

# Outlier detection and disparity refinement in stereo matching

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## Abstract

Disparity estimation in ill-posed regions, such as occlusions, repetitive patterns and textureless regions, is a challenging problem in stereo matching. The initial disparities obtained in these regions tend to be regarded as outliers that must be detected and addressed. In this paper, two outlier detection methods are proposed, i.e., the efficient approach and the accurate approach. The efficient approach detects outliers by exploring the disparity map for the left image only and reduces runtime and memory costs. First, the match fixed point jumps (MFPJ) algorithm is proposed as an initial solution to detect outliers. Then, a high-probability outlier detection algorithm is proposed to accomplish denser outlier detection with less noise. The accurate approach first classifies outliers as occlusions or mismatches. Then, 3D label assignment is performed for occlusion outliers and normal-based plane fitting is conducted for mismatch outliers to refine the disparities of the outliers and to achieve an accurate stereo matching result. Evaluations of the Middlebury datasets demonstrate that the proposed methods effectively improve the stereo matching performance.

*Keywords:* Outlier detection, Stereo matching, Match fixed point jumps, Normal-based plane fitting

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## 1. INTRODUCTION

Stereo matching is a fundamental problem in computer vision that has received increasing attention in the literature in recent decades. Consider two

images taken from two horizontal cameras as input. The goal of stereo matching is to identify the disparity  $d$  of one point at position  $(x, y)$  in the left image such that the corresponding point appears at position  $(x - d, y)$  in the right image. Once the disparity  $d$  is obtained, the depth  $z$  of the pixel can be computed in the 3D scene as follows:  $z = fB/d$ , where  $f$  is the focal length of the camera and  $B$  is the baseline length (the distance between the centers of the two cameras). Scharstein and Szeliski [1] describe a taxonomy for stereo matching. According to the taxonomy, a stereo algorithm consists of four steps: matching cost computation, cost aggregation, disparity computation (optimization) and disparity refinement. One challenging problem in stereo matching is to find the corresponding points in ill-posed regions, e.g., occlusions, textureless regions, repetitive patterns and slanted regions. Many attempts have been made to accurately find the corresponding points. These studies mainly focus on the definition of the matching cost function [2, 3, 4] or the subsequent optimization [5, 6, 7] to enhance the performance of stereo matching. When investigating the above stereo matching algorithms, outlier detection and disparity refinement are nonnegligible components that can further improve the accuracy of stereo matching.

This paper studies outlier detection and the disparity refinement of outliers in stereo matching. Two approaches are proposed, i.e., the efficient approach and the accurate approach. The efficient approach detects outliers by exploring the disparity map for the left image only, thereby reducing the runtime and memory costs by approximately one-half. Compared with traditional efficient outlier detection methods, the proposed approach yields dense and accurate outlier detection results. The accurate approach first classifies outliers as occlusions or mismatches, and the classification guides the following disparity refinement step. Two refinement strategies are presented in the accurate approach, i.e., 3D-label assignment for occlusion outliers and normal-based plane fitting for mismatch outliers. As a result, the stereo matching performance is enhanced by the specific disparity refinement strategies.

The remainder of this paper is organized as follows. Closely related work

is reviewed in Section 2. The matching cost initialization and disparity op-  
35 timization are described in Section 3. The efficient and accurate approaches  
are described in detail in Section 4. The experimental results and discussion  
are presented in Section 5. Finally, conclusions are drawn and further research  
directions are discussed in Section 6.

## 2. RELATED WORK

40 A large volume of studies on stereo matching have been published; we review  
the closely related literature.

Kong and Tao [8] first utilize the sum of absolute differences (SAD) or the  
sum of squared differences (SSD) to compute the matching cost. Zabih and  
Woodfill [9] use the census transform, which considers not only the information  
45 of the current pixel itself but also that of its neighbors, to define the matching  
cost. Mei et al. [2] combine the absolute differences (AD) and census transform  
to define a robust matching cost, namely, the AD-census matching cost. The  
normalized cross-correlation (NCC) [10] is another robust matching cost that is  
invariant to linear variation in illumination. Žbontar and LeCun [11] compute  
50 the matching cost by training a convolutional neural network (CNN), and their  
data-driven similarity measurement outperforms most traditional hand-crafted  
metrics. Many recent algorithms [12, 13, 14] utilize this data-driven matching  
cost and achieve state-of-the-art stereo matching results. In this paper, we also  
adopt the matching cost trained by a CNN as the input.

55 In general, the initial matching cost is sensitive and noisy. Thus, the costs  
are usually aggregated in a support region. Yoon and Kweon [15] aggregate  
the matching costs via an adaptive weight. The weight is based on color and  
spatial differences between a neighboring pixel and the center pixel in a fixed  
window. Zhang et al. [16] propose cross-based cost aggregation in an adaptive  
60 support region with a constant weight. He et al. [17] propose a guided image  
filtering algorithm that is a constant-time and edge-aware filter that can produce  
accurate stereo matching results. These methods implicitly make the frontal-

parallel surface assumption that all the points belonging to the support region share the same disparity. However, this assumption is sometimes violated in practice. Bleyer et al. [18] use Patch-Match stereo matching with a slanted support window to overcome this problem. Lu et al. [19] propose a Patch-Match filter algorithm to accelerate the original Patch-Match algorithm. The approach adopts the super-pixel as the unit of computational cost and achieves a time complexity of approximately  $O(1)$ . Tanai et al. [12] infer per-pixel 3D plane labels on a pairwise Markov random field by using local expansion moves and achieve an accurate stereo matching result.

After the cost aggregation step, the initial disparity map can be computed using a winner-take-all (WTA) strategy or some other disparity optimization strategy. The initial disparity map contains many outliers, and extensive research has been conducted on outlier detection in stereo matching. Outliers in stereo matching are mainly caused by occlusions or mismatches. Five outlier (mainly occlusion) detection methods are summarized in a previous survey [20]: bi-modality (BMD) [21, 22], match goodness jumps (MGJ) [23], ordering (ORD) [24, 25] constraint, occlusion constraint (OCC) [26], and left-right checking (LRC) [27, 28, 29]. BMD, MGJ, ORD and OCC do not require computation of the disparity map for the right image, rendering these methods memory-and-time efficient, especially for large stereo images. However, the outlier detection results of this type of method are usually sparse and noisy and are not suitable for highly accurate stereo matching. In this paper, a high-probability outlier detection algorithm, in which the detection results are denser, less noisy and more accurate, is proposed to overcome this problem. LRC utilizes additional information of the disparity map from the right image. LRC can detect outliers densely, and the outliers can also be classified as occlusions and mismatches. However, the classification is not always accurate because LRC detects outliers only at the pixel level. Some outliers (especially occlusion outliers) tend to appear regionally. The proposed accurate outlier detection first classifies outliers reasonably and accurately and then processes occlusion outliers and mismatch outliers with different disparity refinement algorithms and achieves accurate

stereo matching.

