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Plausible cloth animation using dynamic bending model

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Abstract

Simulating the mechanical behavior of a cloth is a very challenging and important problem in computer animation. The models of bending in most existing cloth simulation approaches are taking the assumption that the cloth is little deformed from a plate shape. Therefore, based on the thin-plate theory, these bending models do not consider the condition that the current shape of the cloth under large deformations cannot be regarded as the approximation to that before deformation, which leads to an unreal static bending. [This paper introduces a dynamic bending model which is appropriate to describe large out-plane deformations such as cloth buckling and bending, and develops a compact implementation of the new model on spring-mass systems. Experimental results show that wrinkles and folds generated using this technique in cloth simulation, can appear and vanish in a more natural way than other approaches.]

Keywords: Cloth animation; Physically based animation; Bending

1. Introduction

The computer animation researches based on physical principles for simulating cloth have been widely conducted in the past two decades. The pioneering work of Terzopoulos et al. [1] provided a general physically based modeling method for elastic objects, which employs the finite-element and the finite-difference methods to solve the dynamic governing equations so that the deformation of objects like cloth, rubber, and metal can be stimulated. After the work of Terzopoulos et al. [1], many approaches have been dedicated to improve the simulation of cloth either in verisimilitude or in efficiency aspects, which include the methods by particle or spring-mass systems [2–9], the simulations by solid mechanics of continuum [1,10–15], the numerical schemes to improve the computational stability and speed [6,8,16–19], the collision handling approaches [20–25], and the realistic rendering of cloth materials [26]. However, none of the current existing approaches solves the problems of static bending or static buckling addressed in the following section, which make cloth simulation unrealistic and inaccurate.

In this paper, we decouple the buckling deformation into two different types: shearing buckling and structural buckling, where the former one is usually caused by stretching and the latter is mainly produced by compression. A new dynamic bending model is derived from the thin-shell theory to describe the structural buckling. An implementation of the dynamic bending model by the dynamic stiffness method has been developed on the widely used spring-mass systems [4,8].

The rest of the paper is organized as follows. We first present the problems in modeling bending deformations in the existing cloth simulation approaches by an example. Then we introduce a dynamic bending analysis based on the thin-shell theory to solve these problems, which can model large bending deformation in cloth animation more efficiently and accurately. After that, we give an implemen-

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tation of our new dynamic stiffness method on traditional mass-spring models. Finally, the modified numerical integration method and some examples and discussions are given before the conclusions are drawn.

2. Problems in the existing cloth simulation approaches

Through the study of fabric materials, it can be found that the cloth usually shows a strong resistance to stretch and shear, but a weak resistance to bending. This is the major reason why textile fabrics can generate folds and wrinkles more easily than other sheet materials (e.g., paper and leather). [For example, applying a compressive force on the surface of cloth will produce structural buckling immediately so that forms folds and wrinkles.] In the deformation of buckled cloth, bending is more important to the shape of the cloth than stretch and shear. Therefore, how to accurately model the strongly resisted stretch and shear and the weakly resisted bending is most important for generating a realistic cloth simulation. The structural buckling has been well investigated in [1–6,8,27]. However, their static analysis of cloth buckling based on the thin-plate theory and small deformation elasticity can produce static bending models which are only suitable for simulating objects with small bending deformations but clothes. Therefore, we focus on the dynamic analysis of cloth bending and buckling in this paper.

The behavior of bending is the most important factor that affects the generation and vanishing of wrinkles and folds in cloth simulation. Before analyzing the cloth, let us consider another soft material – paper. A piece of thin paper with the shape as shown in Fig. 1(a) will be folded when applying a vertical compression force on two sides of the paper. However, if the paper is rolled up as shown in Fig. 1(b) and (c), it shows more and more strong resistance on bending. In summary, the bending deformation along one direction of a thin-sheet material will change the bending stiffness along other directions. Besides, we also observe that for a paper in a rectangular shape, it is easier to yield it along the longer direction than the shorter one. Study on the material properties of cloth shows that we cannot neglect this variation of bending in cloth simulation neither, and this variation in bending stiffness will greatly affect the realism of cloth animation. However, neither the simulation by the simple spring-mass systems as in Ref. [4] nor the computation by the more complex immediate buckling model in Ref. [8] considered about this bending variation. In their approaches, the bending model is derived from the thin-plate theory which assumes that the new shape is just warped from a planar plate so that a constant bending stiffness is adopted. However, taking this assumption limits the computational model to be only true to small warping. When this model is used to simulate cloth animation which always involves large warping, the generated wrinkles and folds are fake. Our new dynamic bending model is derived from the thin-shell theory, which is more appropriate to model large warping deformation. We also introduce a dynamic stiffness method to implement the dynamic bending model on the spring-mass systems, which is compact and easy to integrate into the current available cloth simulator.

