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Parametric Modeling and Generation of Mandala Thangka Patterns

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ABSTRACT

The mandala thangka, as a religious art in Tibetan Buddhism, is an invaluable cultural and artistic heritage. However, drawing a mandala pattern of thangka style is both time- and effort-consuming and requires mastery due to intricate details. Retaining and digitizing this heritage is an unresolved research challenge to date. In this paper, we propose a parametric approach to model and generate mandala thangka patterns to address this issue. Specifically, we construct parameterized models of three stylistic elements used in the interior mandalas of Nyingma school in Tibetan Buddhism according to their geometric features, namely the star, crescent, and lotus flower motifs. Varieties of interior mandala patterns are successfully generated using these parameterized motifs based on the hierarchical structures observed from hand-drawn mandalas. Moreover, we design a user interaction tool which can flexibly generate stylized mandala patterns with arbitrary shapes and colors. The experimental results show that our approach can efficiently generate beautifully-layered colorful traditional mandala patterns used in Buddhism and stylized mandala patterns used in modern art design, which significantly reduce the time and effort in manual production and, more importantly, contributes to the digitization of this great heritage.

1. Introduction

The thangka, or scroll painting, usually depicting a Buddhist deity, scene, or mandala, is a special art of Tibetan Buddhism. As a religious art, thangka retains high cultural and artistic values. In 2006 the Tibetan Thangka was recorded in the first list of national intangible cultural heritages in China and, in 2009, Regong Arts was inscribed on the Representative List of the Intangible Cultural Heritage of Humanity by the UNESCO. Mandala is an important subject in thangka, which is a “psychocosmogram” like a map of a psychological or spiritual space representing the universe [1,2]. In various spiritual traditions, mandalas may be employed as a spiritual guidance tool for focusing attention of practitioners and for mediation. In addition to its religious significance, mandalas have favorable psychological effects, as noted in [3].

In its most common form, a mandala thangka appears as a series of concentric circles. It depicts deities which are enclosed in the square structure situated concentrically within these circles, as shown in Figure 1. Drawing a mandala thangka by hand is both time- and effort-consuming, since it first needs to be sketched onto the canvas in the right proportions following the ancient grid patterns, after which the long painting procedure starts, with fine brushes. The time required to finish drawing a mandala thangka usually ranges from one to several months, depending on the complexity of the mandala thangka. An efficiently computer-aided generation approach would not only significantly reduce the time and effort in manual production, but also provide a quantitative mea-

Fig. 1. Examples of scanned mandala thangka patterns.

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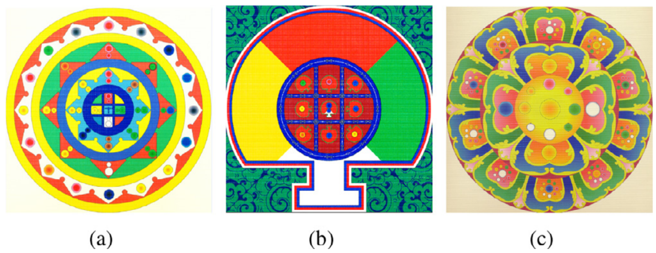


Fig. 2. Examples of scanned and cropped interior mandala patterns: (a) Star pattern; (b) Crescent pattern; (c) Lotus flower pattern.

sure of the composition rules in mandala patterns, which has significance in digitization development of modern art design.

Although mandalas are intricate, they are highly geometric and regular, so it is possible to model mandalas mathematically. The modelled mandalas can be further applied to multiple areas: structural analysis in image analysis and retrieval; modern artistic pattern and card design; mental clarity and spiritual growth in psychology.

In Buddhism there are more than 700 deities, each having his or her own mandala. Modeling a large number of mandalas is beyond the scope of this work. We focus on modeling the central part, called *the interior mandala*, which is the most important part in any mandala thangka. Specifically, in this paper we propose a computer-aided generation approach by parametric modeling three stylistic motifs used in the interior mandalas of Nyingma school in Tibetan Buddhism [4]: star (Figure 2(a)), crescent (Figure 2(b)), and lotus flower motifs (Figure 2(c)). The framework of our proposed approach is shown in Figure 3. The main contributions of our work comprise three aspects:

(1) We construct models of parameterized motifs according to geometric features in star, crescent, and lotus flower types, which can better quantify the geometric structure and explore the design space of these motifs.
(2) We place these parameterized motifs in a hierarchical structure of concentric circles outside-in to generate final interior mandala patterns, where parameter values are determined by a uniformly-spaced reference grid. Comparative results show that our approach can efficiently generate beautifully-layered colorful interior mandalas comparable with hand-drawn mandalas.
(3) We design an interaction tool, through which the user can set parameters and colors of the motifs and arrange them in different rings to flexibly generate stylized mandala patterns with arbitrary shapes and colors. Moreover, we analyze the usage statistics recorded while placing different motifs on the mandala patterns. This part is an extension of our earlier work [5].

Our work provides a reference for computer-aided generation of both traditional mandala patterns used in Buddhism and stylized mandala-like patterns used in modern art design, which contributes to the digitization of this great heritage. The rest of this article is organized as follows: Section 2 summarizes previous studies on computer generation approaches of traditional art patterns. We then introduce in detail the parametric modeling process of star (Section 3), crescent (Section 4), and lotus flower motifs (Section 5). Section 6 presents the generation approach of interior mandala patterns used in Buddhism. The user interaction tool is described in Section 7, which can generate varieties of stylized mandala-like patterns. Finally, we conclude and discuss our future work in Section 8.

2. Related Work

Previous works in computer generation of traditional art patterns can be classified mainly into the following categories: Islamic geometric patterns, Indian Kolam patterns, Celtic knots, Chinese paper-cut patterns, Uygur fabric patterns, Japanese textiles, and painting artworks. Some works had focused on extracting pictorial style elements, and adopting mathematical models or heuristic grammars to parametrically encode shape features, construction rules, or stylized layout, to generate new visual art forms with similar styles by placing graphical entities in terms of aesthetic conventions, spatial constraints, or configuration styles. Some other works had utilized deep neural network techniques to automatic emulation of particular artists' styles for application to re-rendering of paintings and photos.

Islamic geometric patterns are built on stars, squares, and circles, typically repeated, overlapped, and interlaced to form intricate connected patterns. Earlier works [6–8] used symmetry groups to analyze forms of organization and structures of Islamic star patterns. There was some research in constructing Islamic geometric patterns by the theory of using the "strap work" approach, which used two simple tools i.e. a straightedge (ruler) and a pair of compasses. Based on this method, the patterns were created out of shapes such as triangles, squares, circles, and polygons that were formed by straight lines and arcs produced by the compass angle, which were then transformed into stars and overlapping lattices [9,10]. By using polygonal network as the base, the polygons-in-contact technique was adopted to create a variety of Islamic geometric patterns [11–13]. The modular design system based on the star, cross, and traditional Islamic pattern modules was introduced in [14–17] to generate different families of geometric star and rosette patterns. Based on shape rules, [18–22] applied the parametric shape grammar approach to generate Islamic geometric patterns.