95     Outlier detection is akin to confidence measure detection in stereo vision.  
One point in the image tends to be an outlier when the confidence value of  
that point is low. Hu and Mordohai [30] evaluate several confidence measures  
in stereo vision. Some of the approaches in [30] compute the confidence value  
of a single point by comparing the minimum cost and the second minimum cost  
100 [31, 32, 33], whereas some approaches explore the shape of the cost curve [33].  
Other approaches compute the confidence value by treating the value assigned  
to each potential disparity as a probability for the disparity [34, 35]. Yoon et al.  
[36] use the distinctiveness of points as a confidence measure. Hu and Mordohai  
[30] also propose a measure called left-right difference (LRD) that considers  
105 both the two smallest minima of the costs and the consistency of the minimum  
costs across two images to obtain a robust confidence measure.

All these methods detect outliers based on ‘abnormal phenomena’. These  
phenomena are distinguished by the matching cost (e.g., MGJ) or the disparity  
(e.g., BMD, ORD and the visibility constraint), while LRC identifies outliers by  
110 detecting disparities in the left image that are in conflict with the disparities of  
the corresponding points in the right image. In this paper, the aim is to design  
a method to detect outliers more efficiently and accurately and to facilitate the  
subsequent disparity refinement step is designed to produce a more accurate  
stereo matching result.

### 115 **3. MATCHING COST COMPUTATION AND DISPARITY OPTI- MIZATION**

Fig. 1 illustrates the whole algorithm of the proposed method. The details  
of each part of the proposed method will be described in the following sections.

#### *3.1. Matching cost computation*

Let  $I^L$  and  $I^R$  be the left image and the right image, respectively. The  
purpose of the proposed method is to estimate the disparities of the left image

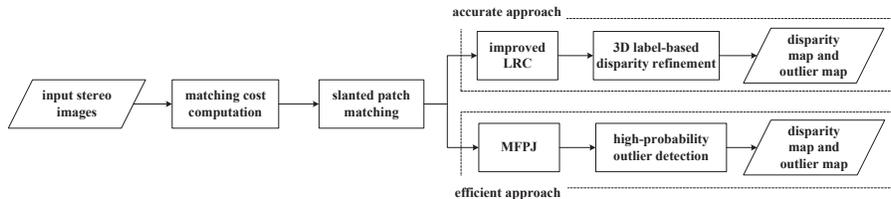


Figure 1: The flowchart of the proposed method

robustly. First, we initialize the matching cost. The position of a point in the image is denoted by a lowercase letter, such as  $p$  or  $q$ . A point  $p(p_x, p_y)$  in the left image matches the point  $q(p_x - d, p_y)$  in the right image with disparity  $d$ , and the point-wise matching cost is denoted by  $C(p, d)$ . We adopt the convolutional neural network (CNN) approach in [11] to compute the matching cost:

$$C(p, d) = C_{CNN}(R^L(x, y), R^R(x - d, y)) \quad (1)$$

120 where  $C_{CNN}(\cdot, \cdot)$  is the predicted similarity measure obtained by the trained CNN. The cost is computed between the  $11 \times 11$  image patch  $R^L$  centered at pixel  $p$  of  $I^L$  and the image patch  $R^R$  centered at the corresponding pixel of  $I^R$ .

### 3.2. Slanted patch matching

The initial matching cost is not generally robust. For accurate stereo matching, we aggregate the matching costs in slanted support windows [18]. For each point  $p(p_x, p_y)$ , the goal is to identify a fitting plane such that the disparity of  $p$  can be computed as follows:

$$d_p = a_p \cdot p_x + b_p \cdot p_y + c_p \quad (2)$$

125 where  $a_p$ ,  $b_p$  and  $c_p$  are three parameters of the plane. The tuple  $(a_p, b_p, c_p)$  is called a 3D label. We then utilize the local expansion moves proposed in [12] to optimize each pixel's 3D label in both the left and right images and obtain the disparity maps  $D^L$  and  $D^R$ , respectively.

## 4. OUTLIER DETECTION AND DISPARITY REFINEMENT APPROACHES

130 Two approaches are proposed in this section to address the trade-off between the computing time and accuracy. The efficient approach, which does not need to compute the disparity map for the right image, is tuned for runtime and memory consumption and reduces the time and memory expenses by approximately one-half. The accurate approach improves the initial LRC outlier detection map  
135 by distinguishing occlusion and mismatch outliers more reasonably. Then, two different disparity refinement approaches, i.e., 3D label assignment for occlusion outliers and normal-based plane fitting for mismatch outliers, are proposed for better stereo matching performance.

### 4.1. The efficient approach

140 In this section, we introduce an efficient outlier detection approach. First, match fixed point jumps (MFPJ), which utilizes the cost volume information of the left image to detect initial outliers, is proposed. Then, a high-probability outlier detection algorithm is presented to obtain a denser and less noisy outlier detection result.

#### 145 4.1.1. Match fixed point jumps

In this step, the outliers are detected by using the proposed MFPJ. In MFPJ, the confidence of a point  $p$  is defined as follows:

$$C_{MFPJ}(p) = \min_{p_n \in \mathcal{N}} \{C(p_n, d_n)\} - C_1(p, d) \quad (3)$$

where  $C_1(p, d)$  is the minimum matching cost between point  $p$  and its corresponding pixel  $p'$ ,  $p_n$  is a neighboring point of  $p$  in a window  $\mathcal{N}$  centered at  $p$ , and  $C(p_n, d_n)$  is the matching cost between  $p_n$  and  $p'$ . The confidence defined in Equation 3 indicates that if the point's minimum cost  $C_1(p, d)$  is relatively  
150 high, then a high probability exists that the point does not have a good matching point, indicating that the point is likely to be an outlier (the confidence is low). Note that this method differs from LRD in [30], which considers the

consistency of the minimum costs across the two images. We utilize only the left  
 cost volume information and do not search the whole disparity range (which is  
 155 usually greater than 100 in current stereo pairs). Thus, the method is efficient.

Many confidence measures, such as MGJ [20], consider the relative magni-  
 tude of a point’s minimum costs, for example by comparing the relative magni-  
 tude of the current point’s minimum cost and the neighboring points’ minimum  
 costs. However, the proposed method differs from MGJ to some extent. Fig.  
 160 2 illustrates the difference between MFPJ and MGJ. MGJ compares the cur-  
 rent point’s minimum cost and the neighboring points’ minimum costs, whereas  
 MFPJ compares the current point’s minimum cost and the costs of matching  
 the neighboring points to a fixed point.