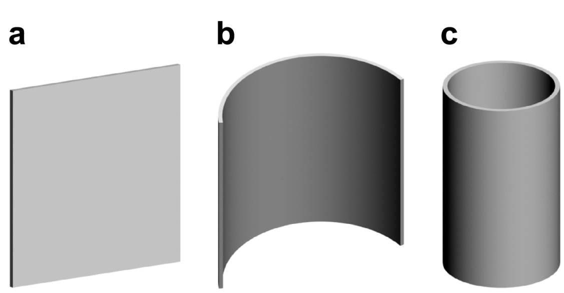

Fig. 1. Different bending stiffness along the vertical direction is formed by the same piece of soft and thin material in three different shapes. (a) Very weak bending stiffness; (b) medial bending stiffness; (c) strong bending stiffness.

3. Bending analysis based on thin-shell theory

The material of cloth is in general thin and soft so that folds and wrinkles will be generated even if very small external loadings are added. Recent researches on cloth simulation show that bending simulation is the most important part for a realistic cloth animation as the cloth is almost inextensible in many cases [27]. There are several methods in solid mechanics to model the bending behavior of thin and soft elastic models accurately (e.g., thin-shell elements in Finite Element Analysis). However, these approaches are seldom adopted in the computer animation applications mainly for two reasons: (1) the computation is too time-consuming, and (2) it is hard to interact with other processes like collision handling.

In the particles (or spring-mass system) based approaches, the bending is usually converted into a linear deformation and is simulated by adding a linear spring linking the particles on two triangles sharing the same edge. For the discretization with quadrilateral faces, such a linear bending spring is added on the diagonal of two facets sharing only one node. Such simplification leads to the drawback in twofold: (1) linear springs can only present the one-dimensional relationship between stress and strain but cannot mimic the two-dimensional deformation, and (2) using a linear spring can only simulate the relationship between the bending deformation and the forces in small deformation as the relationship is actually nonlinear in large rotation and bending. Therefore, the simulation result by using linear bending springs is far from reality. Furthermore, the bending deformation for cloth animation is dynamic, which is much different from the static deformation from one equivalent status to another. During the simulation, we need to consider about the variation of bending stiffness led by the change of shape (e.g., the same material but with different shapes as shown in Fig. 1 will have different bending stiffness).

The widely employed thin-plate theory in the previous cloth simulation approaches can only model the static bending deformation, and it is based on the assumption that every shape is just slightly deformed from a thin-plate. Thus, it is not appropriate to employ the thin-plate theory to simulate cloth which is usually with large rotation and deformation. To solve this problem, we adopt the thin-shell theory to simulate the dynamic bending in this paper. Computations governed by the thin-shell theory are based on the current shape of the objects. Simply, the bending deformation can be formulated as an equation of moment, bending stiffness k^b, and curvature κ as

$$M_x = k^b\kappa = (EI_x)\kappa \quad (1)$$

where x represents the principal inertia axis, I_x denotes the inertia moment around the axis, and E is Young's modulus. The curvature κ can be evaluated by the formula given in Ref. [8]. In general, there are two inertial axes on the cross-section of an object, and both of them pass the centroid of the cross-section. To be simple, in the later part of this paper, we will call the inertia moment around the principal inertia axis as inertia moment in short. From Eq. (1), we know that $k^b = EI_x$. Although E is a constant Young's modulus in bending, the value of I_x varies when the shape of the cloth is changed. How to model the variation of I_x is the key problem for modeling the dynamic bending deformation, which will be detailed below.

Fig. 2(a) shows a plate before deformation, and Fig. 2(b) shows its shape after bending deformation. Without loss of generality, we assume that the plate is warped around the $\overline{cd}$ axis and there is no deformation along the $\overline{cd}$ direction. In order to analyze the variation of inertia moment before versus after bending, we extract the cross-section of the plate by the $x-o-z$ plane. The thickness (h) of fabrics is usually between 0.15 and 2 mm, and each quadrilateral facet after tessellation is with the width (w) between 1 and 2 cm. The width-thickness ratio (w/h), which is an important parameter in the later analysis of bending, usually falls in the range between 5 and 50. Choosing different values for w/h will greatly affect the bending deformation result as it changes the value of inertial moment.