Indian Kolam patterns consist of a symmetric matrix of dots and curving lines which wind around the dots on the geometry. A tiling-based approach using diamond-shaped tiles placed corner to corner to construct square loop Kolam was presented in [23]. Gopalan and Vanleeuwen [24] proposed a topological method which could generate all possible Kolams for any spatial configuration of dots. Other approaches included encoding Kolams using graph, picture and array grammars [25–27], converting Kolam patterns into numbers and linear diagrams [28], L- and P-systems [29], extended pasting schemes [30], gestural lexicons [31], stroke chain-code [32], and knot theory [33].

Celtic knots are a variety of knots and stylized graphical representations used extensively in the Celtic style of Insular art for decoration. This knotwork is analogous to closed loops of rope that cross over and under one another, becoming entangled. Kaplan and Cohen [34] presented a technique for automating the construction of Celtic knotwork and decorations. In their work, all possible Celtic knots were created by planar graphs. After defining the layout of a knotwork by graph, the system automatically oriented the threads around any configuration of user defined breakpoints or graph angles to generate the resulting knotwork. The construction of wild knots was introduced in [35] by recursive application of a simple generator set of predefined cord segments, as an embellishment to knotwork designs in the Celtic style. Taylor [36] used trigonometric parameterizations to generate decorative knots based on plane curves interlaced by using one or more sin functions in the 3D dimension.

For Chinese paper-cutting patterns, Uygur fabric patterns, and Japanese textiles, Liu et al. [37] studied the cyclic and dihedral symmetries of different annuli in paper-cutting designs, and synthesized new designs with different rotational orders. Li et al. [38] designed a set of tools for annotating animated 3D surfaces

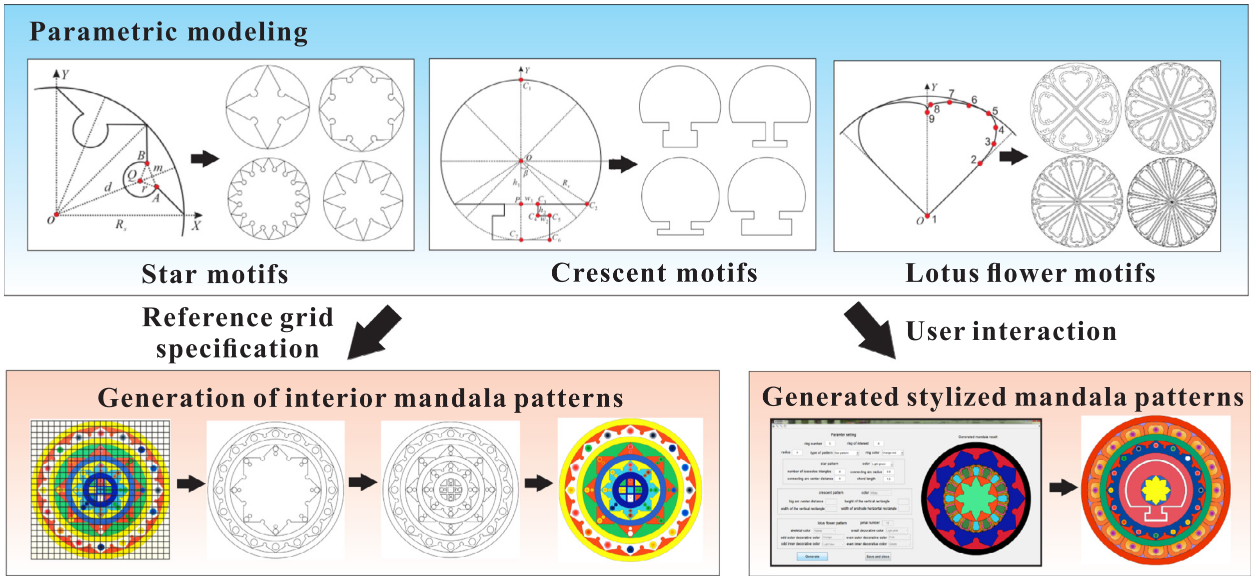


Fig. 3. The framework of our parametric modeling and generation of mandala thangka patterns.

with holes derived from traditional paper-cutting motifs. By using independent patterns as basis, a library of complex paper-cutting patterns was established in [39]. Zhao et al. [40] proposed a method of automatic generation of Uygur fabric patterns based on configuration styles such as hexagonal, brick-shaped and diamond structure tile.

Sano and Yamamoto [41] presented a three-dimensional computer-aided design system which could measure the body form of a customer and simulate Japanese kimono textiles. A fashion system utilizing a set of Japanese textile patterns through the interactive genetic algorithm to produce clothes designs was designed in [42]. In their work, clothes components and fabric patterns were designed and coded separately, then the system combined clothes components and fabric pattern set, using the interactive genetic algorithm and laws based on fashion design.

There has also been research on computer generation of painting artworks. The mathematical structures, smooth shape transitions, and duality property of M.C. Escher's artworks have been extensively studied. Related studies have addressed analysis of mathematical structures in Escher's artworks, and automatically generating Escher-like tessellations via constrained optimization of Kaplan and Salesin [43], 44], or Sky-and-Water transmutations [45]. Lin et al. [46] introduced a system to create a variety of Escher-like transmutations, by using the techniques for initializing a tile pattern with dual figure-ground arrangement, shape matching, and shape warping. Zheng et al. [47] and Lian et al. [48] proposed layered modeling approaches to generate Pollock's drip and Picasso's cubism style paintings, respectively. In their works, Zheng et al. [47] modeled the drawing process of Pollock's drip paintings as background layer, irregular shape layer, line layer, and water drop layer, and Lian et al. [48] divided the generation process of Picasso's cubism style paintings into abstract objects layer, texture layer, cubism layer, concrete objects layer, and shadow layer. Each layer was composed of basic components which were randomly located and colored. Abstract paintings in the style of Malevich, Kandinsky, and Miro were randomly generated by Tao et al. [49], Zhang and Yu [50], Xiong and Zhang [51], which extracted and parametrically encoded pictorial elements and composition rules in each type of paintings. Recently, some applications such as the Becasso [52], Prisma, DeepArt, and Ostagram have been widely used in daily life, which utilized artificial intelligence techniques to transfer ordinary photos into styles of masters. Gatys et al. [53] introduced a convolutional neural network algorithm of artistic style transfer, which generated new art images with similar styles that combined the semantic content of an arbitrary photograph with the appearance of numerous famous artworks. Chen et al. [54] proposed StyleBank, which was composed of multiple convolution filter banks and each filter bank represented one style. To transfer an image to a specific style, the corresponding filter bank was convolved with the intermediate feature embedding produced by a single autoencoder, which enabled new interesting style fusion effects, like linear and region-specific style transfer. A new technique for visual attribute transfer across images that may had very different appearance with perceptually similar semantic structure was proposed by Liao et al. [55]. They validated the effectiveness of their method in style/texture transfer, color/style swap, and sketch/painting to photo.

Patterns in the interior mandalas of Buddhism differ from aforementioned patterns in both geometric shape and composition structure. Poelke et al. [56] created mandala-like patterns by creating polynomials with L-symmetric zero set and applying the classical Schwarz reflection principle, but the resultant patterns differed dramatically from those in the interior mandala patterns both in shape variation regularity and hierarchical structure. It is therefore necessary to construct parameterized motifs according to geometric features in each pattern and place them in a hierarchical structure to generate different interior mandala patterns.