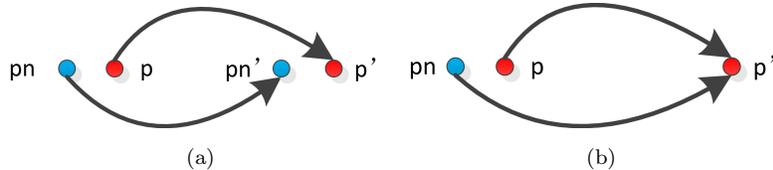


Figure 2: An illustration of MGJ and MFPJ.  $p$  and  $p'$  denote corresponding points, as well as  $p_n$  and  $p'_n$ . (a) MGJ defines the confidence using the minimum costs among the point pair  $(p, p')$  and its neighboring pairs (for example,  $(p_n, p'_n)$ ); (b) MFPJ defines the confidence using the minimal costs among the point pair  $(p, p')$  and the costs of the pairs composed of the neighboring point  $(p_n$  for instance) and the fixed point  $p'$ , i.e.,  $(p_n, p')$ .

However, if significant photometric variation exists between the matched  
 165 features, MGJ tends to detect a false signal [20]. Fig. 3 shows an example.  
 In the circled region, strong photometric variation exists near the edges of the  
 books on the bookshelf. The detection result of MGJ is noisy, while that of the  
 proposed MFPJ method is reasonable.

The reason that MFPJ achieves a better result is because in regions with  
 170 significant photometric variation, points with different colors may have different  
 magnitudes of minimum costs, which may mislead outlier detection. Thus, MGJ  
 tends to pick up wrong signals. When the MFPJ method determines whether  
 one point is an outlier, the corresponding point in the right image is fixed, and

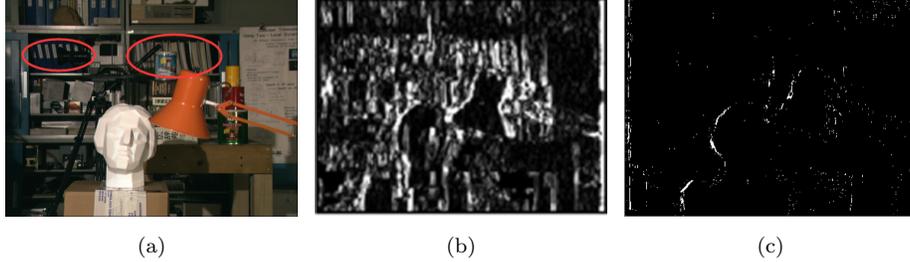


Figure 3: Comparison of the outlier detection results of MGJ and MFPJ. (a) Raw image; (b) the outlier detection result of MGJ, where the bright points denote outliers, and (c) the outlier detection result of MFPJ, which has substantially less noise than the MGJ result

only the costs of the neighboring points matched with this point are adopted  
 175 for comparison. Thus, wrong signals are not picked up in this case.

Let  $M$  denote the outlier map detected by MFPJ. For each point, when the confidence is lower than a threshold, the point is regarded as an outlier, as follows:

$$M(p) = \mathbb{1} \{C_{MFPJ}(p) < \eta\} \quad (4)$$

where  $\mathbb{1} \{\cdot\}$  is the indicator function, and  $\eta$  is a threshold. The choice of all the parameters will be discussed in Section 5.4. In the following sections, we improve this outlier map and achieve more accurate outlier detection results.

#### 4.1.2. High-probability outlier detection

180 In Section 4.1.1, we demonstrate that the MFPJ strategy has advantages over MGJ. However, the detection result is still relatively noisy and sparse, especially for some large-sized images, as shown in Fig. 4(b). Actually, this is a universal problem with outlier detection methods that explore the disparity map for the left image only. In this section, a high-probability outlier detection  
 185 method is proposed to address this problem.

Since the detection result of MFPJ is still noisy and sparse, the defect should be remedied. Ordering constraint (ORD) [20] can be adopted to detect the ordering of points in both the left and the right images. The basic idea of this

algorithm is that the order of the points in a scanline in the left image should be  
 190 consistent with that of the corresponding points in the right image. Any point  
 that violates this rule is regarded as an outlier. The ORD detects a point's  
 order from the disparity map  $D^L$  computed in Section 3.2. The detection result  
 is shown in Fig. 4(c). For some occlusion pixels (circled), the disparities still  
 obey the ordering constraint and the ORD cannot detect them. However, the  
 195 outlier map of the ORD has less noise than that of MFPJ; thus, this map can  
 be adopted as a guidance map to improve the result of MFPJ.

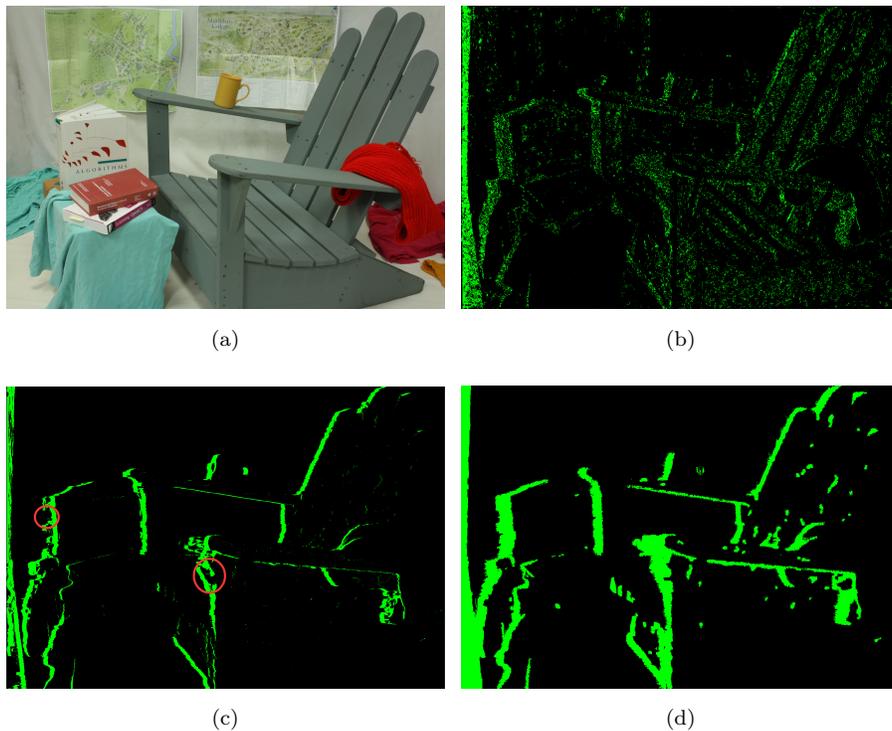


Figure 4: Outlier detection results. (a) Raw image of Adirondack; (b) the outlier map detected by MFPJ is still noisy and sparse; (c) in the outlier map detected by the ORD, some occlusion outliers shown in the circled regions are missed; and (d) the high-probability outlier detection result. The outlier map is denser than the MFPJ and ORD maps