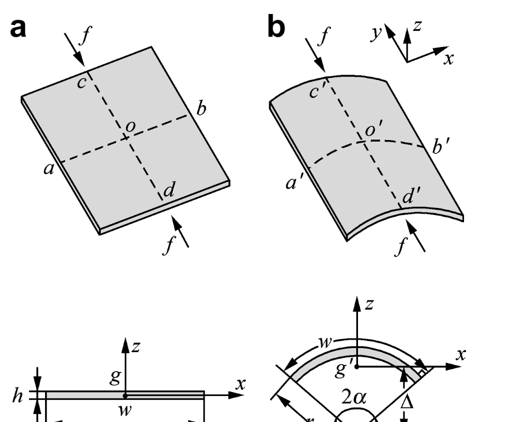

Fig. 2. The bending deformation in different configurations. (a) Bending from a plate; (b) bending from a warped shell that is approximated by a cylinder, where g and g' are the centroid, and x is the inertia axis.

We assume that the buckling of fabrics after bending is much greater than the thickness of fabrics, which simplifies the computation of inertial moments after bending. Therefore, the inertial moments for the thin-shell (i.e., the facets) in the shape as shown in Fig. 2(a) and (b) can be computed by

$$\overline{I}_x^0 = \frac{1}{12}wh^3 \quad (2)$$

$$\overline{I}_x^\alpha \approx \frac{1}{16}\left(\frac{w^3h}{\alpha^3}\right)\left[2\alpha + \sin 2\alpha - \frac{4\sin^2\alpha}{\alpha}\right](0 < \alpha \leqslant \pi) \quad (3)$$

where α is the angle of bending. The ratio between the inertia moments before and after bending is defined as

$$\eta = \frac{\overline{I}_x^\alpha}{\overline{I}_x^0} = \frac{3}{4\alpha^3}\left(\frac{w}{h}\right)^2\left[2\alpha + \sin 2\alpha - \frac{4\sin^2\alpha}{\alpha}\right]. \quad (4)$$

Note that the computation of $\overline{I}_x^\alpha$ in Eq. (3) neglects the thickness of the shell, to consider this, the practical inertia moment of the shell with the bending angle α is approximated by

$$I_x^\alpha = (1+\eta)\overline{I}_x^0 + \delta\alpha \approx (1+\eta)\overline{I}_x^0 \quad (0 \leqslant \alpha \leqslant \pi) \quad (5)$$

according to the solid mechanics theory, where $\delta\alpha$ is a small compensation relating to α. Note that when $\alpha = 0$, both $\delta\alpha$ and η are zero so that Eq. (5) is consistent with Eq. (2).

The value of η is related to both the bending angle α and the width-thickness w/h. In Fig. 3, we show the relationship between them. In the cloth simulation, the value of w/h is fixed during the whole procedure of computation as it only relates to the tessellation and the type of fabrics. α is a variable as it will change from time to time. Although we cannot clearly know the relationship between η and α from Eq. (4), it is however not difficult to find that the value of η is nearly linear to α when $\alpha \in [15°, 150°]$. Besides, we know that $\eta \propto (w/h)^2$ from Eq. (4). During the cloth simulation, large deformations are easily generated on fabrics, which means that the values α vary from place to place, and so does the inertial moment I_x. From Fig. 3, we know that

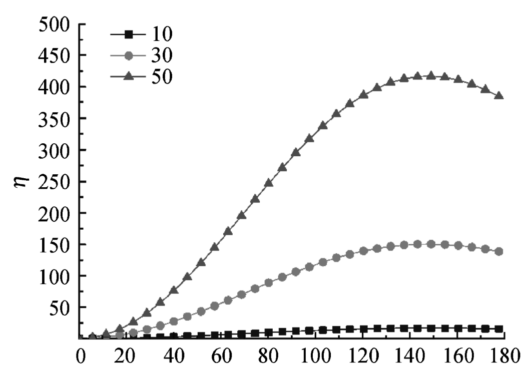

Fig. 3. η affected by the value of α and w/h.

the greater value is assigned for w/h, the greater variation is given on I_x^α with the change of α. In the following section, we develop a dynamic stiffness method to simulate this variation of bending stiffness.

4. Implementation on particle system

4.1. Particle system and definitions of forces

A representative particle model – spring-mass system is commonly used in cloth simulation [4,8,12,22]. In such systems, the mass of each patch is discretized onto particles (i.e., mass-point), which are connected by different types of springs to simulate forces on particles to resist stretching, shearing and bending deformation. Different material properties are reflected by different spring stiffness coefficients. Here, we adopt the connectivity structure of interacting mass as shown in Fig. 4 to simulate stretch, shear and bending, which is similar to Ref. [8].

