Multi-resolution Terrain Rendering Using Summed-Area Tables

Shi Li, Chuankun Zheng, Rui Wang*, Yuchi Huo, Wenting Zheng, Hai Lin, Hujun Bao
State Key Lab of CAD&CG, Zhejiang University, HangZhou, 310058, China

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ABSTRACT

Due to the fundamental weaknesses of level-of-detail (LOD) control and rich details in the Geometry Clipmaps, we propose a multi-resolution terrain rendering algorithm that utilizes summed-area tables (SATs) to facilitate the rendering of terrain with better geometric and shading details. First, our algorithm introduces a novel geometric error bound on the screen-based terrain rendering approach that juggling low rendering throughput and better LOD control. Geometric errors are estimated in real-time from SATs, enabling error-bounded geometry clipmap. Second, we utilize Spherical Gaussian (SG) functions to approximate lighting and bidirectional reflection distribution functions (BRDFs), and efficiently calculate outgoing radiance with self-occlusions of the terrain. SATs are utilized to enable the mipmapping of visibility and normal maps. We demonstrate the improvements of our method with experiments on accuracy and efficiency.

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Appendix A. Terrain Integration

The outgoing radiance on point $x$ toward direction $\omega_o$ is calculated as [2]:

$$ L_o(x, \omega_o) = \int_{S^2} L_i(x, \omega_i) \rho(x, \omega_i, \omega_o) \langle n(x), \omega_i \rangle d\omega_i, \quad (A.1) $$

where $\omega_i$ is the incident direction, $S^2$ represents the hemisphere, $L_i(\omega_i)$ denotes the incident radiances, $\rho$ shows the BRDF, $n(x)$ stands for normal vector at point $x$, the operator $\langle \cdot, \cdot \rangle$ gives the dot product of two vectors, clamped to zero if it is negative. Considering the filtering of nearby samples on the heightmap get the an antialiased integration over the footprint $\Omega$ around the point $x$ [3]:

$$ L_o(\Omega, \omega_o) = \frac{1}{|\Omega|} \int_{\Omega} \int_{S^2} L_i(y, \omega_i) \rho(y, \omega_i, \omega_o) V(y, \omega_i) \langle n(y), \omega_i \rangle d\omega_i dy, \quad (A.2) $$

where $|\Omega|$ denotes the surface area of $\Omega$ and $V(y, \omega_i)$ is the visibility term representing whether local geometry cast shadows at point $y$ from direction $\omega_i$.

Han et al. [4] assumed that $L(y, \omega) \approx L_i(x, \omega)$ for all $y \in \Omega$ and $\omega \in S^2$ when $\Omega$ is small neighbor around $x$ to simplify Equation as:

$$ L_o(\Omega, \omega_o) = \int_{S^2} L_i(x, \omega_i) \tilde{\rho}(\omega_i, \omega_o) d\omega_i, \quad (A.3) $$

where $\tilde{\rho}$ is the effective BRDF [4] given by

$$ \tilde{\rho}(\omega_i, \omega_o) = \frac{1}{|\Omega|} \int_{\Omega} \rho(y, \omega_i, \omega_o) V(y, \omega_i) \langle n(y), \omega_o \rangle dy \quad (A.4) $$

Previous works [5, 6] have proved that separating the shadow term from the normal and BRDF can achieve an accurate approximation of Equation [A.5] as:

$$ \tilde{\rho}(\omega_i, \omega_o) = \bar{\rho}_n(\omega_i, \omega_o) \tilde{\rho}_s(\omega_i), $$

$$ \bar{\rho}_n(\omega_i, \omega_o) = \frac{1}{|\Omega|} \int_{\Omega} \rho(y, \omega_i, \omega_o) \langle n(y), \omega_o \rangle dy, \quad (A.5) $$

$$ \tilde{\rho}_s(\omega_i) = \frac{1}{|\Omega|} \int_{\Omega} V(y, \omega_i) dy. $$

*Corresponding author: Tel.: +86-0571-88206681-430; e-mail: rwang@cad.zju.edu.cn (Rui Wang)
Thus, we can derive our shading equation with Equation \( \text{A.3} \) and \( \text{A.5} \):\[
L_\omega(\Omega, \omega_n) = \int_{S^2} L_r(x, \omega_i) \tilde{p}_n(\omega_i, \omega_n) \tilde{p}_i(\omega_i) d\omega_i.
\tag{A.6}
\]