Let  $O$  denote the outlier map detected by ORD. For each outlier  $p(p_x, p_y)$  in  $M$ , we compute the numbers of outliers in the local window centered at  $p$  in  $M$

and in  $O$ , respectively, and define the probability of the point  $p$  being an outlier as:

$$P(p_x, p_y) = \frac{\alpha \cdot N_M(p_x, p_y) + (1 - \alpha) \cdot N_O(p_x, p_y)}{N} \quad (5)$$

where  $N_M(p_x, p_y)$  and  $N_O(p_x, p_y)$  are the numbers of outliers in  $M$  and  $O$  centered at  $(p_x, p_y)$ ,  $N$  is the size of the local window, and  $\alpha$  is a constant value. We define point  $p$  as a high-probability outlier when  $P(p_x, p_y)$  is greater than a  
 200 threshold  $\mu$ . The parameters will be analyzed in Section 5.4

To achieve a dense outlier detection result, we fill the black points that are surrounded by high-probability outliers since these points also tend to be outliers. For each black point, we search the high-probability outliers along the horizontal and vertical directions in a window centered on the black point. We  
 205 mark the black point as an outlier if and only if four high-probability outliers are found in the window. Fig. 4(d) illustrates the final outlier detection results by using the proposed efficient approach.

#### 4.2. The accurate approach

In this section, an accurate approach is proposed to detect outliers and refine  
 210 the outliers' disparities more accurately than with the efficient approach. In this approach, first, the outlier map detected by the LRC is adopted as the input. Then, the outliers are classified as occlusions or mismatches. Finally, a 3D label-based disparity refinement is proposed to achieve accurate stereo matching.

##### 4.2.1. Improved outlier classification

The LRC checks the disparity consistency of corresponding points and is formulated as follows [29]:

$$|D^L(\mathbf{p}) - D^R(\mathbf{p} - D^L(\mathbf{p}))| \leq 1 \quad (6)$$

215 A point that violates Equation 6 is regarded as an outlier. The detected outliers are further classified as occlusions and mismatches. If one point violates Equation 6 for the current disparity but the point has any other candidate disparities

that are satisfied with Equation 6, then the point is a mismatch; otherwise, the point is an occlusion [11].

220 However, the classification of mismatch and occlusion outliers via LRC is not always accurate. Fig. 5(b) illustrates an outlier map detected by LRC. In some occlusion regions, the outliers are classified as mismatches, which will have a negative impact on the subsequent disparity refinement. **Through observation, we find that occlusion points tend to appear together rather than in isolation,**  
225 **as shown in Fig. 5(d).** Thus, for each mismatch outlier  $p$ , we will reclassify it as an occlusion outlier if the ratio of the number of occlusion points in a window centered at  $p$  is greater than a threshold  $\kappa$ , which will be discussed and determined in the Section 5.4. The improved outlier map obtained via the improved classification is illustrated in Fig. 5(c), which shows better classification than  
230 that of LRC.

#### 4.2.2. 3D label assignment for occlusion outliers

In this subsection, each occlusion point is assigned a reliable 3D label to enhance the stereo matching performance. For each occlusion point  $p$ , its closest reliable points (not outliers) are searched along the horizontal and vertical  
235 directions. One of these reliable points is selected to assign  $p$  a reliable 3D label. However, the search should be in the neighborhood of  $p$  since points that are far away tend to lie in different planes. Thus, if a discontinuous boundary is met, then the search should stop. The discontinuous boundaries can be determined via both texture edges and disparity edges. First, a texture edge map  
240 and a disparity edge map are detected by the Canny operator and Sobel operator respectively, as illustrated in Fig. 6(b) and Fig. 6(c). Clearly, the detected texture edges are redundant for discontinuous boundaries. We alleviate this problem with the help of detected disparity edges. For each texture edge point,  
245 we search the corresponding neighboring points in the disparity edge map. If the ratio of the number of neighboring disparity edge points is higher than a threshold  $\rho$ , which will be also discussed and determined in Section 5.4, then the texture edge point will be regarded as a real discontinuous boundary point.

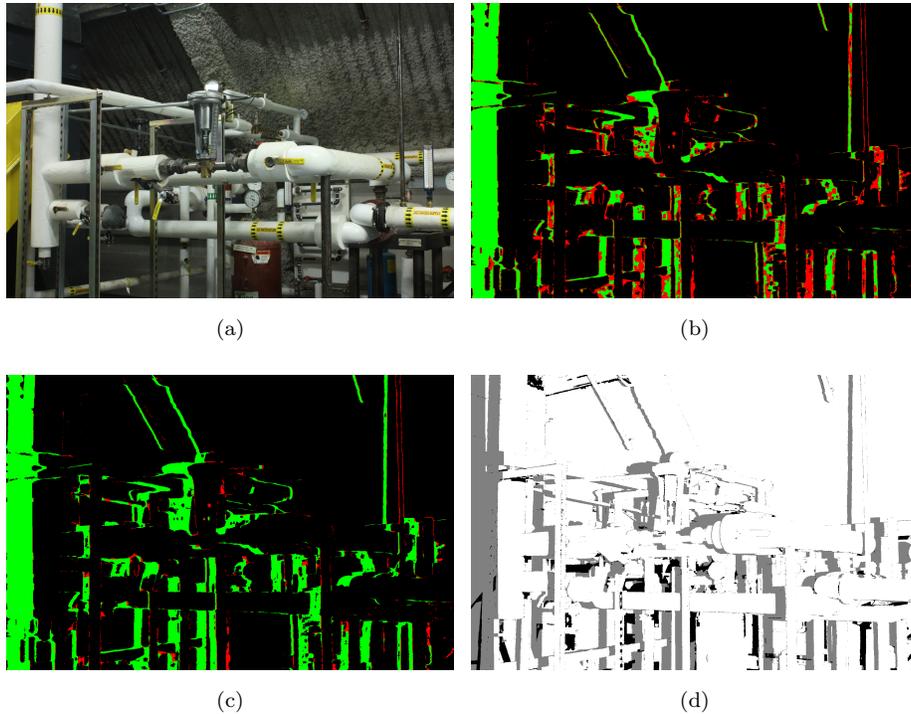


Figure 5: Outlier map comparison between the LRC and the improved LRC. (a) Raw image of Pipes; (b) the outlier map classified by the LRC, where the green and red points denote occlusion and mismatch outliers, respectively; (c) the outlier map classified by the improved LRC; and (d) the ground truth of the non-occlusion mask, where the gray points denote occlusion points. The improved LRC classifies occlusion outliers more properly. (Best viewed in color)

Fig. 6(d) illustrates the result of discontinuous boundaries.

