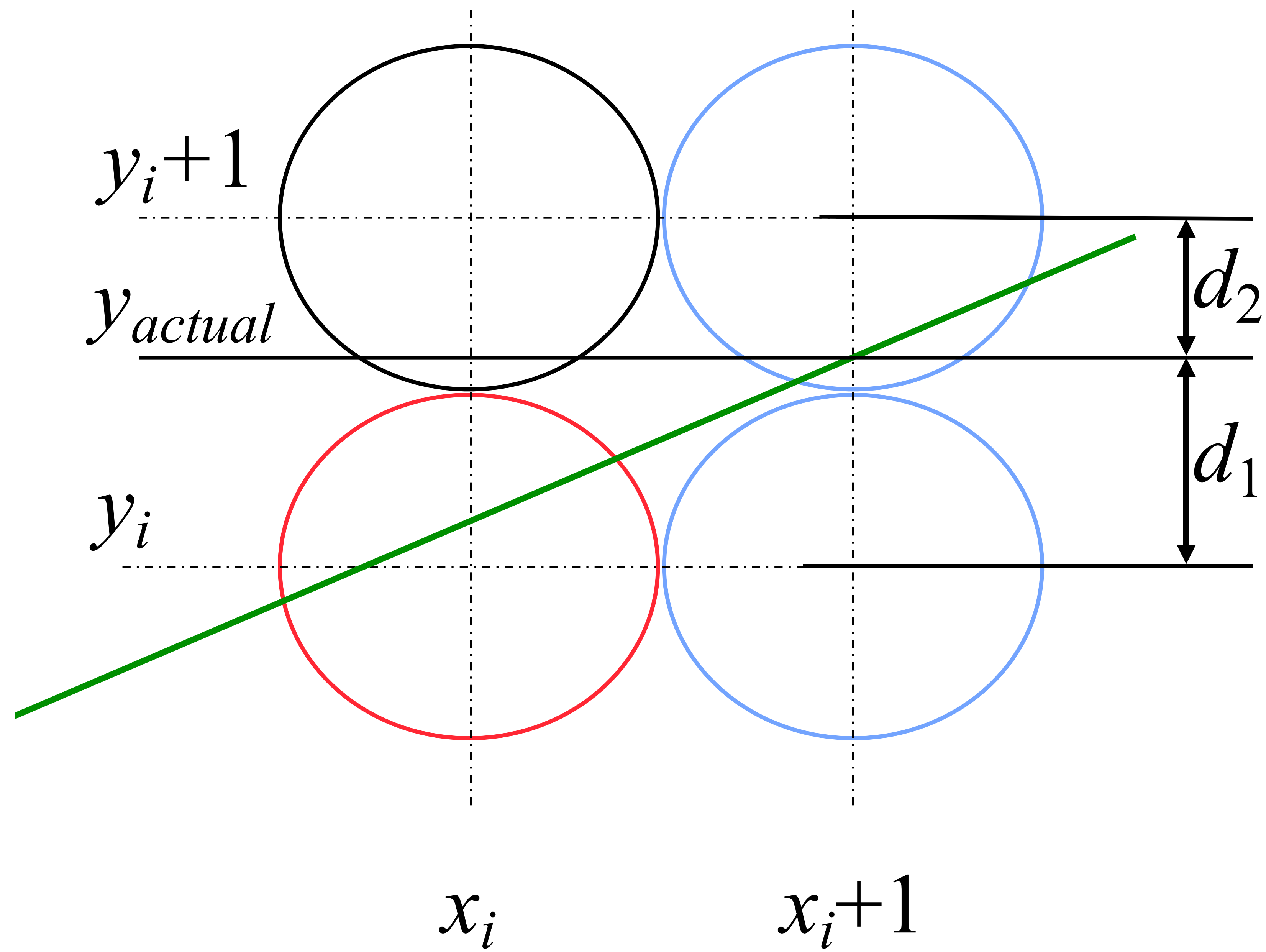


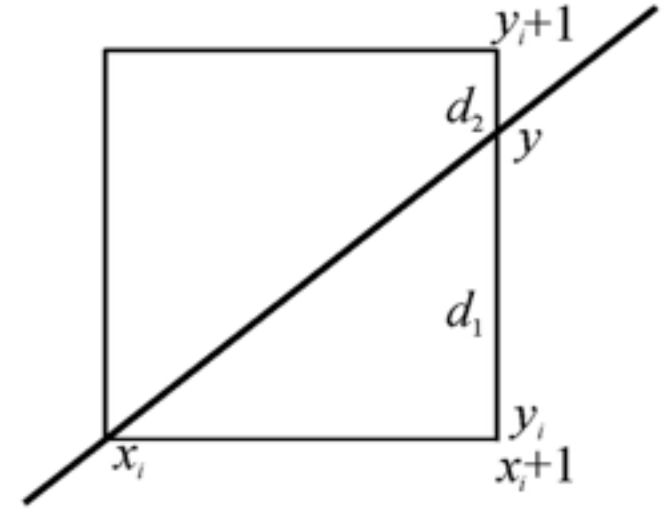




$$y_i = mx_i + c$$

$$where, \qquad m = \frac{y2 - y1}{x2 - x1}$$

$d_1>d_2? \Rightarrow y_{i+1}=y_i$ or $y_{i+1}=y_i+1$



$$y = m(x_i + 1) + b \qquad (2.1)$$

$$d_1 = y - y_i \qquad (2.2)$$

$$d_2 = y_i + 1 - y \qquad (2.3)$$

If $d_1-d_2>0$, then $y_{i+1}=y_i+1$, else $y_{i+1}=y_i$

substitute (2.1)、(2.2)、(2.3) into d_1-d_2,

$$d_1-d_2= 2y - 2y_i -1= 2dy/dx*x_i + 2dy/dx + 2b-2y_i-1$$

on each side of the equation, * dx, denote (d_1-d_2) dx as P_i, we have

$$P_i = 2x_i dy - 2y_i dx + 2dy + (2b-1)dx \qquad (2.4)$$

Because in first octant dx>0, we have $\text{sign}(d_1-d_2)=\text{sign}(P_i)$

If $P_i>0$, then $y_{i+1}=y_i+1$, else $y_{i+1}=y_i$

$P_{i+1} = 2x_{i+1}dy-2y_{i+1}dx+2dy+(2b-1)dx$, note that $x_{i+1}=x_i+1$