4.2. Dynamic stiffness method

As aforementioned, the structural buckling is the most important reason for the formation of wrinkles and folds on fabrics. However, no existing approach considers about the variation of bending stiffness led by the change of inertia moment. This section will introduce how we compute the dynamic bending stiffness on buckled fabrics.

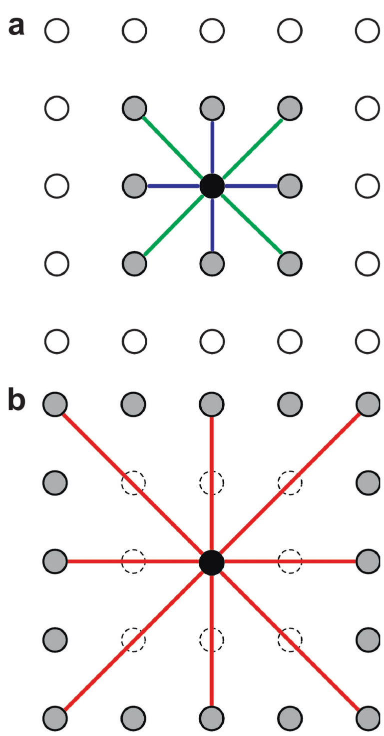

Fig. 4. Structure connectivity of the mass-spring model for stretch, shear and bending. (a) Stretch interactivities are simulated by horizontal and vertical springs (in blue color) and the shearing deformation is generated by the diagonal springs (in green) linking mass-points in the same grid; (b) the springs linking mass-points not in the same grid (in red color) are used to simulate the bending deformation.

Without loss of generality, we consider the grids around the particle P_0 as illustrated in Fig. 5. Assume that $\overline{P_1P_0P_2}$ is a line segment consisting of two horizontal springs linking to P_0, it has been buckled into a curve $\widehat{P_1P_0P_2}$ in the simulation. The dynamic bending stiffness coefficient along the curve $\widehat{P_1P_0P_2}$ will be computed as below. In Fig. 5(b), the points P_0, P_1 and P_2 and their two vertical neighbors form three planes with unit normal vectors as $\boldsymbol{n}_{i(i=0,1,2)}$. If the curves passing P_0, P_1 and P_2 are approximated by arcs, the shape of the cross-sections will become planar arch. Hence, the inertia moments $I_{i(i=0,1,2)}$ of these three cross-sections can be computed by Eq. (5). However, as P_1P_0 and P_2P_0 in general are not perpendicular to these cross-section planes, the inertia moment at P_0 needs to be computed by projecting I_is. In summary, the bending stiffness at P_0 along $\widehat{P_1P_0P_2}$ is computed by

$$k_0^b \approx \lambda_1 k_{10}^b + \frac{1}{2}\lambda_0(k_{01}^b + k_{02}^b) + \lambda_2 k_{20}^b \tag{6}$$

with

$$k_{ij}^b = EI_i|\boldsymbol{n}_i \cdot \boldsymbol{l}_{ij}| \quad (i \in \{0,1,2\}, ij \in \{01,02,10,20\}) \tag{7}$$

where $\boldsymbol{l}_{ij}$ are unit vectors as shown in Fig. 5(b). In Eq. (6), the weights λ_i should satisfy $\lambda_0 + \lambda_1 + \lambda_2 = 1$. The purpose of setting λ_is is to give users the flexibility to specify the smoothness of bending along $\widehat{P_1P_0P_2}$. For those thick fabrics, we let $\lambda_0 = \lambda_1 = \lambda_2 = 1/3$ to mimic smooth bending. For simulating the thin fabrics, we should increase the value of λ_0 so that the bending can vary fast along $\widehat{P_1P_0P_2}$.

