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Some slides adapted from Noah Snavely, Lingqi Yan

Panorama (全景图)







360度VR





GoPro odyssey



图像拼接





How to combine two images?



How to combine two images?



What is image warping? How to compute it?







Image Warping

• **image filtering**: change *intensity* of image





• image warping: change shape of image





Parametric (global) warping



p = (x,y)





p' = (x',y')

• Transformation T is a coordinate transformation:

p' = T(p)

• Examples:



translation



rotation



aspect

Scale



Scale Transform



$$\begin{aligned} x' &= sx\\ y' &= sy \end{aligned}$$

Scale Matrix



Scale (Non-Uniform)



$$\begin{bmatrix} x'\\y' \end{bmatrix} = \begin{bmatrix} s_x & 0\\0 & s_y \end{bmatrix} \begin{bmatrix} x\\y \end{bmatrix}$$

Reflection Matrix



Horizontal reflection:

$$\begin{aligned} x' &= -x \\ y' &= y \end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear Matrix



Hints:

Horizontal shift is 0 at y=0 Horizontal shift is a at y=1 Vertical shift is always 0 $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Rotate (about the origin (0, 0), CCW by default)



Rotation Matrix



$$\mathbf{R}_{\theta} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Linear Transforms = Matrices

(of the same dimension)

$$x' = a x + b y$$
$$y' = c x + d y$$

$$\begin{bmatrix} x'\\y' \end{bmatrix} = \begin{bmatrix} a & b\\c & d \end{bmatrix} \begin{bmatrix} x\\y \end{bmatrix}$$
$$\mathbf{x}' = \mathbf{M} \mathbf{x}$$

Translation



Translation??



$$x' = x + t_x$$
$$y' = y + t_y$$

Why Homogeneous Coordinates

• Translation cannot be represented in matrix form

$$\begin{bmatrix} x'\\y' \end{bmatrix} = \begin{bmatrix} a & b\\c & d \end{bmatrix} \begin{bmatrix} x\\y \end{bmatrix} + \begin{bmatrix} t_x\\t_y \end{bmatrix}$$

(So, translation is NOT linear transform!)

- But we don't want translation to be a special case
- Is there a unified way to represent all transformations? (and what's the cost?)

Solution: Homogenous Coordinates

Add a third coordinate (w-coordinate)

- 2D point = $(x, y, 1)^{T}$
- 2D vector = (x, y, 0)^T

Matrix representation of translations

$$\begin{pmatrix} x'\\y'\\w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x\\0 & 1 & t_y\\0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x\\y\\1 \end{pmatrix} = \begin{pmatrix} x+t_x\\y+t_y\\1 \end{pmatrix}$$

What if you translate a vector?

Homogenous Coordinates

Valid operation if w-coordinate of result is 1 or 0

- vector + vector = vector
- point point = vector
- point + vector = point
- point + point = ??

In homogeneous coordinates,

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} \text{ is the 2D point } \begin{pmatrix} x/w \\ y/w \\ 1 \end{pmatrix}, \ w \neq 0$$

2D Transformations

$$\mathbf{S}(s_x, s_y) = \begin{pmatrix} s_x & 0 & 0\\ 0 & s_y & 0\\ 0 & 0 & 1 \end{pmatrix}$$

Rotation

Scale

$$\mathbf{R}(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0\\ \sin \alpha & \cos \alpha & 0\\ 0 & 0 & 1 \end{pmatrix}$$

Translation

$$\mathbf{T}(t_x, t_y) = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

Affine Transformations

Affine map = linear map + translation

$$\begin{pmatrix} x'\\y' \end{pmatrix} = \begin{pmatrix} a & b\\c & d \end{pmatrix} \cdot \begin{pmatrix} x\\y \end{pmatrix} + \begin{pmatrix} t_x\\t_y \end{pmatrix}$$

Using homogenous coordinates:

$$\begin{pmatrix} x'\\y'\\1 \end{pmatrix} = \begin{pmatrix} a & b & t_x\\c & d & t_y\\0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x\\y\\1 \end{pmatrix}$$

any transformation represented by a 3x3 matrix with last row [001] we call an *affine* transformation

Projective Transformation (Homography)





Change projection plane (pp)



Can generate any synthetic camera view as long as it has **the same center of projection**!