$$P_{i+1} = P_i+2dy-2(y_{i+1}-y_i)\, dx \qquad (2.5)$$

Bresenham algorithm in first octant

1. Initialization $P_0 = 2\,dy - dx$
2. draw (x_1, y_1), $dx=x_2-x_1$, $dy=y_2-y_1$,
   Calculate $P_1=2dy-dx$, $i=1$;
3. $x_{i+1} = x_i + 1$
   if $P_i>0$, then $y_{i+1}=y_i+1$, else $y_{i+1}=y_i$;
4. draw (x_{i+1}, y_{i+1});
5. calculate P_{i+1}:
   if $P_i>0$ then $P_{i+1}=P_i+2dy-2dx$,
   else $P_{i+1}=P_i+2dy$;
6. $i=i+1$; if $i<dx+1$ then goto 3; else end

Bresenham algorithm in first octant

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Question 3: Is it faster than DDA ?

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Question 3: Is it faster than DDA ?

Question 4: What technique ?

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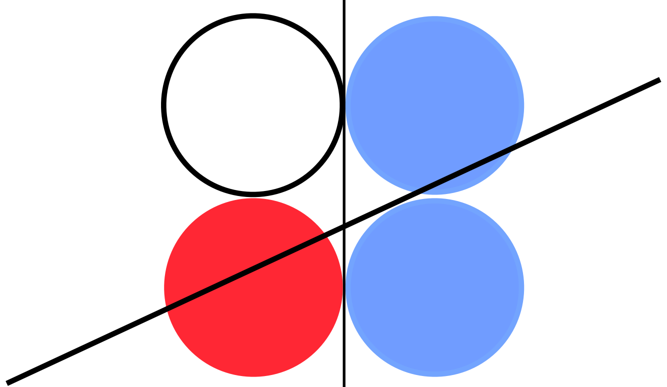