Considering the detected discontinuous boundaries, the search for reliable  
 250 3D labels along the horizontal and vertical directions stops when a reliable 3D  
 label is identified or a discontinuous boundary is met. Then, the disparity of  
 point  $p$  is obtained by assigning  $p$  a reliable 3D label. Here, the point with  
 the lowest disparity value is adopted as the point  $p$ 's filled-in disparity since  
 occlusion points tend to occur in the background.

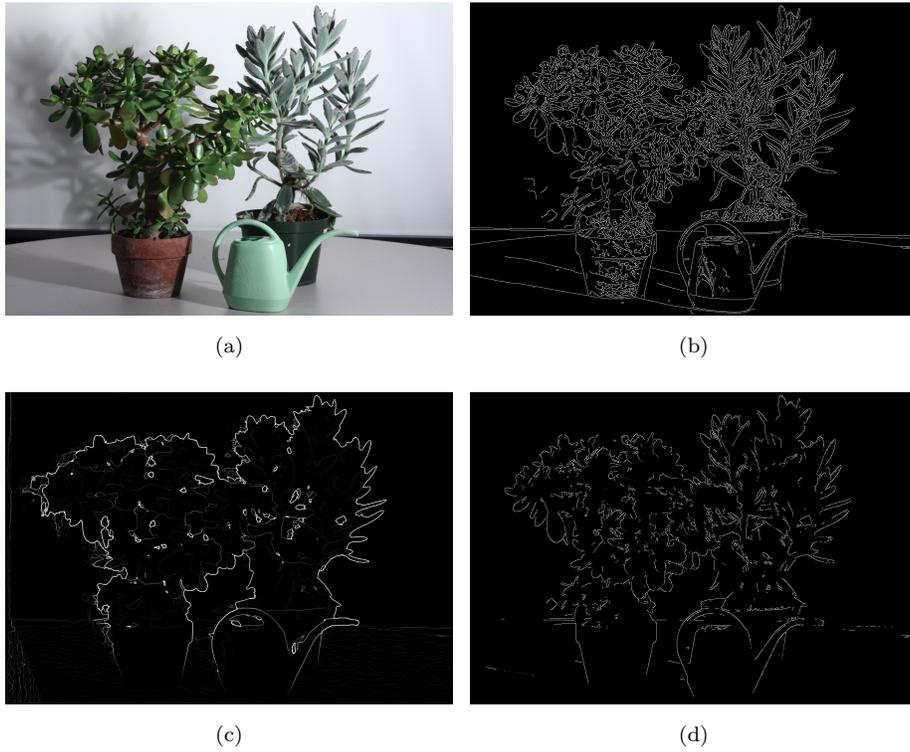


Figure 6: An example of discontinuous boundary detection. (a) Raw image of Plants; (b) the texture edge map detected by the Canny operator; (c) the disparity edge map detected by the Sobel operator; and (d) the discontinuous boundaries

The disparities of mismatch points also need to be refined. Here, a normal-based plane fitting algorithm, which is based on the observation that mismatch points often appear at the slanted planes, is proposed. In some disparity maps, we find that the error pixels are usually at the slanted planes. Thus, refining mismatch outliers with neighboring reliable points using a plane fitting strategy is reasonable. Since we not only have the disparities of the reliable points but also their 3D labels, the information of these 3D labels should be utilized in the plane fitting strategy. For each mismatch point  $p$ , we search its closest reliable points along the horizontal and vertical directions, as in Section 4.2.2. Then,  $k(k \leq 4)$  reliable points and their corresponding 3D labels are obtained. The three components of the reliable point's normal can be computed by  $n_z = 1/\sqrt{a_q^2 + b_q^2 + 1}$ ,  $n_x = -a_q \cdot n_z$ ,  $n_y = -b_q \cdot n_z$ , where  $q$  denotes the reliable point and  $(a_q, b_q, c_q)$  is its 3D label. Then,  $p$ 's normal is defined as the average of the reliable points' normals. A least-squares plane fitting strategy based on  $p$ 's normal is then implemented for the new 3D label of mismatch point  $p$ :

$$\min \sum_{i=1}^n \frac{[a_p \cdot q_{x_i} + b_p \cdot q_{y_i} - d_{q_i} + c_p]^2}{a_p^2 + b_p^2 + 1} \quad (7)$$

$$s.t. \mathbf{n}_p = \sum_{i=1}^k \frac{\mathbf{n}_{q_i}}{k} \quad (8)$$

where  $(q_x, q_y)$  is the position of the point  $q$ ,  $d_q$  is the disparity of  $q$ ,  $(a_p, b_p, c_p)$  is  $p$ 's 3D label, and  $\mathbf{n}_p$  is  $p$ 's refined normal. Note that the disparity of  $p$  is not refined if we do not find any reliable point around it ( $k = 0$ ). The disparity of  $p$  is computed when assigning  $p$  a new 3D label. The proposed method considers the information of neighboring reliable points' 3D labels. In the slanted regions, plane fitting with only disparity information may not be sufficient to obtain an accurate result. With the information of the neighboring 3D labels, the plane fitting strategy is more robust for the refinement of disparities. The experimental results demonstrate that the proposed approach can achieve accurate disparity refinement.

#### 4.2.4. Weighted median filter

Finally, the weighted median filter from [37] is adopted to refine the disparity map. Let  $D^N$  denote the disparity map obtained after normal-based plane fitting. The final disparity map is refined as follows:

$$D^M(p) = \underset{q \in \Omega}{med} \{ \omega(p, q) D^N(q) \} \quad (9)$$

where  $\omega(p, q)$  is a weight factor that depends on the Euclidian distance between  $p$  and  $q$  in RGB color space, and  $\Omega$  is the neighborhood region of  $p$ .  $D^M(p)$  is the final disparity map with our accurate approach.

## 270 5. EXPERIMENTAL RESULTS

In this section, the proposed method is evaluated from various perspectives, including the performance of the accurate approach in Section 5.2, the performance of the efficient approach in Section 5.2, the performance of the confidence measure in Section 5.3, and the parameter analysis in Section 5.4. 275 The MC-CNN-acrt matching cost computation [11] is executed on a personal desktop computer equipped with an Nvidia Titan X graphics card. The other components of our stereo algorithm are executed on a personal computer with an Intel(R) Core(TM) i5-4590 CPU with 3.30 GHz and 16 GB of RAM. The Middlebury stereo dataset version 2014 [38] is adopted to evaluate our method 280 since it is a more challenging dataset than the older version of the Middlebury dataset.