3. Star Motifs

Star motifs are widely used in interior mandalas, which may contain 4, 8, or 16 isosceles triangles inscribed in a circle (red lines in Figure 4(a)), with their angles facing outward. The base of each triangle is stylized with a circular crest, which is called *the connecting arc* (blue curves in Figure 4(a)).

To model the star motif, we first place a circumscribed circle centered at the origin O and set its radius as R_s, and then calculate the coordinates (V_{ix}, V_{iy}) of the isosceles triangle's vertex V_i (red points in Figure 4(a)) with the following formulae:

$$\begin{cases} V_{ix} = R_s \cdot \cos((i-1) * 2\pi / n_s) \\ V_{iy} = R_s \cdot \sin((i-1) * 2\pi / n_s) \end{cases} \tag{1}$$

where $i \in [1, n_s]$, and n_s is the number of isosceles triangles.

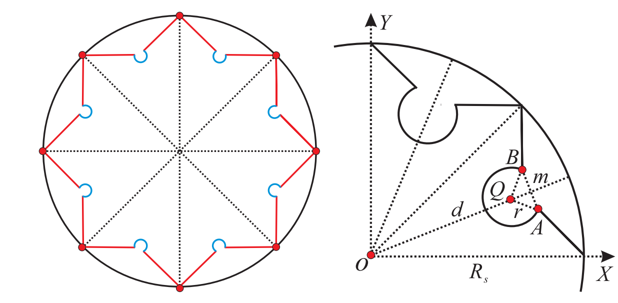


Fig. 4. A star motif: (a) Basic structure; (b) Geometric model.

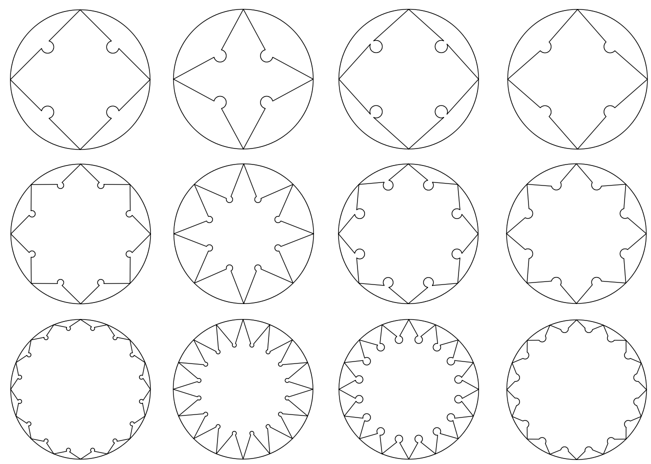


Fig. 5. Variants of star motifs.

Table 1
Coordinates of key points in the crescent motif.

Key points	Coordinates of key points
$\widehat{C_1C_2}(\theta)$	$(R_c\sin(\theta),\ -R_c\cos(\theta)),\ \theta\in[\beta,\ \pi]$
C_3	$(w_1,\ -R_c\cos(\beta))$
C_4	$(w_1,\ -R_c\cos(\beta)-h_2)$
C_5	$(w_1+w_2,\ -R_c\cdot\cos(\beta)-h_2)$
C_6	$(w_1+w_2,\ -R_c)$
C_7	$(0,\ -R_c)$

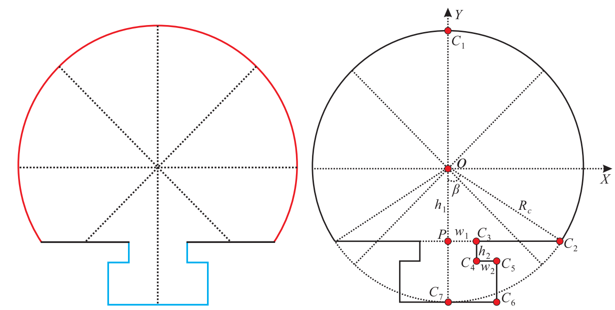


Fig. 6. A crescent motif: (a) Basic structure; (b) Key points in the geometric model.

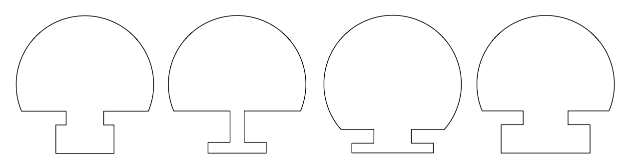


Fig. 7. Variants of crescent motifs.

Assume that the distance between O and the connecting arc's center Q is $d=|OQ|$, the coordinates $(Q_x,\ Q_y)$ of Q are calculated as:

$$\begin{cases} Q_x = d\cdot\cos(\pi/n_s) \\ Q_y = d\cdot\sin(\pi/n_s) \end{cases} \tag{2}$$

Next we determine the size of connecting arcs and their end points coinciding with end points of the base in two adjacent isosceles triangles. To clearly illustrate, we enlarge the connecting arcs in Figure 4(b). Setting the arc radius as r and chord length $|AB|=m$, the deviation angle from $\overline{OQ}$ to $\overline{QA}$ is calculated as $\alpha = \arcsin(m/2r)$.

The connecting arc $\widehat{AB}$ can be defined by the following equations:

$$\begin{cases} \widehat{AB}_x(\theta) = Q_x + r\cdot\cos(\theta+\pi/n_s) \\ \widehat{AB}_y(\theta) = Q_y + r\cdot\sin(\theta+\pi/n_s) \end{cases} \tag{3}$$

where $\theta\in[\alpha, 2\pi-\alpha]$.

Once the arc $\widehat{AB}$ is obtained, we simply rotate $\widehat{AB}$ around O with $2\pi/n_s$ to produce n_s-1 additional arcs so that all adjacent isosceles triangles can be connected by those arcs to form the final star motif. Also, by tuning parameters R_s, n_s, d, r, and m, variants of star motifs can be obtained, as shown in Figure 5.

4. Crescent Motifs

Crescent motifs are usually used in mandalas for dakinis. A crescent motif consists of a relatively large arc (red curve in Figure 6(a)) and an inverted T-shaped structure at the bottom (blue line in Figure 6(a)).

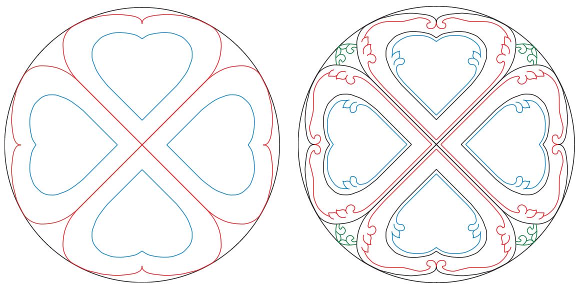
We label several key points on the large arc and the T-shaped structure with $C_1 \sim C_7$, as shown in Figure 6(b). Assume that the arc center is at the origin O and the arc radius is R_c, the distance from O to the chord is $|OP|=h_1$, as shown in Figure 6(b), the deviation angle from $\overline{OP}$ to $\overline{OC_2}$ is calculated as $\beta=\arccos(h_1/R_c)$.