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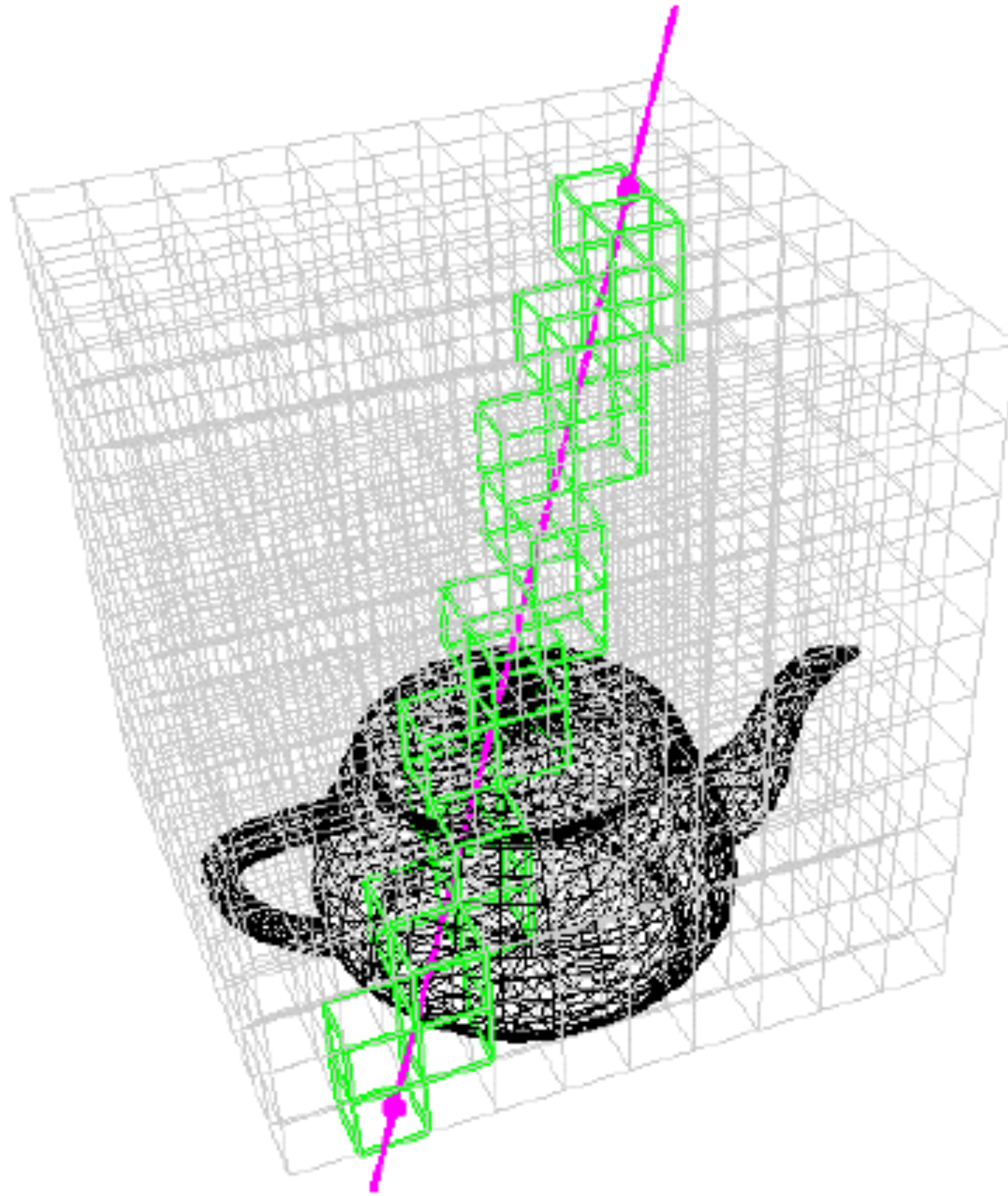
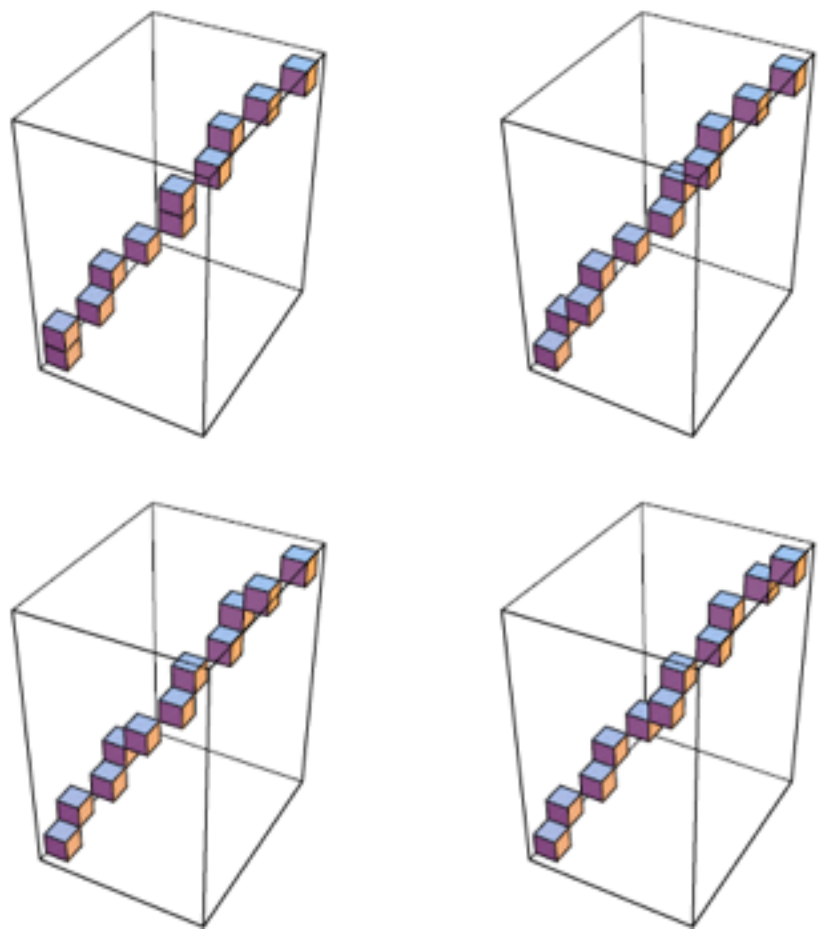
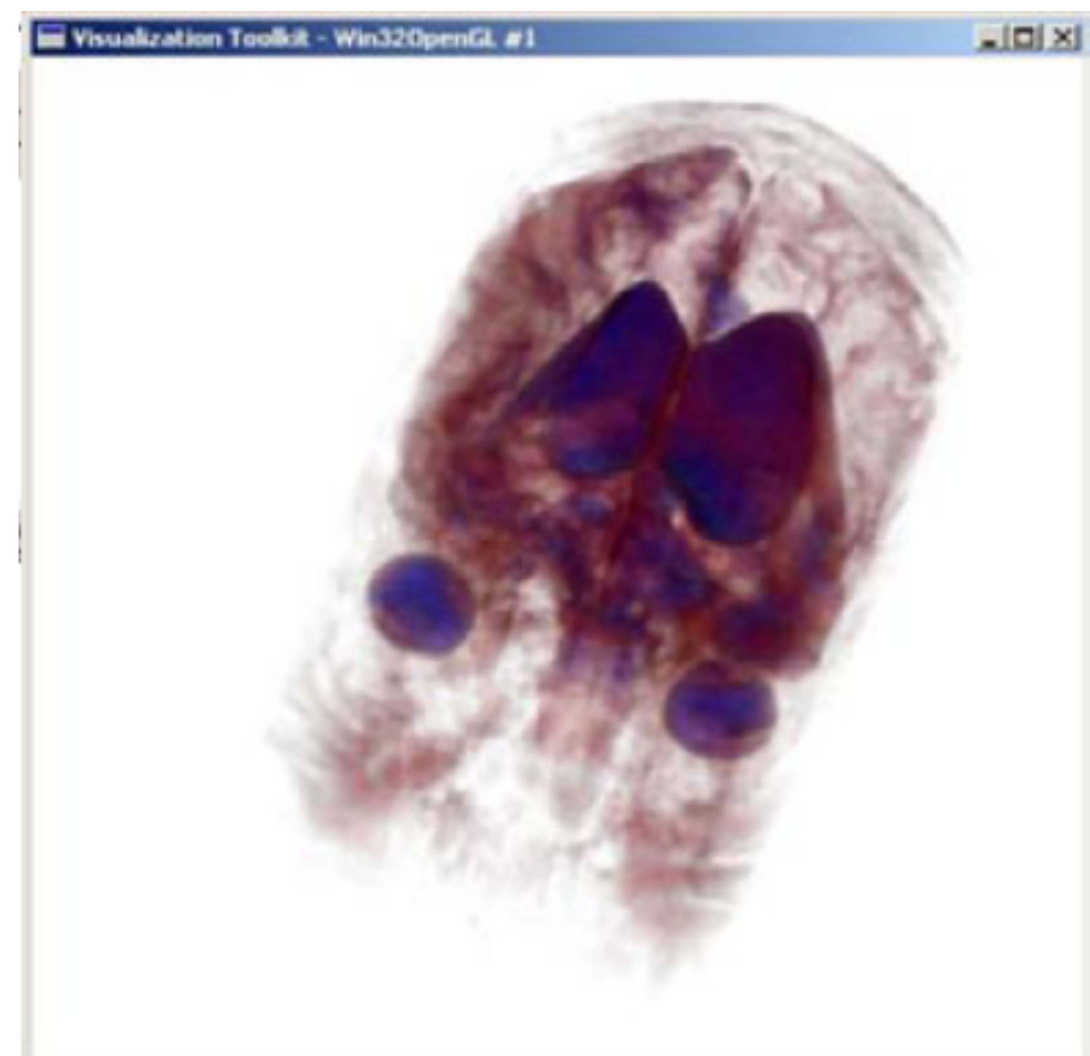
3D DDA and 3D Bresenham



3D DDA and 3D Bresenham algorithm





Scan converting circles

A circle with center (x_c, y_c) and radius r:

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

orthogonal coordinate

$$y = y_c \pm \sqrt{r^2 - (x - x_c)^2}$$