### 5.1. Performance of the accurate approach

Some disparity maps obtained using our accurate approach on the Middlebury stereo dataset are shown in Fig. 7, which can help us to qualitatively 285 evaluate the proposed method. The proposed method can generate accurate disparity maps with sharp edges. Then, the proposed accurate approach is compared with other state-of-the-art methods on the Middlebury stereo benchmark. Here, half-size images are adopted in our test since the memory of GPU

limits the MC-CNN-acrt matching cost computation. The criteria bad 1.0 (%),  
 290 bad 2.0 (%), and bad 4.0 (%) for non-occluded (*nonocc*) regions and the whole  
 image area (*all*) are adopted for the evaluations. The statistical results are  
 given in Table 1. As shown, more reasonable outlier classification and disparity  
 refinement results in more accurate disparity estimation. The proposed method  
 achieves a state-of-the-art result for both *nonocc* and *all* regions. As of May  
 295 2nd, 2018, the proposed accurate approach ranks 3rd in the criterion bad 4.0  
 (%) (*nonocc*) and 2nd in all the other criteria.

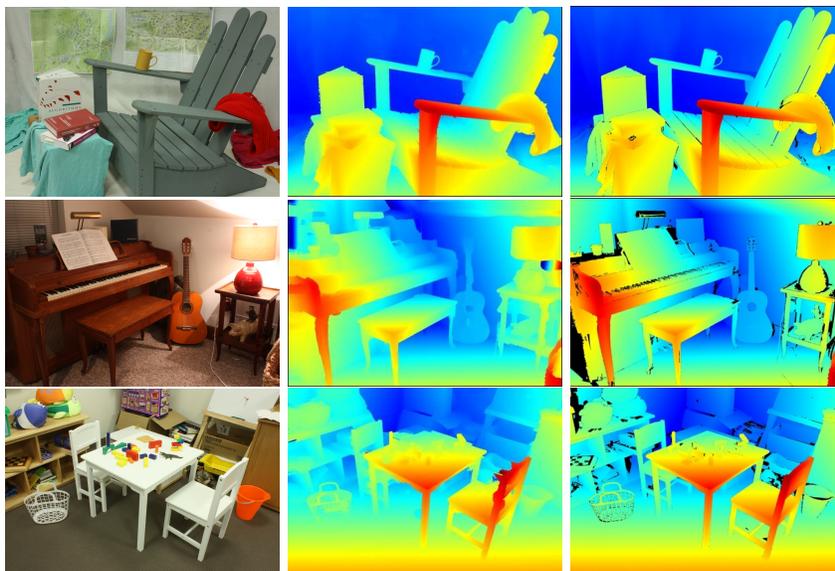


Figure 7: Disparity maps of examples in the Middlebury dataset generated by the proposed accurate approach. From left to right: left images, disparity maps and ground truth

The stereo matching accuracy between the LRC [29] approach and the proposed accurate approach is then compared. The difference is that the former uses LRC to detect outliers while the proposed accurate approach utilizes the  
 300 improved LRC. The criterion bad 2.0% is used to evaluate their performances. As shown in Fig. 8, the proposed accurate approach achieves a relatively superior result to that of the LRC approach since the improved LRC in the accurate approach considers the regional information more appropriately.

	Average Error						Runtime(s)
	bad 1.0		bad 2.0		bad 4.0		
	nonocc	all	nonocc	all	nonocc	all	
Ours-accurate	13.6	19.8	6.30	12.0	3.83	8.58	867
Ours-efficient	14.1	21.1	7.04	13.8	4.73	10.8	411
NOSS	12.9	19.5	5.82	11.9	3.67	8.73	1545
LocalExp [12]	13.7	19.9	6.52	12.1	4.07	8.69	846
SGM-Forest	14.8	21.9	7.01	13.7	3.73	9.55	64.6
3DMST [13]	15.1	21.5	7.08	12.9	4.43	9.22	167
APAP-Stereo	20.9	27.3	7.53	14.3	4.50	10.8	117
FEN-D2DRR [39]	16.7	23.8	7.89	14.1	3.98	8.53	73.3
PMSC [40]	16.4	23.0	8.20	14.2	5.15	10.0	579
LW-CNN [41]	16.6	25.7	8.31	17.8	4.89	14.1	224
MeshStereoExt [42]	18.4	26.2	9.32	16.8	5.53	12.0	133
MCCNN-Layout	18.0	27.0	9.34	18.6	5.21	14.3	300
OVOD	17.6	24.2	9.65	15.8	5.67	10.8	59.9
NTDE [43]	18.1	25.8	9.94	16.9	6.13	11.8	128
MC-CNN-acrt [11]	18.4	27.7	10.1	19.7	6.34	15.7	106

Table 1: Comparison of the proposed method and the state-of-the-art stereo methods against the Middlebury benchmark 3.0

We also evaluate the effect of normal-based plane fitting in Section 4.2.3.

305 We compare the performances of plane fitting with and without the normal constraint. The statistical results are shown in Fig. 9. Here, the criterion bad 2.0% is adopted. Clearly, plane fitting with the normal constraint performs better for most sequences than trivial plane fitting since the proposed method utilizes more information. A more reasonable 3D label can be obtained for

310 mismatch outliers when using the normal constraint.

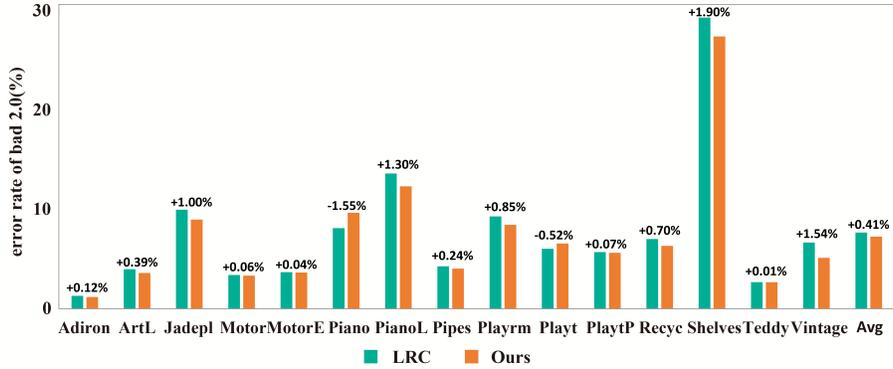


Figure 8: Comparison of the LRC approach and the proposed accurate approach using the criterion bad 2.0%. For each pair of bars, the baseline is “Ours”

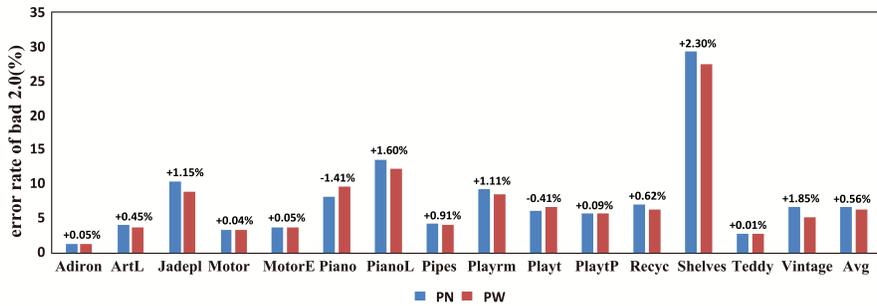


Figure 9: Comparison of plane fitting without the normal constraint (PN) and plane fitting with the normal constraint (PW). The criterion bad 2.0% is adopted, and for each pair of bars, the baseline is “PW”

## 5.2. Performance of the efficient approach

We first compare the runtimes of the proposed efficient approach and the outlier detection approach that computes the disparity maps for both the left and the right images, e.g., LRC. No refinement is conducted in this experiment since we consider only the runtimes for obtaining the outlier maps. Table 2 compares the runtimes of the proposed efficient approach and LRC and shows that the proposed efficient approach consumes approximately half of the runtime of LRC since the efficient approach does not need to compute the disparity map for the right image.