We set the following two parameters $w_1=|PC_3|$ and $h_2=|C_3C_4|$, with which the width and height of the vertical rectangle in the T-shaped structure can be determined. The width of the protruded horizontal rectangle in the T-shaped structure is determined by $w_2=|C_4C_5|$. All key points can be determined by the formulae given in Table 1.

We connect $\widehat{C_1C_2}$, C_3, C_4, C_5, C_6, and C_7 to generate the right part of the crescent motif, and then flip it over horizontally to obtain the left part of the motif. Moreover, variants of crescent motifs can be generated by tuning parameters R_c, h_1, h_2, w_1, and w_2, as shown in Figure 7.

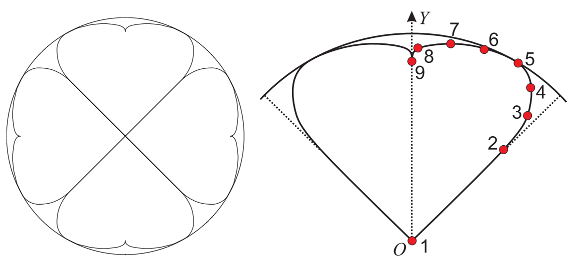
5. Lotus Flower Motifs

Lotus flower motifs are used in mandalas for deities in the lotus flower family. They may contain 4, 8, 10, or 16 petal motifs from the inner core to the outer rings (Figure 2(c)). Here we take the 4-petal motif as an example to describe how to model the lotus flower motifs. The model is divided into skeletal and shape levels: the skeletal motif globally defines the structure of the entire petal motif and locally defines petal contours, as shown by Figure 8(a), where each petal motif has an outer line depicting petal's contour (red curves) and an internal line depicting petal's texture (blue curves). Decorative shapes are then added inside the outer and internal lines in the skeletal motifs to form the final petal motif, as indicated by red and blue curves in Figure 8(b), respectively. Small decorative shapes are finally added between every two adjacent petal motifs (green curves in Figure 8(b)). Next, we describe the construction of the skeletal motifs and decorative shapes in detail.



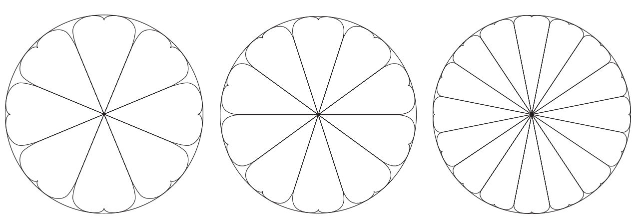
(a) (b)

Fig. 8. Model of a lotus flower motif: (a) Skeletal motifs; (b) Decorative shapes.



(a) Skeletal motifs (b) Control points

Fig. 9. Construction of 4-petal skeletal motifs.



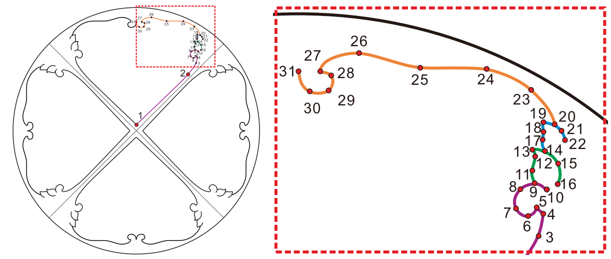
(a) 8 petals (b) 10 petals (c) 16 petals

Fig. 10. Skeletal motifs with different number of petals.

5.1. Skeletal Motifs

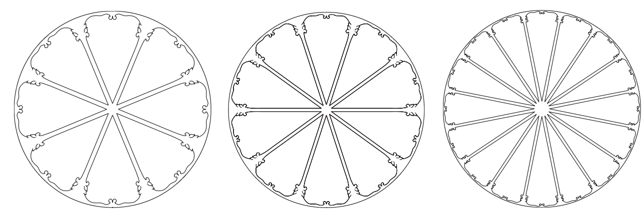
In the 4-petal motif, a petal is restricted to a quarter circle in a unit circle, as shown in Figure 9(a). Since the petal shape is symmetric about its central axis, we can model the right half of the petal by first specifying 9 control points as the original set of control points (red points in Figure 9(b)) and then interpolating those points with a B-spline. The positions of 9 control points are expressed with polar coordinates. We use a three-dimensional array to store the radius r_i, the angle θ_i and the relative angle $\Delta_i = |\theta_{ref} - \theta_i|$, where $i = 1, 2, .., 9$, and θ_{ref} is the angular coordinate of a reference control point taken from the given control points. In this example we take the 9th point as the reference point.

The left half of each petal can be obtained by adding azimuth of the petal's central axis to Δ_i to obtain the azimuth of control points flipped over, and keeping the radius coordinates of control points flipped over unchanged, as shown in Figure 9(b). For motifs with $n_p > 4$ petals, we modify relative angles Δ_i in the original control point set by $\Delta_i' = \Delta_i * 4/n_p$, a new set of control points with polar coordinates $(r_i, \theta_{ref} - \Delta_i')$ define petal's shape which is narrower than that defined with the original set of control points. We can use this new set of control points to generate the motifs with n_p petals in a similar manner. Figure 10 shows skeletal motifs with 8, 10, and 16 petals.



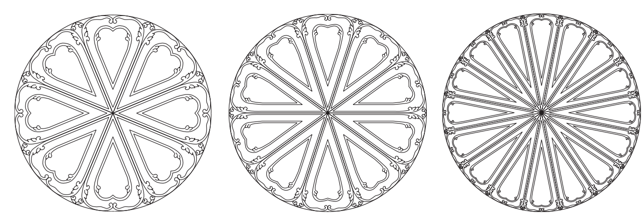
(a) Decorative shapes (b) Control points

Fig. 11. Construction of 4-petal decorative shapes.



(a) 8 petals (b) 10 petals (c) 16 petals

Fig. 12. Decorative shapes with different number of petals.



(a) 8 petals (b) 10 petals (c) 16 petals

Fig. 13. Lotus flower motifs with different number of petals.

5.2. Decorative Shapes

Skeletal motifs described in the previous section define the structure of lotus flower motifs. Decorative shapes need to be added inside the skeletal motifs to obtain the final motifs. As shown in Figure 11(a), the right half of the decorative shape is divided into 4 segments — the purple, green, blue, and orange curves in the right half of the quarter circle — as indicated by the red rectangle in Figure 11(a) and its enlarged portion in Figure 11(b). We specify 31 control points all together for those 4 segments, and then proceed to interpolate those points associated with each segment with the B-spline to generate corresponding decorative shapes. Figure 12 shows decorative shapes with 8, 10, and 16 petals.