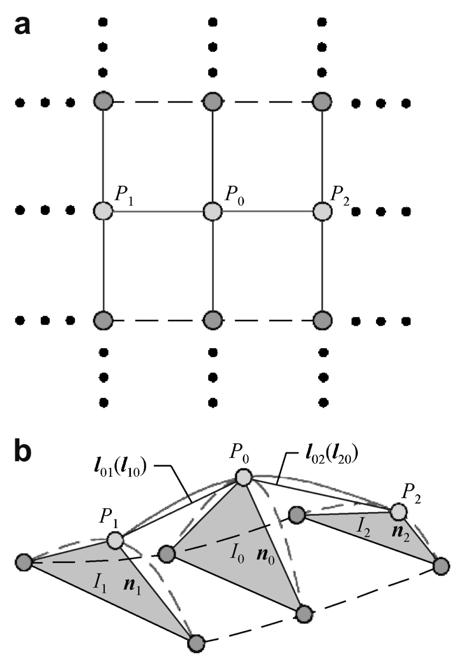

Fig. 5. The illustration for how to compute the dynamic bending stiffness. (a) A part of spring-mass system before deformation; (b) the buckled surface for this part of spring-mass system.

5. Nonlinear numerical integration scheme

The physically based cloth simulation is formulated as a partial differential equation (PDE), which can numerically be solved as an ordinary differential equation (ODE) after discretization as given below

$$0 = \boldsymbol{M}\ddot{\boldsymbol{x}} - \left(-\frac{\partial E}{\partial x} + \boldsymbol{F}\right) \tag{8}$$

where $\boldsymbol{M}$ is the mass matrix of the spring-mass system, the vector $\boldsymbol{x}$ contains the positions of all the particles in the system, $\boldsymbol{F}$ represents all the non-conservative forces (e.g., frictions, external constrains, etc.), and $-\partial E/\partial \boldsymbol{x}$ denotes all the conservative forces including gravity, stretch, shear, and bending. Each conservative force is associated with a type of potential energy computed by the current status of the spring-mass system. The dynamic bending model developed in this paper is nonlinear to the position of particles so that it is difficult to derive the explicit formula for the derivatives of the bending force to particle positions. Thus, we use the nonlinear implicit method to solve the differential equation in our implementation. More specifically, the simulation of every frame is further subdivided into several time steps such that each time step is dt. Within a pre-assigned range, the value of dt is adaptively adjusted according to the accuracy of simulation and the stability of numerical computation as that in Ref. [6]. In each time-step, the implicit-Euler method is conducted so the above