	Andiron	ArtL	Jadepl	Motor	MotorE	Piano	PianoL	Pipes
LRC	1032.58	344.22	1024.09	1036.02	950.79	829.83	955.22	984.52
Ours	466.78	169.63	493.49	506.67	498.43	504.52	538.34	388.49
	Playrm	Playt	PlaytP	Recyc	Shelvs	Teddy	Vintge	Average
LRC	854.91	730.95	849.15	953.54	994.28	361.90	1192.00	876.93
Ours	394.63	373.78	455.35	491.03	476.83	177.6	474.69	410.88

Table 2: Runtime (s) comparison of the LRC and the proposed efficient approach on the Middlebury stereo dataset version 2014

320 To test how the efficient outlier detection approach influences the accuracy of stereo matching, we refine the disparities of the outliers uniformly using the 3D label assignment algorithm in Section 4.2.2 since the proposed efficient approach does not classify outliers as occlusions and mismatches. The statistical results of the efficient approach are shown in Table 1. The efficient approach performs well  
325 in terms of accuracy after the disparity refinement step, and it is also efficient in terms of time and memory.

Note that the size of the Middlebury dataset is relatively large, and some global optimization algorithms [12], which are time-consuming are used to compute the disparity maps here. Subsequently, the runtime difference between  
330 LRC and the proposed efficient approach is large. In some real-time embedded multimedia systems, obtaining the disparity map for the right image is relatively cheap, and the runtime overhead is therefore little [44]. Nevertheless, the proposed efficient approach balances the computing time and accuracy well, especially for some time-consuming algorithms.

### 335 5.3. Performance of the confidence measure

In this sub-section, the performance of the proposed efficient approach is evaluated further and compared with some other efficient outlier detection approaches in terms of confidence measure. First, some notations are given. Let

$d_1(p) = D^L(p)$  denote the predicted disparity at position  $p$ , let  $c_1(p) = C_1(p, d)$  denote the smallest matching cost, and let  $c_2(p)$  denote the second smallest matching cost.

We compare several confidence measures: (1) the naive version of peak ratio (PKRN) [30], i.e., the ratio between the second smallest matching cost and the smallest cost,  $C_{PKRN} = c_2/c_1$ ; (2) curvature (CUR) [33],  $C_{CUR} = -2c(d_1) + c(d_1 + 1) + c(d_1 - 1)$ ; (3) maximum likelihood measure (MLM) [34],  $C_{MLM} = e^{-c_1/\sigma_{MLM}^2} / \sum_d e^{-c_d/\sigma_{MLM}^2}$  (4) LRD [30]; and the proposed efficient approach. We use the area under the curve (AUC) measure in [30] to evaluate the performance of these confidence measures. The average AUC of the Middlebury training dataset is shown in Table 3. The proposed method performs better than the other confidence measures. Some detection results are also illustrated in Fig. 10. Clearly, the outlier detection result of the proposed efficient approach has less noise than that of the other approaches and achieves dense detection results.

	PKRN	CUR	MLM	LRD	Ours
AUC	9.35	11.55	14.85	11.19	7.75

Table 3: The average AUC for the Middlebury dataset for different confidence measures. The proposed method outperforms other confidence measures in terms of the AUC

To further evaluate the occlusion detection effectiveness of the proposed efficient approach, two criteria are utilized: the hit rate and the false-positive rate of occlusions [20]. The Middlebury dataset with manually labeled occlusions is adopted here. Table 4 shows the hit rate and false-positive rate of occlusions for various detection methods. The proposed approach achieves the highest hit rate and the lowest false-positive rate.

#### 5.4. Parameter analysis

In this sub-section, the sensitivity of the parameters used in two approaches are analyzed and carefully discussed. First, all the parameter settings are listed in Table 5. Then, the sensitivity of the parameters is evaluated.

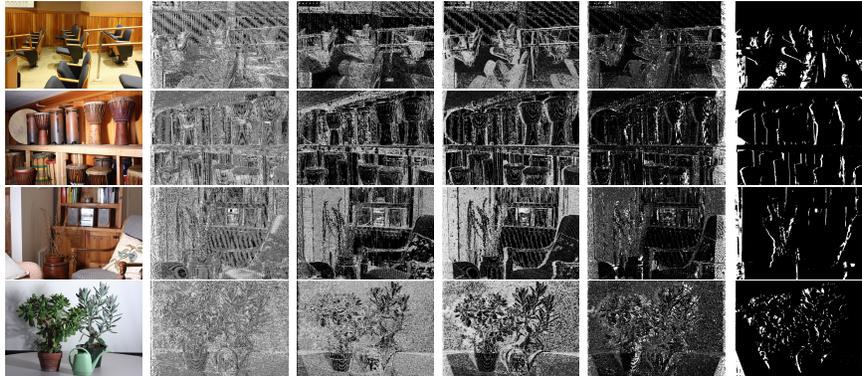


Figure 10: Comparison of various outlier detection approaches. From left to right: raw images, confidence maps of MLM, confidence maps of CURVE, confidence maps of PKRN, confidence maps of LRD, and confidence maps of the proposed method. Bright pixels indicate low confidence, i.e., a tendency to be an outlier. Our confidence maps (outlier detection maps) have less noise and achieve dense detection results

	PKRN	CUR	MLM	LRD	Ours
Hit rate	0.589	0.531	0.331	0.716	0.761
False-positive rate	0.172	0.251	0.280	0.130	0.110

Table 4: The hit rate and false-positive rate of occlusions [20] for the Middlebury dataset for different confidence measures

#### 5.4.1. Parameter analysis of the efficient approach

365 In the efficient approach, the parameter  $\eta$  in Equation 4 controls the threshold of whether one point is an outlier when using MFPJ. Fig. 11(a) illustrates the sensitivity analysis result for the Middlebury 3.0 dataset. The AUC measure [30] is adopted to evaluate the outlier detection performance of different parameter values. The proposed MFPJ method is insensitive to  $\eta$  in a large

$\eta$	$\alpha$	$\mu$	$\kappa$	$\rho$
-0.1	0.15	0.8	0.6	0.2

Table 5: The parameter settings of the proposed method

370 parameter setting range. Note that in this experiment, the other parameters  
 are fixed as in Table 5, except for the parameter  $\eta$ . The following experiments  
 for the other parameter analyses are conducted similarly.