5.3. Complete Lotus Flower Motifs

With all models constructed for different motifs in the lotus flower motifs, we can generate a complete lotus flower pattern with the following procedure. First, we specify n_p as the petals number in the pattern, and then divide the unit circle into n_p sectors. In the first sector, we proceed to generate the skeletal motifs for both outer and inner motifs in the petal and add decorative shapes inside skeletal motifs to obtain a complete petal motif. The petal motifs in the remaining sectors of the unit circle can be generated by rotating the petal motif in the first sector around the center with $2\pi/n_p$. By changing n_p, lotus flower motifs with different petals can be generated, as shown in Figure 13. Since the lotus flower motif is restricted to a unit circle, we can scale up the radial

Table 2
Statistics associated with 5 interior mandala patterns generated with our system. Here R_r is the radius of a ring in the interior mandala pattern; w is the grid size; R_s, n_s, d, r, and m are the radius of a circumscribed circle, number of isosceles triangles, distance from the connecting arc center to the circle center, connecting arc radius, and chord length in the star motif, respectively; R_c, h_1, w_1, h_2, and w_2 are the large arc radius, distance from chord to big arc center, width, height of the vertical rectangle, and width of the protruded horizontal rectangle in the T-shaped structure in the crescent motif, respectively; R_f and n_p are the radius of a circumscribed circle and number of petals in the lotus flower motif, respectively.

Image	Image size (pixel)	Generation time (s)	Ring number	Parameter values
Figure 14(d)	2212 × 2212	22.02	1 – 5	$R_r(12w, 11w, 7w, 3w, 2w)$
			2 – 3	$R_s(11w, 7w)$ $n_s(8, 4)$ $d(8.26w, 4.57w)$
				$r(0.50w, 0.55w)$ $m(0.77w, 0.73w)$
Figure 15(d)	1516 × 1516	44.46	1 – 8	$R_r(12w, 11w, 9w, 8w, 6w, 5w, 3w, 2w)$
			2,	$R_s(11w, 8w, 5w)$ $n_s(16, 8, 8)$
			4,	$d(9.77w, 6.04w, 3.77w)$ $r(0.32w, 0.37w, 0.23w)$
			6	$m(0.46w, 0.60w, 0.37w)$
Figure 16(d)	974 × 974	28.73	5 – 6	$R_r(6w, 5.25w)$
			1 –	$R_c(12w, 11.75w, 11.5w, 11.25w, 1.05w, 0.93w)$
			4, 6	$h_1(6.67w, 6.48w, 6.19w, 6w, 0.58w, 0.46w)$
				$w_1(2.63w, 2.34w, 2.05w, 1.76w, 0.22w, 0.1w)$
				$h_2(1.68w, 2.29w, 2.9w, 3.51w, 0.14w, 0.22w)$
				$w_2(1.75w, 1.75w, 1.75w, 1.75w, 0.14w, 0.14w)$
Figure 17(d)	1146 × 1146	46.75	1 – 3	$R_r(12w, 8w, 4w)$
			1 – 2	$R_f(12w, 8w)$ $n_p(10, 4)$
Figure 18(d)	575 × 575	22.41	1 – 5	$R_r(12w, 9.6w, 7.2w, 4.8w, 2.4w)$
Figure 18(e)	1150 × 1150	45.16		
Figure 18(f)	2300 × 2300	94.38	1 – 4	$R_f(12w, 9.6w, 7.2w, 4.8w)$ $n_p(16, 8, 8, 4)$
Figure 18(g)	4600 × 4600	192.42		

coordinates of all control points in the entire motif by R_f to obtain the lotus flower motif within the circumscribed circle of radius R_f.

6. Generation of Interior Mandala Patterns

With models available for the star, crescent, and lotus flower motifs, it is possible for us to generate corresponding interior mandala patterns composed of related motifs. Globally the interior mandala pattern has a hierarchical structure of concentric rings. On each ring, corresponding motifs are added.

To generate an interior mandala pattern within a specified area, such as a square of side length W, we need to first determine the distances between neighboring rings in the square. Since the radius of each ring varies from mandala to mandala, we first overlay a uniformly-spaced grid of size $w = W/24$ as the reference grid over a mandala, as thangka artists do when they draw mandalas manually, and then generate each ring according to its radius denoted by R_r in the reference grid from the outer rings to the inner core. We choose the outside-in order to generate mandalas since it prevents the overlay of the outer patterns on the inner patterns associated with each ring during the rendering phase.

Once all rings in an interior mandala pattern are generated, we add proper motifs with parameter values indicated by the reference grid on the corresponding rings and color them according to the hand-drawn interior mandalas. Finally, small circles depicting different deities are added inside motifs, with flat or gradient colors specified interactively. Our system is executed on a platform of Core i5-4590 3.30 GHz CPU and PC with 8 GB memory, and implemented in Matlab R2013b, which is a multi-paradigm numerical computing environment and proprietary programming language developed by MathWorks. It allows data visualization, plotting of functions and data, implementation of algorithms, creation of user interfaces, etc. Moreover, it supports developing applications with graphical user interface (GUI) features and includes GUIDE (GUI development environment), which provides a platform with which users can design interfaces and interact with electronic devices through graphical icons and visual indicators efficiently. We choose 5 interior mandala patterns from [4] as the hand-drawn images and simulate them with our models. The image size, generation time, ring number, and parameter values associated with 5 interior mandala patterns generated with our system are shown in Table 2. The table shows that our approach can efficiently generate colorful interior mandala patterns compared with hand-drawn mandala images.

Figures 14 and 15 present the generation process of the central parts of Black Yamaraja and Zhitro Narag Dongprug mandalas with star motifs, respectively. The pattern in Black Yamaraja mandala consists of 5 rings, with an 8 pointed star on the 2nd ring and 4 pointed star on the 3rd ring. The pattern in Zhitro Narag Dongprug mandala consists of 8 rings, with a 16 pointed star on the 2nd ring, an 8 pointed star on the 4th and 6th rings. The area surrounded by the 8th ring is divided into 9 parts.

Figure 16 shows the generation process of the central part of Yangdag Nine Crescent mandala with crescent motifs. There are 4 concentric large crescent motifs and 9 small crescent motifs with different orientations in the 9 parts divided in the central circle.

Figures 17 and 18 present the generation process of the central parts of Dumdo's Auxiliary Yangsem and Dumdo's Auxiliary Nyanthos mandala pattern, respectively. The pattern in Dumdo's Auxiliary Yangsem mandala consists of 3 rings, with lotus flower motif of 10 petals on the 1st ring and 4 petals on the 2nd ring. The pattern in Dumdo's Auxiliary Nyanthos mandala consists of 5 rings, with lotus flower motif of 16 petals on the 1st ring, 8 petals on the 2nd and 3rd rings, and 4 petals on the 4th ring. Besides, we have generated a few more examples with the same drawing patterns and different resolutions for comparison, as shown in Figure 18(d)-Figure 18(g), and corresponding generation times have been added at the last row in Table 2.

7. Generation of Stylized Mandala-Like Patterns

In the previous section, we obtained the traditional interior mandala patterns used in Buddhism with rigid composition, proportions, and colors. Since in terms of design space, mandala patterns used in Buddhism are subsets of mandala-like patterns used in modern art design. With parametric motifs available for the star, crescent, and lotus flower types, it is possible for us to generate stylized mandala-like patterns composed of these motifs with arbitrary shapes and colors.