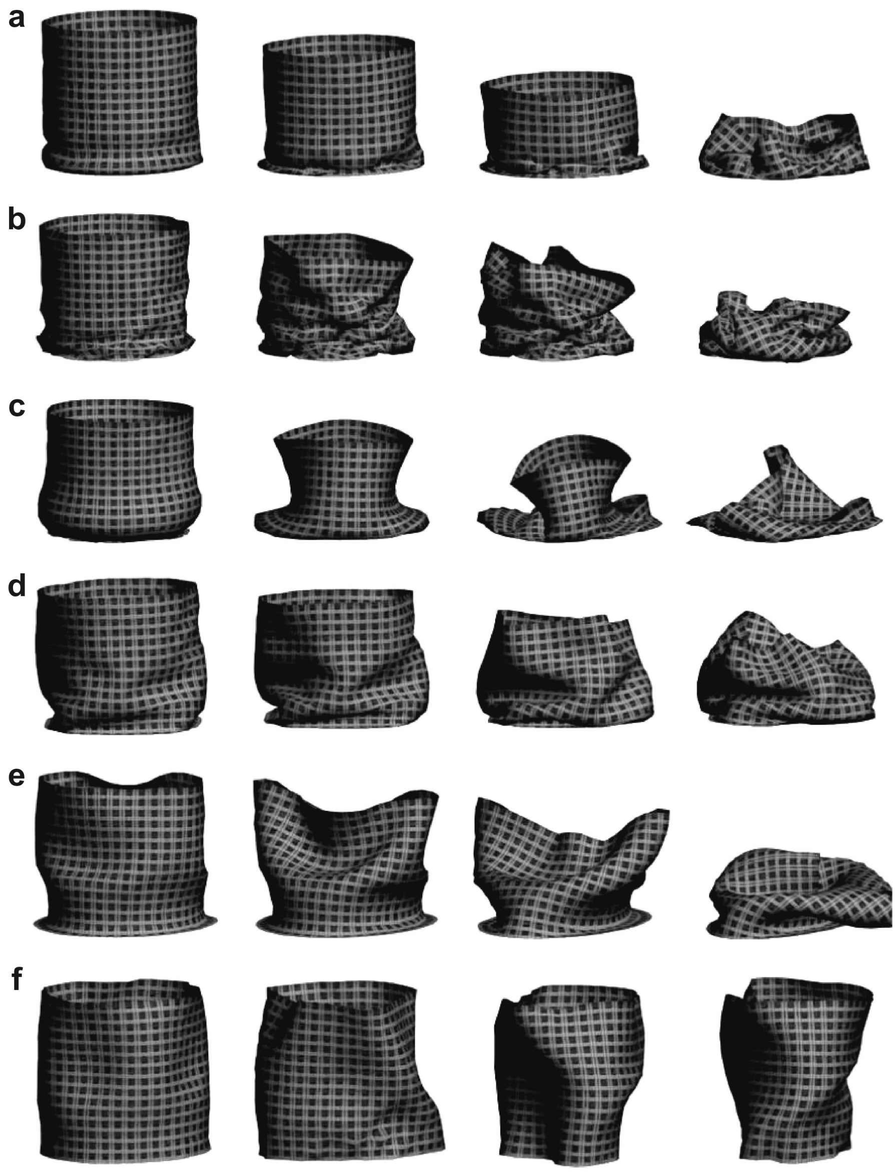

Fig. 6. Comparison of the bending deformations of a cylinder-shape cloth falling down by its own weight – all are with the same grids. (a) The result by Ref. [8], (b) the result by Ref. [4] with small Young's modulus in bending, (c) the result by Ref. [4] with large Young's modulus bending, (d) our result by the dynamic stiffness method – the small Young's modulus as (b) is used in bending and $w/h = 6$ is chosen, (e) our result with $w/h = 12$ and the large Young's modulus in bending as (c), and (f) our result by increasing the value of w/h from 6 to 24 while keeping all other parameters in (d).

Fig. 7. Screen-shots from the catwalk simulation of Qipao.

ODE is further converted into the formula below, which is solved by the nonlinear conjugate gradient method.

$$\begin{pmatrix} \boldsymbol{v}^{t+\mathrm{d}t} \\ \boldsymbol{x}^{t+\mathrm{d}t} \end{pmatrix} = \begin{pmatrix} \boldsymbol{v}^{t} + h\boldsymbol{M}^{-1}\boldsymbol{f}^{t+\mathrm{d}t} \\ \boldsymbol{x}^{t} + h\boldsymbol{v}^{t+\mathrm{d}t} \end{pmatrix} \tag{9}$$

6. Results

The model presented in this paper has been implemented in a prototype cloth simulation system by C++ and OpenGL. The simulation results from our model were compared with the results from the conventional spring-mass system [2–4] and the immediate buckling model [8]. Both are based on our implementation.

The first test is given to test the performance of the dynamic bending modeling (as shown in Fig. 6), where a piece of square cloth is sewed into a cylinder-shaped and falls down by its own weight. For the simulations shown in Fig. 6 (animations (a)–(f)), the time step was fixed to 0.02 s throughout the animation. Fig. 6(a) gives the simulation result of Ref. [8], where the fabric has almost no resistance to its weight as the compression of bending trusses leads to immediate buckling (i.e., the cloth is warped immediately). Fig. 6(b) gives the simulation from the conventional spring-mass system with small bending stiffness where the shape change of fabric has no effect on its bending performance. The wrinkles shown in Fig. 6(b) are unreal as they appear almost at any place on the cloth. How about if we increase the bending stiffness as shown in Fig. 6(c). The result is more like rubber instead of woven fabrics. The simulation from our model is more real as the dynamic bending is considered (see Fig. 6(d) and (e), where Fig. 6(d) employs the same Young's modulus in bending as Fig. 6(b), and Fig. 6(e) employs the same Young's modulus in bending as Fig. 6(c)). In the modeling of dynamic bending, not only the shape of the fabrics but also the tessellation has influence on the simulation result. As shown in Fig. 6(f), when increasing the width-thickness ratio w/h from 6 to 24 while keeping the same values for other parameters, the cloth will hardly fall to the ground (i.e., it acts more stiffly). It is easy to find that wrinkles can be generated even if such great bending stiffness is chosen.

Lastly, the screen-shots from our catwalk simulation of Qipao with the time step of 0.01 s have been shown in Fig. 7, where the wrinkles and folds are generated and vanished very naturally.

7. Conclusion

This paper presents a physical model for cloth simulation, which considers one cue that is seldom discussed in the existing work in the literature – the dynamic bending. The dynamic bending is caused by the change of inertia moments during the simulation. After the mechanical analysis of large buckling deformations of cloth, we introduced the method about how to implement our new physical model in a modified spring-mass system. Comparisons to other approaches show that our model can capture the physical property of woven fabrics more accurately, and the wrinkles and folds can be generated more naturally in the cloth simulation based on our model.

Acknowledgments

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