Fun with homography

Original image



St.Petersburg photo by A. Tikhonov

Virtual camera rotations





Projective Transformation (Homography)

$$\begin{bmatrix} x'_{i} \\ y'_{i} \\ 1 \end{bmatrix} \approx \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_{i} \\ y_{i} \\ 1 \end{bmatrix}$$

Maybe nonzero
$$x'_{i} = \frac{h_{00}x_{i} + h_{01}y_{i} + h_{02}}{h_{20}x_{i} + h_{21}y_{i} + h_{22}}$$
$$y'_{i} = \frac{h_{10}x_{i} + h_{11}y_{i} + h_{12}}{h_{20}x_{i} + h_{21}y_{i} + h_{22}}$$

 We usually constrain the length of the vector [h₀₀ h₀₁ ... h₂₂] to be 1, which means the degree of freedom is 8

Summary of 2D transformations



Translation



Affine



Projective



2 unknowns

6 unknowns

8 unknowns

Inverse Transform

$$T^{-1}$$

 T^{-1} is the inverse of transform T in both a matrix and geometric sense



Implementing image warping

Given a coordinate transform (x',y') = T(x,y) and a source image f(x,y), how do we compute an transformed image g(x',y') = f(T(x,y))?





Forward Warping

Send each pixel f(x) to its corresponding location (x',y') = T(x,y) in g(x',y')



Forward Warping

• What if pixel lands "between" pixels?



Inverse Warping

 Get each pixel g(x',y') from its corresponding location (x,y) = T⁻¹(x,y) in f(x,y)



Inverse Warping

• What if pixel lands "between" pixels?



Answer: interpolate color values from neighboring pixels
Interpolation

Nearest neighbor

- Copies the color of the pixel with the closest integer coordinate



Interpolation

Weighted sum of four neighboring pixels



Interpolation

- Possible interpolation filters:
 - nearest neighbor
 - bilinear
 - bicubic
 - sinc



Questions?

Part II Image Stitching

How to compute transformation?

$$\begin{bmatrix} x'\\y'\\1 \end{bmatrix} \cong T \begin{bmatrix} x\\y\\1 \end{bmatrix}$$



- 1. Image matching (each match gives an equation)
- 2. Solve T from the obtained matches

Affine transformations

$$\begin{bmatrix} x'\\y'\\1 \end{bmatrix} = \begin{bmatrix} a & b & c\\d & e & f\\0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x\\y\\1 \end{bmatrix} = \begin{bmatrix} ax+by+c\\dx+ey+f\\1 \end{bmatrix}$$

- How many unknowns?
- How many equations per match?
- How many matches do we need?

Affine transformations

• For each match, we have

$$x' = ax + by + c$$
$$y' = dx + ey + f$$

• Matrix form $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$

Affine transformations

• For n matches



How to solve t?

• Least squares: find **t** that minimizes

$$||\mathbf{At} - \mathbf{b}||^2$$

• To solve, form the *normal equations*

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{t} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$
$$\mathbf{t} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Projective transformations



 $\begin{array}{c} \mathbf{P'} & \textbf{Homography} & \mathbf{P} \\ \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} \sim H_{10} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$

Solving for homographies

$$\begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$x'_{i} = \frac{h_{00}x_{i} + h_{01}y_{i} + h_{02}}{h_{20}x_{i} + h_{21}y_{i} + h_{22}}$$
$$y'_{i} = \frac{h_{10}x_{i} + h_{11}y_{i} + h_{12}}{h_{20}x_{i} + h_{21}y_{i} + h_{22}}$$

 $\begin{aligned} x_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{00}x_i + h_{01}y_i + h_{02} \\ y_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{10}x_i + h_{11}y_i + h_{12} \end{aligned}$

Solving for homographies

 $\begin{aligned} x_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{00}x_i + h_{01}y_i + h_{02} \\ y_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{10}x_i + h_{11}y_i + h_{12} \end{aligned}$





Defines a least squares problem: minimize $\|Ah - 0\|^2$

- Since $\, h \,$ is only defined up to scale, solve for unit vector $\, \, \hat{h} \,$
- Solution: $\hat{\mathbf{h}}$ = eigenvector of $\mathbf{A}^T \mathbf{A}$ with smallest eigenvalue
- Works with 4 or more points



Robustness

• Let's consider a simpler example... linear regression



Problem: Fit a line to these datapoints

• How can we fix this?