We design an user interaction tool built on geometric features of star, crescent, and lotus flower motifs used in mandalas, which can be used for exploring the design space of these motifs, as

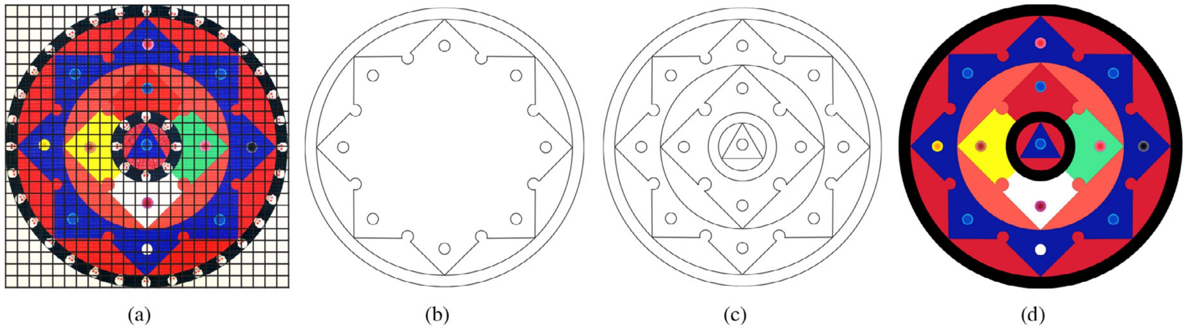


Fig. 14. Generation process of central part of Black Yamaraja mandala: (a) Hand-drawn mandala with a grid overlayed; (b) 1^{st} and 2^{nd} rings; (c) The result after adding 3^{rd} ~ 5^{th} rings; (d) Colored interior mandala pattern.

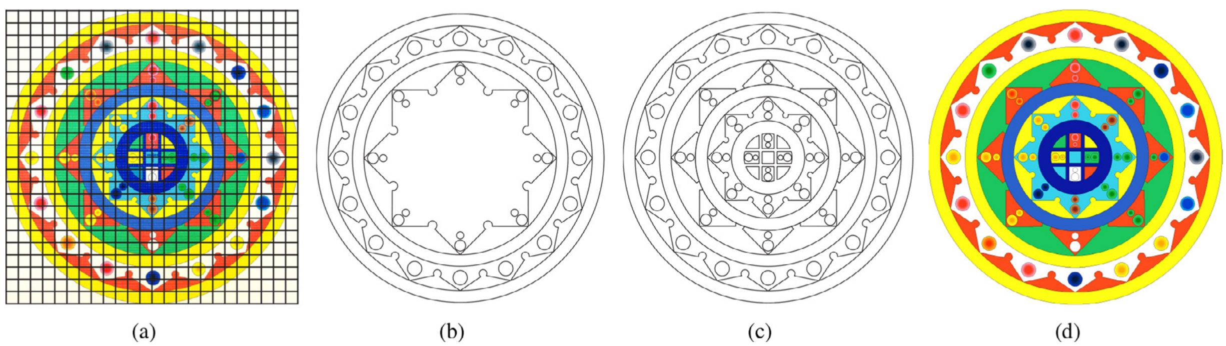


Fig. 15. Generation process of central part of Zhitro Narag Dongprug mandala: (a) Hand-drawn mandala with a grid overlayed; (b) 1^{st} ~ 4^{th} rings; (c) The result after adding 5^{th} ~ 8^{th} rings; (d) Colored interior mandala pattern.

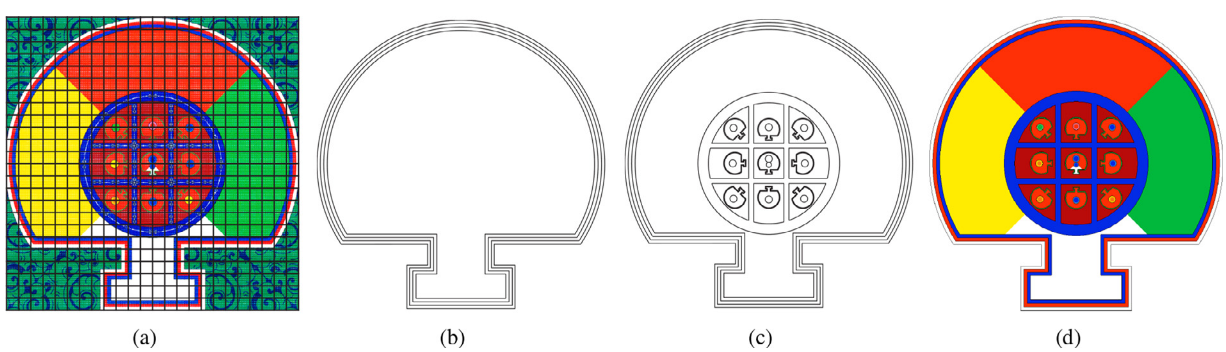


Fig. 16. Generation process of central part of Yangdag Nine Crescent mandala: (a) Hand-drawn mandala with a grid overlayed; (b) 4 concentric large crescent motifs; (c) The result after adding 9 small crescent motifs with different orientations; (d) Colored interior mandala pattern.

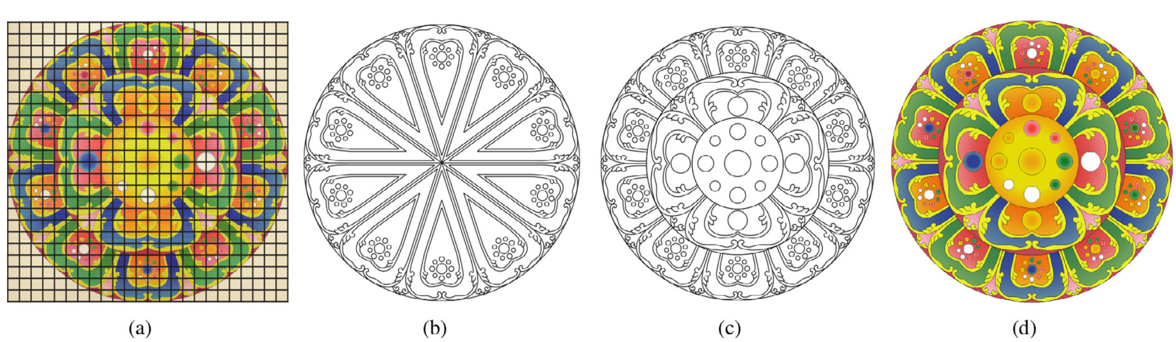


Fig. 17. Generation process of central part of Dumdo's Auxiliary Yangsem mandala: (a) Hand-drawn mandala with a grid overlayed; (b) 1^{st} ring; (c) The result after adding 2^{nd} and 3^{rd} rings; (d) Colored interior mandala pattern.