When detecting high-probability outliers in Equation 5, the parameter  $\alpha$   
 controls the numbers of outliers detected by MFPJ and the ORD. Fig. 11(b)  
 375 shows the AUC using different  $\alpha$  on Middlebury 3.0 dataset. When using only  
 the ORD method ( $\alpha$  equals to 0), the AUC is relatively high since the ORD  
 has its own limitations as stated in Section 4.1.2. When using only the MFPJ  
 method ( $\alpha$  equals to 1), the AUC is lower than that of the ORD but is still  
 relatively high. The performance of outlier detection is better when these two  
 380 methods are combined, and the high-probability outlier detection performance  
 is stable for the parameter  $\alpha$  in a large range.

Parameter  $\mu$  controls the threshold of whether one point is a high-probability  
 outlier. As shown in Fig. 11(c), the proposed high-probability outlier detection  
 method is insensitive when  $\mu$  varies from 0.5 to 0.8.

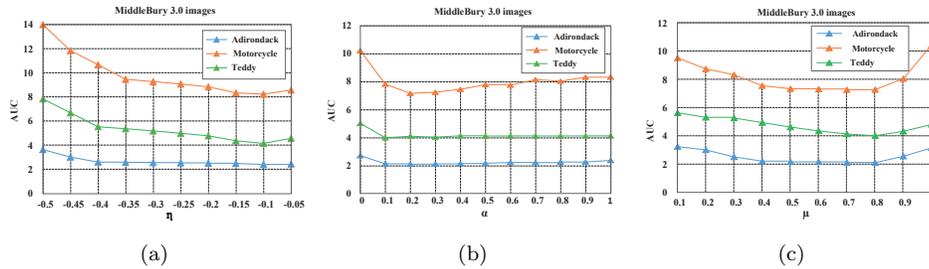


Figure 11: Sensitivity analysis of the parameters in the efficient approach on the Middlebury 3.0 dataset

385 *5.4.2. Parameter analysis of the accurate approach*

In the accurate approach, parameter  $\kappa$  controls the ratio between the occlu-  
 sions and mismatches in the improved LRC in Section 4.2.1. If one mismatch  
 is surrounded by occlusions, and the ratio of the occlusions is greater than the  
 threshold  $\kappa$ , then the mismatch is reclassified as an occlusion. To evaluate the

390 detection performance of the improved LRC when using different  $\kappa$ , two criteria  
 are used: the hit rate and the false-positive rate of occlusions [20]. Fig. 12  
 illustrates the hit rate and the false-positive rate of occlusions at various  $\kappa$ . The  
 hit rate of occlusions decreases when  $\kappa$  increases, as shown in Fig. 12(a). The  
 false-positive rate of occlusions also decreases when  $\kappa$  increases, as illustrated  
 395 in Fig. 12(b). Both the hit rate and the false-positive rate are stable regardless  
 of the variation of  $\kappa$ .

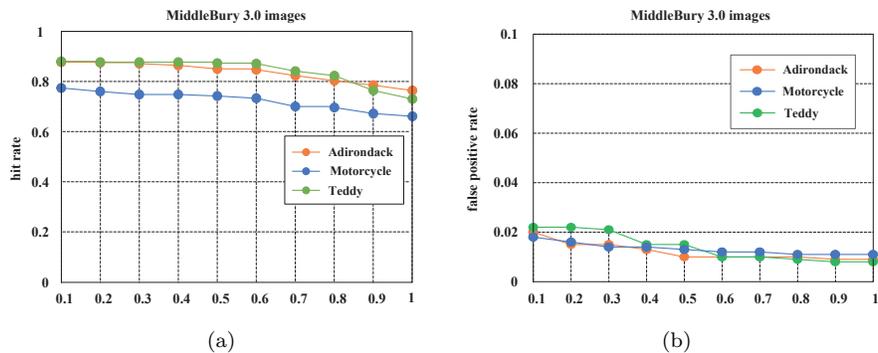


Figure 12: The hit rate and false-positive rate of occlusions [20] with various  $\kappa$

Parameter  $\rho$  is the threshold of whether one texture edge point is regarded  
 as a real discontinuous boundary point in Section 4.2. Fig. 13 shows the error  
 rate of bad 2.0 (%) in the nonocc regions and in all regions of the Middlebury  
 400 dataset. The real discontinuous boundary is an auxiliary means to confine  
 the search region of reliable points, and the final stereo matching result is not  
 sensitive to  $\rho$  in the range from 0.05 to 0.3. Note that when  $\rho$  is too large  
 to detect real discontinuous boundaries, the result will be influenced since the  
 search region is not confined.

## 405 6. CONCLUSION

Outlier detection is an important component of stereo matching. Good outlier  
 detection will increase the robustness and accuracy of the stereo matching  
 algorithm. The disparities in outlier regions can be discarded or filled by some

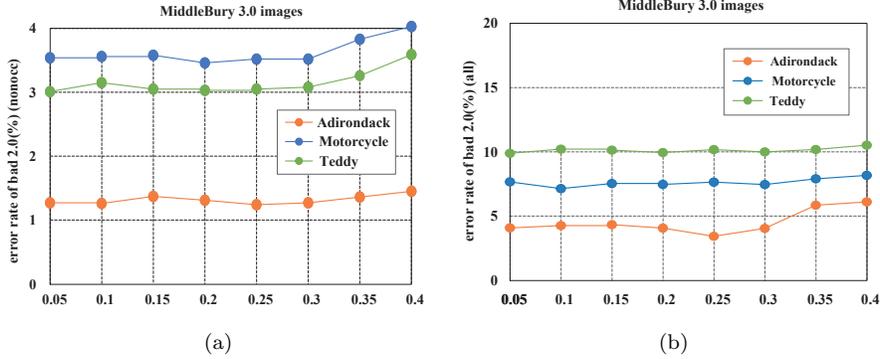


Figure 13: The error rate of bad 2.0 (%) in the nonocc regions are in the all regions with different  $\rho$

reliable disparities depending on different application conditions. The efficient  
 410 outlier detection approach detects outliers at a relatively low cost, i.e., approx-  
 imately half of the time and memory expenses. The accurate outlier detection  
 approach classifies outliers reasonably, providing a better guidance for disparity  
 refinement.

One possible future direction is to investigate how to utilize the outlier de-  
 415 tection results to guide other stereo matching steps, such as matching cost  
 computation and cost aggregation, to obtain a more robust stereo matching  
 result.

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