RANSAC



RANSAC









Select one match at random, count inliers



Select another match at random, count inliers



Output the translation with the highest number of inliers

RANSAC

- Idea:
 - All the inliers will agree with each other on the translation vector;
 - The outliers will disagree with each other
 - RANSAC only has guarantees if there are < 50% outliers
 - "All good matches are alike; every bad match is bad in its own way."

– Tolstoy via Alyosha Efros

RANSAC

- General version:
 - 1. Randomly choose *s* samples
 - Typically s = minimum sample size that lets you fit a model
 - 2. Fit a model (e.g., transformation matrix) to those samples
 - 3. Count the number of inliers that approximately fit the model
 - 4. Repeat *N* times
 - 5. Choose the model that has the largest set of inliers

Final step: least squares fit







输入图像





特征匹配





RANSAC计算变换矩阵





固定第一幅图,变换第二幅图



- Graphcut
- Poisson Image Editing



重叠的图像

简单的接缝







最大流最小割算法

多项式时间

Panoramas

• Now we know how to create panoramas!

1) Warp all images to a reference image; 2) merge them



Rotation about vertical axis



- What if our camera rotates on a tripod?
- What's the structure of H?

 $H = KRK^{-1}$

Do we have to project onto a plane?


Full Panoramas

• What if you want a 360° field of view?



Cylindrical projection



Cylindrical projection



Cylindrical panoramas



- Steps
 - Reproject each image onto a cylinder
 - Blend
 - Output the resulting mosaic

Cylindrical image stitching



- What if you don't know the camera rotation?
 - Solve for the camera rotations
 - Note that a rotation of the camera is a **translation** of the cylinder!

Assembling the panorama



• Stitch pairs together, blend, then crop

Problem: Drift



- small (vertical) errors accumulate over time
- apply correction so that sum = 0 (for 360° pan.)

Full-view (360°) panoramas







Questions?

基于单视图的三维重建周晓巍

3D Navigation (Free Viewpoint)



3D Navigation (Free Viewpoint)

- Need 3D models
- Difficult to obtain high-quality models



3D Navigation (Free Viewpoint)

Can we do it from a single photograph?





St. Jerome in his Study, H. Steenwick





Flagellation, Piero della Francesca



video by Antonio Criminisi

Some preliminaries: projective geometry



3D to 2D: perspective projection

Projection:



Vanishing point and line tell us a lot about camera position and orientation

Figure 23.4

A perspective view of a set of parallel lines in the plane. All of the lines converge to a single vanishing point.

Ames Room





Vanishing points



- Vanishing point
 - projection of a point at infinity

Vanishing points (2D)



Vanishing points



- Properties
 - Any two parallel lines have the same vanishing point v
 - The ray from **C** through **v** is parallel to the lines
 - v tells us the direction of the lines
 - An image may have more than one vanishing point

Vanishing lines



- Multiple Vanishing Points
 - Any set of parallel lines on the plane define a vanishing point The union of all of these vanishing points is the *horizon line*
 - also called vanishing line
 - Note that different planes define different vanishing lines

Vanishing lines



- Multiple Vanishing Points
 - Any set of parallel lines on the plane define a vanishing point The union of all of these vanishing points is the *horizon line*
 - also called vanishing line
 - Note that different planes define different vanishing lines



- Properties
 - \mathbf{P}_{∞} is a point at *infinity*, **v** is its projection $\mathbf{V} = \mathbf{\Pi} \mathbf{P}_{\infty}$
 - They depend only on line *direction (angle between the line and optical axis)*
 - Parallel lines $P_0 + tD$, $P_1 + tD$ intersect at P_{∞}

Computing vanishing lines



- Compute I from two sets of parallel lines on ground plane
- Properties
 - I is intersection of horizontal plane through C with image plane
 - I depends on the orientation of the camera
 - All points at same height as **C** project to **I**
 - points higher than C project above I, vice versa





"Tour into the Picture" (SIGGRAPH '97)

•Create a 3D "theatre stage" of five billboards

•Use camera transformations to navigate through the scene





The idea

- Many scenes (especially paintings), can be represented as an axisaligned box volume (i.e. a stage)
- Key assumptions:
 - All walls of volume are orthogonal
 - Camera view plane is parallel to back of volume
 - Camera up is normal to volume bottom
- How many vanishing points does the box have?
 - Three, but two at infinity
 - Single-point perspective
- Can use the vanishing point
- to fit the box to the particular
- Scene!