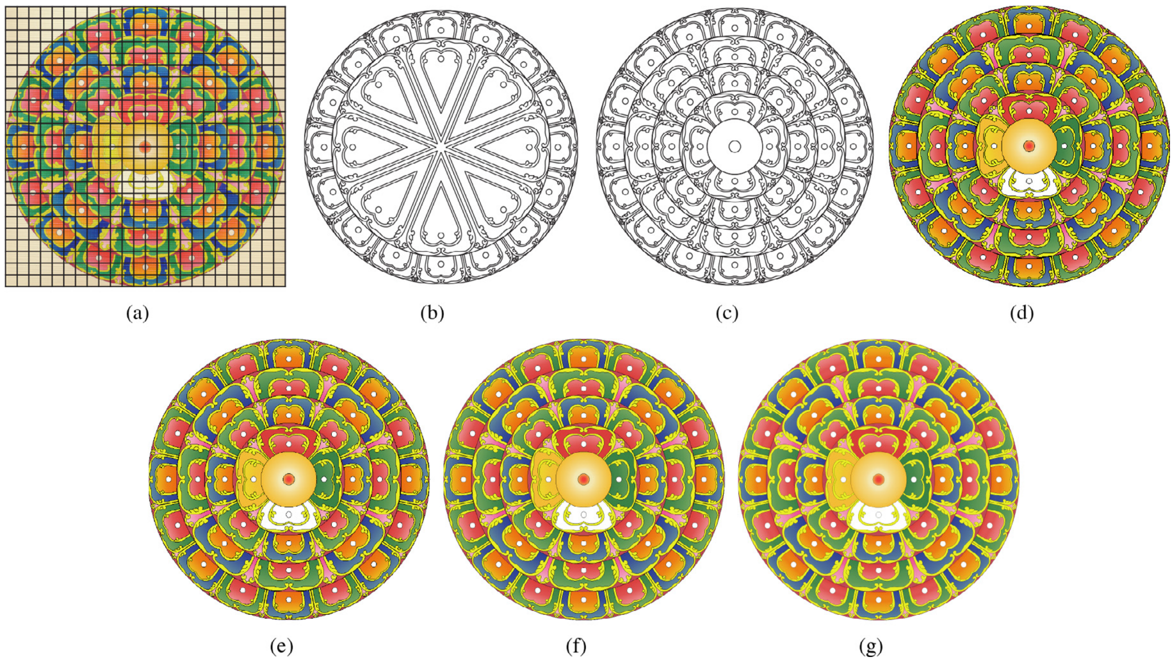


Fig. 18. Generation process of central part of Dumdo's Auxiliary Nyanthos mandala: (a) Hand-drawn mandala with a grid overlayed; (b) 1st and 2nd rings; (c) The result after adding 3rd ~ 5th rings; Colored interior mandala patterns with (d) 575x575; (e) 1150x1150; (f) 2300x2300; (g) 4600x4600 resolutions.

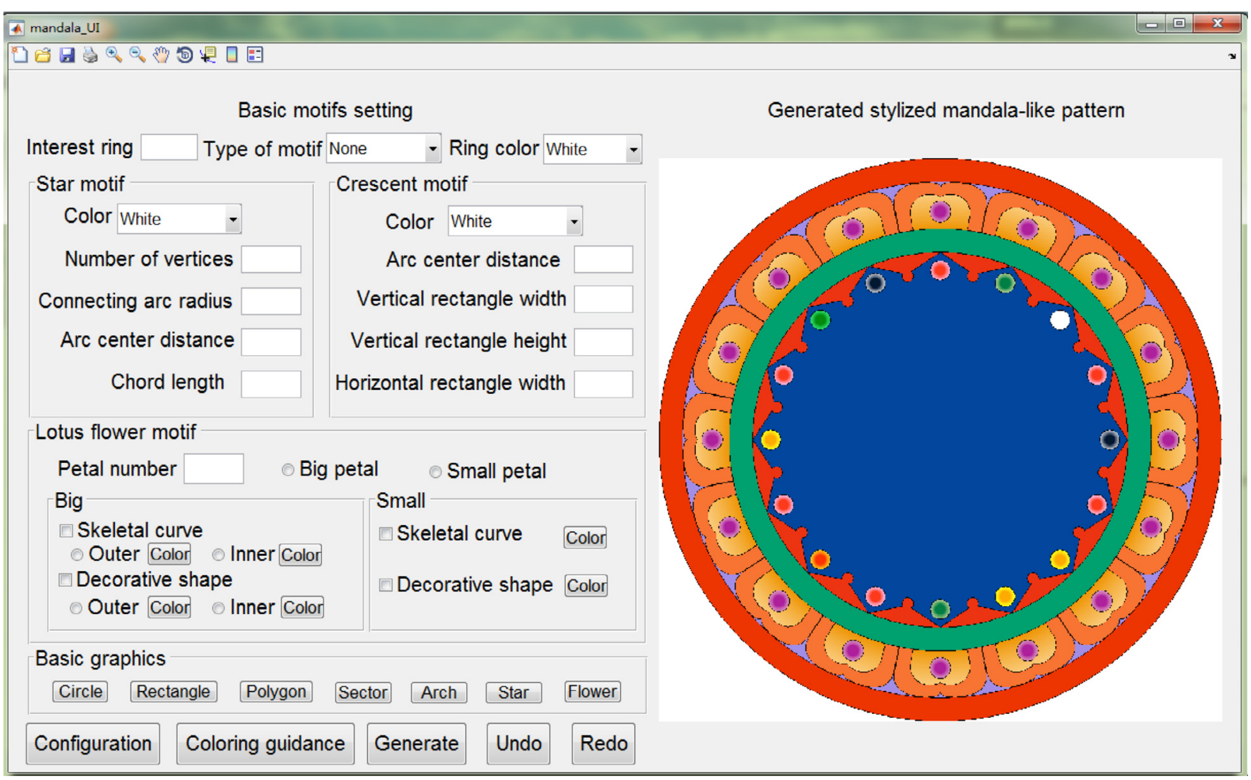


Fig. 19. User interface.

shown in Figure 19. Within our interface, the left part is the pattern editor, in which the user can set basic structure of the mandala pattern, different parameters and colors associated with corresponding motifs, and the latest generated pattern in each step is shown in the right window.

The user firstly clicks on the "Configuration" button, which specifies the total number of rings and associated radius of each ring in the mandala pattern. Then for each drawing step, the user can specify the ring of interest, type of motifs (none, star, crescent, lotus flower) filled in the ring, and choose the ring color from the drop-down box. The relevant parameters and colors associated with corresponding motifs in each module can be edited by the user. For instance, for the lotus flower motif, the user can edit the petal number, whether to add skeletal curves or decorative shapes (outer or inner) in big or small petals, and corresponding filling colors (flat or gradient). Finally, the user can interactively add basic

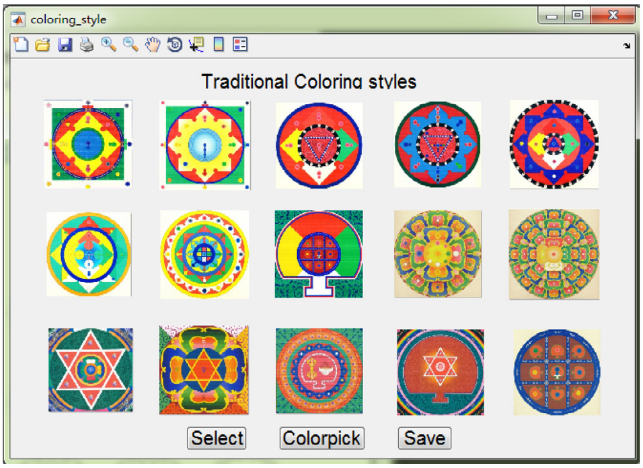


Fig. 20. Coloring styles in traditional mandala thangka patterns.