**Appendix B. vMF based Approximation**

In order to efficiently calculate the integration, we use different approaches to approximate \( \tilde{p}_n \) and \( \tilde{p}_i \). For \( \tilde{p}_n \), a specular BRDF term can be represented using the vMF distribution as follows:\[
\tilde{p}_n(\omega_i, \omega_n) = \frac{1}{N} \sum_{j=0}^{N} vMF(\omega_i; \mu_j, \kappa_j) \tag{B.1}
\]

where \( \omega_n = (\omega_i + \omega_o) / \|\omega_i + \omega_o\| \) is the half-vector, \( N \) is the number of discrete texels in footprint \( \Omega \), \( \mu_j \in S^2 \) is the direction of a lobe, and \( \kappa_j \in R^+ \) is the glossiness of a lobe. When \( \kappa_j >> 1 \), Equation \( \text{B.1} \) can be approximated with a normalized SG function:\[
vMF(\omega; \mu, \kappa) = \frac{\kappa}{4\pi \sinh(\kappa)} e^{\kappa (\mu \cdot \omega - 1)} \tag{B.2}
\]

Similarly, Wang et al. \[7\] adopted SG functions to represent a diffuse BRDF by substituting \( \tilde{p}_n(y, \omega_o, \omega_o) = \frac{1}{2} \) in Equation \( \text{A.5} \):\[
\tilde{p}_n(\omega_i, \omega_n) = \frac{1}{N} \sum_{j=0}^{N} 1.170 e^{2.133(\omega_i \cdot n(y_j) - 1)} \tag{B.3}
\]

Therefore, we could give a unified expression to represent the average BRDF based on vMF distributions as follows:\[
\tilde{p}_n(\omega_i, \omega_n) = \frac{1}{N} \sum_{j=0}^{N} vMF(\omega_i, \omega_n; \mu_j, \kappa_j) \tag{B.4}
\]

**Appendix C. vMF Lobes Merging**

Because the large number of vMF lobes in \( \rho_n \) at each footprint can cause heavy integration overhead at runtime, we adapt vMF merging technique to reduce the size \[6\]. Xu et al. \[6\] introduced an unnormalized vector \( r \in R^3 \) to characterize a vMF distribution. These vectors can be merged into one or multi merged vMF lobes to accelerate the integration over the pixel footprint. Specifically, the mapping between a vMF and a vector \( r \) can be calculated as follows:\[
\frac{r}{\|r\|} = \mu \frac{3\|r\| - \|r\|^3}{1 - \|r\|^2} = \kappa \tag{C.1}
\]

Therefore, we could replace the presentation \( vMF(\omega; \mu, \kappa) \) in Equation \( \text{B.4} \) with \( vMF(\omega; r) \).\[
\tilde{p}_n(\omega_i, \omega_n) = \frac{1}{N} \sum_{j=0}^{N} vMF(\omega_n; r_j) \tag{C.2}
\]

The merging of multiple lobes, i.e., vector \( r_j \), can be calculated by a simple average of vectors:\[
\tilde{r}_M = \frac{1}{N} \sum_{j=0}^{N} r_j \tag{C.3}
\]

where \( \tilde{r}_M \) is the merged lobe. The original parameters(\( \mu \) and \( \kappa \)) of a vMF distribution can be obtained from \( \tilde{r}_M \) (see Xu et al. \[6\] for more details).

Thus, we use a similar method to acquire the average BRDF over the pixel footprint \( \Omega \) from Equation \( \text{C.2} \)\[
\tilde{p}_n(\omega_i, \omega_n) = vMF(\omega_i, \omega_n; \tilde{r}_M) \tag{C.4}
\]

with \( \tilde{r}_M \) is the average vMF distribution.

**Appendix D. Additional Result**

In order to demonstrate the shading effects of our error-bound method, we construct a heightmap with more cliffs and details using a terrain generator with Perlin Noise. As shown in Figure \( \text{D.1} \), our method depicts mountain gullies and ridges vividly, while Geometry Clipsmaps \[8,9\] overly smoothes the mountains’ contour lines.

### References

Fig. D.1. Comparison of geometric details between Our Methods, Geometry Clipmaps [8] and Ge et al. [9]. The RMSE values are shown in the bracket.