Fitting the box volume





- User controls the inner box and the vanishing point placement (# of DOF???)
- Q: What's the significance of the vanishing point location?
- A: It's at eye level: ray from COP to VP is perpendicular to image plane.

Comparison of how image is subdivided based on two different camera positions. You should see how moving the vanishing point corresponds to moving the eyepoint in the 3D world.


Comparison of two camera placements – left and right. Corresponding subdivisions match view you would see if you looked down a hallway.



2D to 3D conversion

• First, we can get ratios



2D to 3D conversion

- Size of user-defined back plane must equal size of camera plane (orthogonal sides)
- Use top versus side ratio to determine relative height and width dimensions of box
- Left/right and top/bot ratios determine part of 3D camera placement



Depth of the box



- Can compute by similar triangles (CVA vs. CV'A')
- Need to know focal length f (or FOV)
- Note: can compute position on any object on the ground
 - Simple unprojection
 - What about things off the ground?

DEMO

- Now, we know the 3D geometry of the box
- We can texture-map the box walls with texture from the image





DEMO

- Now, we know the 3D geometry of the box
- We can texture-map the box walls with texture from the image





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Foreground Objects

- Add vertical rectangles for each foreground object
- Can compute 3D coordinates P0, P1 since they are on known plane.
- P2, P3 can be computed as before (similar triangles)





(a) Specifying of a foreground object

(b) Estimating the vertices of the foreground object model



(c) Three foreground object models

Foreground DEMO (and video)









Single View Modeling using Learning

Depth estimation using CNNs

input



Figure 3: Qualitative comparison of Make3D, our method trained with l_2 loss ($\lambda = 0$), and our method trained with both l_2 and scale-invariant loss ($\lambda = 0.5$).

Depth Map Prediction from a Single Image using a Multi-Scale Deep. David Eigen, Christian Puhrsch, Rob Fergus. NIPS 2014. Cited by 1482

3D-R2N2

通过Encoder-3DLSTM-Decoder 的网络结构建立2D images -to -3D voxel model 的映射。



(a) Images of objects we wish to reconstruct (b) Overview of the network

3D-R2N2: A Unified Approach for Single and Multi-view 3D Object Reconstruction. Christopher B. Choy, Danfei Xu, JunYoung Gwak, Kevin Chen, Silvio Savarese. ECCV 2016. Cited by 584.

Point Set Generation Network

主要思想:

利用深度网络通过单张图 像直接生成点云,解决了 基于单个图片对象生成**3D** 几何的问题。



Figure 6. Visual comparison to 3D-R2N2. Our method better preserves thin structures of the objects.

A Point Set Generation Network for 3D Object Reconstruction from a Single Image, Haoqiang Fan, Hao Su, Leonidas Guibas. CVPR 2017. Cited by 476.)

Pixel2Mesh

主要思想:用一个椭球作为任意物体的初始形状,然后逐渐将这个形状变成目标物体。不借助点云、深度或者其他更加信息丰富的数据,而是直接从单张彩色图片直接得到 3D mesh



Fig. 2. The cascaded mesh deformation network. Our full model contains three mesh deformation blocks in a row. Each block increases mesh resolution and estimates vertex locations, which are then used to extract perceptual image features from the 2D CNN for the next block.

Pixel2Mesh: Generating 3D Mesh Models from Single RGB Images, Nanyang Wang, Yinda Zhang, Zhuwen Li et.al. ECCV 2018, cited by 184.

Mesh R-CNN



于Mask R-CNN的增 强网络,输入一个图 像,检测图像中的所 有对象,并输出所有 对象的类别标签,边 界框、分割掩码以及 三维三角形网格。

Mesh R-CNN是基

3D Meshes

3D Voxels

Figure 1. Mesh R-CNN takes an input image, predicts object instances in that image and infers their 3D shape. To capture diversity in geometries and topologies, it first predicts coarse voxels which are refined for accurate mesh predictions.

Mesh R-CNN. Georgia Gkioxari, Jitendra Malik, Justin Johnson. 2020

Learning-based SFM

- Learning without ground-truth depth information
- Modeling the learning target with video sequences



Single Image Pop-Up from Discriminatively Learned Parts

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Learning to Estimate 3D Human Pose and Shape from a Single Color Image

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Questions?