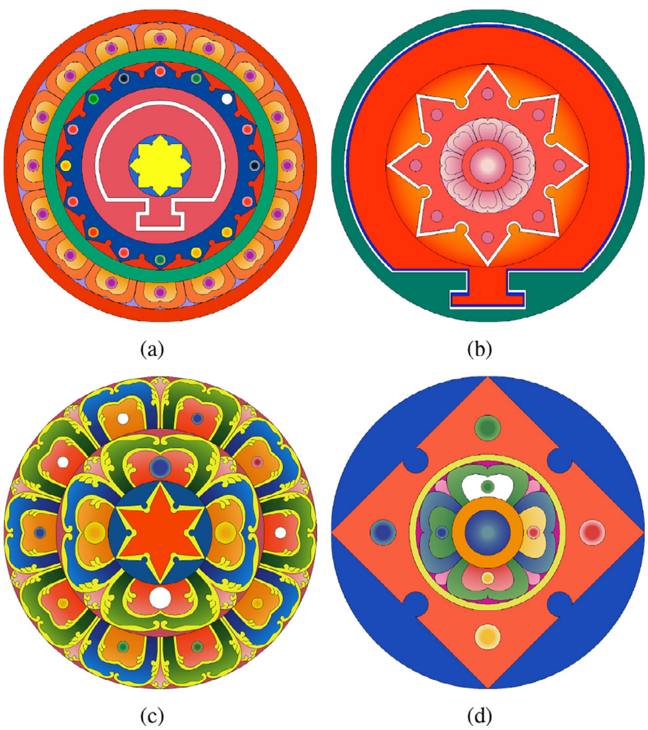


Fig. 21. Generated stylized mandala patterns.

graphics (circle, rectangle, polygon, sector, arch, star, flower) to the desired position inside the motifs by clicking on the corresponding buttons, with arbitrary sizes, shapes and colors. After setting up all the items in each step, the user clicks on the "Generate" button and the right window displays the current generated result. If the user has second thoughts about the current drawing step, then he/she can click on the "Undo" button, and the right window redisplays the previously generated result and the subject can redraw. The user can also click on the "Redo" button for returning to the latest generated status. In order to better assist users to color mandala patterns with religious styles, in our drawing tool, the user can click on the Coloring guidance button, then the system will display some recommended coloring styles in traditional mandala thangka patterns, the user can choose one or more styles and guide the coloring process by clicking on the "Colorpick" button to pick the colors in different regions, as shown in Figure 20.

Table 3
Usage statistics for three operations

Pattern	t_{st} (s)	t_{cr} (s)	t_{lf} (s)	t_{gc} (s)
Figure 21(a)	5 - 10	5 - 10	10 - 20	20 - 30
Figure 21(b)	5 - 10	10 - 15	10 - 20	5 - 10
Figure 21(c)	5 - 10	-	20 - 30	20 - 30
Figure 21(d)	3 - 6	-	15 - 25	10 - 20

Figure 21 shows four stylized mandala patterns generated by our user interaction tool.

Figures 21 (a) and 21(b) show two mandala patterns which combine star, crescent and lotus flower motifs together. The pattern in Figures 21(a) consists of 8 rings, with lotus flower motif of 16 petals with skeletal curves on the 2nd ring, a 16 pointed star motif on the 4th ring, an 8 pointed star motif on the 8th ring, crescent motifs on the 6th and 7th rings. The pattern in Figures 21(b) consists of 8 rings, with crescent motifs on the 2nd, 3rd, and 4th rings, an 8 pointed star motif on the 5th ring and lotus flower motif of 8 petals with skeletal curves on the 6th ring.

Figures 21 (c) and 21(d) show another two generated results which combine star and lotus flower motifs together. The pattern in Figures 21(c) consists of 4 rings, with lotus flower motif of 10 petals on the 1st ring, 4 petals on the 2nd ring, 6 pointed star motifs on the 3rd and 4th rings. The pattern in Figures 21(d) consists of 5 rings, with a 4 pointed star motif on the 1st ring and lotus flower motif of 4 petals on the 3rd ring.

Table 3 presents some usage statistics recorded while placing different motifs on the stylized mandala patterns in Figure 21. The reported timings in the table include:

t_{st} = iterative time spent editing the star motif;
t_{cr} = iterative time spent on editing the crescent motif;
t_{lf} = iterative time spent on editing the lotus flower motif;
t_{gc}= iterative time spent on adding the basic graphics;

In Table 3, ' - ' means that there is no relevant operation needed for the corresponding pattern. We did not list exact times, but instead listed time ranges spent on corresponding operations. We did this because some editing operations may vary even for different motifs. For example, placing lotus flower motifs on the mandala pattern usually takes longer time than other motifs because it involves more interactions such as choosing whether to add big or small petals, setting the filling colors of outer and inner skeletal curves, and decorative shapes of each petal, respectively.

8. Conclusion and Future Work

This paper has introduced a parametric modeling and generation approach of mandala thangka patterns. The experimental results show that our method can efficiently generate beautifully-layered colorful interior mandala patterns comparably with hand-drawn mandalas. In addition to mandalas used in Buddhism, stylized mandala patterns that are in any number of shapes, sizes, and colors can be modelled by our user interaction tool, which is an interesting digital contents creation tool that encourages exploration of mandala design space. Our modeled mandala patterns can be used for a wide range of applications, including coloring book for children and adults, textile pattern, card, and package design, etc. Thus, our work provides a reference for computer-aided generation of both traditional mandala patterns used in Buddhism and mandala-like patterns used in modern art design.

Currently our parametric modeling and generation approaches are only suitable for mandala patterns with star, crescent, and lotus flower motifs. In the future, we plan to explore the computer generation methods for more patterns with different motifs and styles, decorative patterns in surrounding regions other than the interior area, and also 3D printing techniques of the mandala thangka designs using colored sand-like particles. Since the motivation of our project is proposing an approach of parametric modeling of hand-drawn traditional mandala thangka patterns, the generation durations and interaction cannot be considered real-time. In the future, as computer hardware accelerates and our drawing and rendering algorithms further improves, maybe our drawing tool can be used to generate mandala thangka styles in real-time. Besides, by organizing these elements in mandalas together, we can research on explorers to progressively generate, browse, and render "infinite" mandalas, with arbitrary nesting and detail, and have the development of such tools on mobile devices/tablet for better portability, demonstration, and education purpose. Moreover, investigating aesthetic rules of coloring mandala patterns and develop automatic coloring models in our drawing tool can be another interesting research topic.

Declaration of Competing Interest

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature or kind in any product, service or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled, "Parametric Modeling and Generation of Mandala Thangka Patterns".

CRediT authorship contribution statement

Jiajing Zhang: Conceptualization, Methodology, Software, Writing - original draft, Visualization. **Kang Zhang:** Data curation, Investigation, Visualization. **Ren Peng:** Validation, Resources. **Jinhui Yu:** Conceptualization, Writing - review & editing, Supervision, Funding acquisition.

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